

# **ESO Cosmic Duologue: The MOND perspective**

**Federico Lelli**  
**Cardiff University**

# Dynamics of Galaxies

(Oort 1932, Rubin+1978, Bosma 1978)



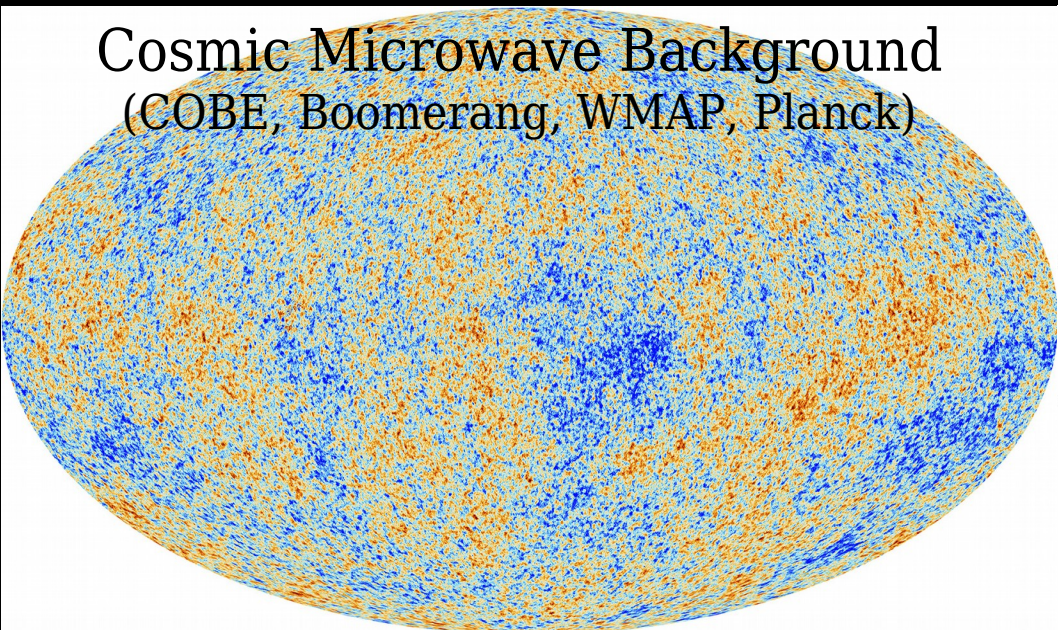
# Dynamics of Galaxy Clusters

(Zwicky 1933, Smith 1936)

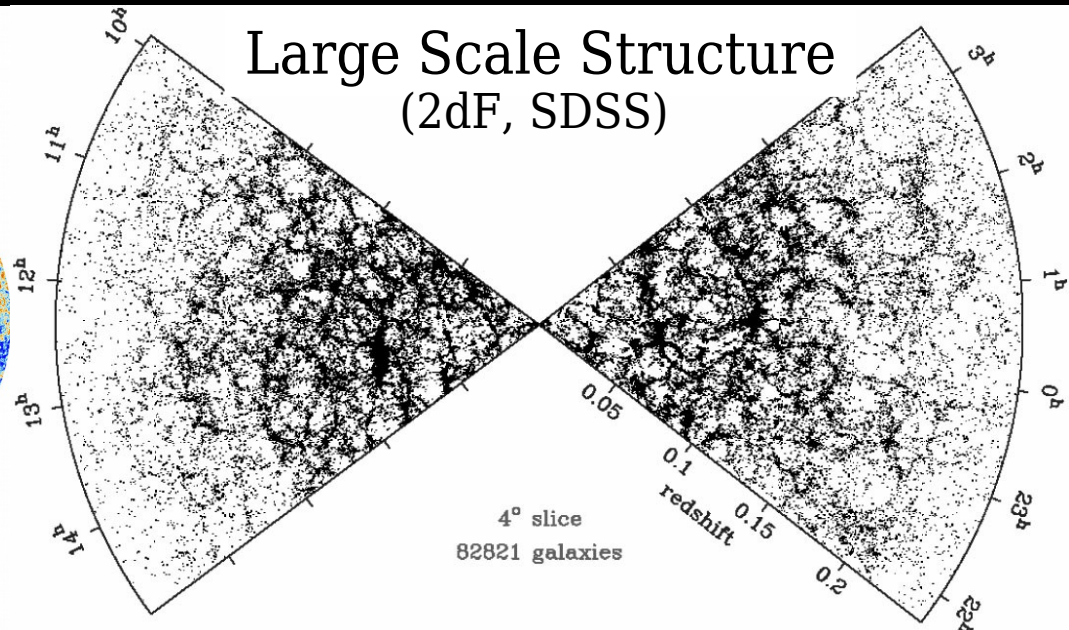


## What do we *really* mean with Dark Matter?

Cosmic Microwave Background  
(COBE, Boomerang, WMAP, Planck)



Large Scale Structure  
(2dF, SDSS)



# Dynamics of Galaxies

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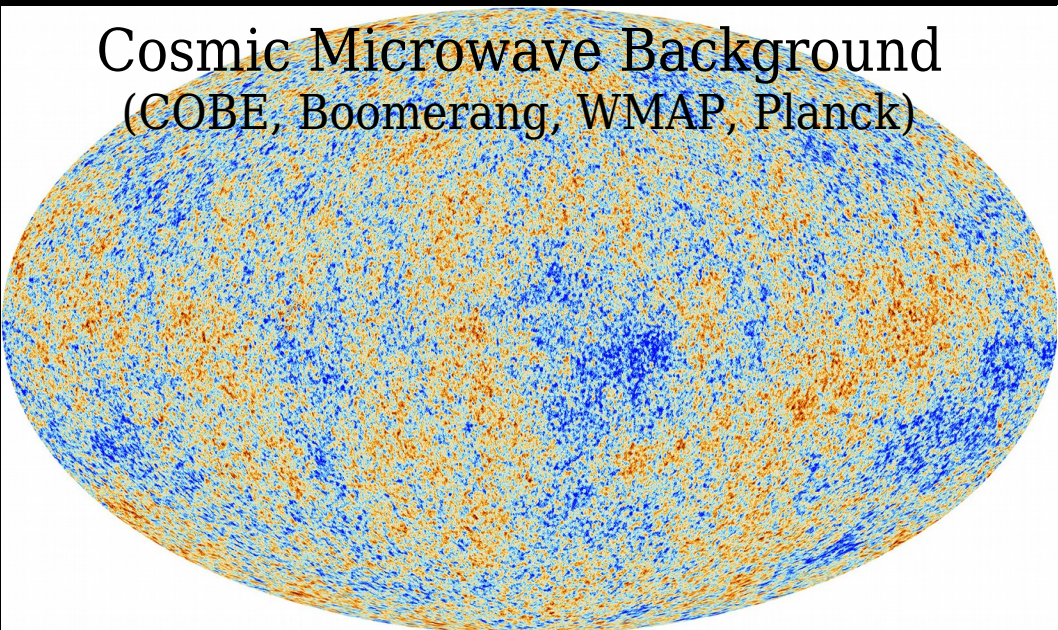
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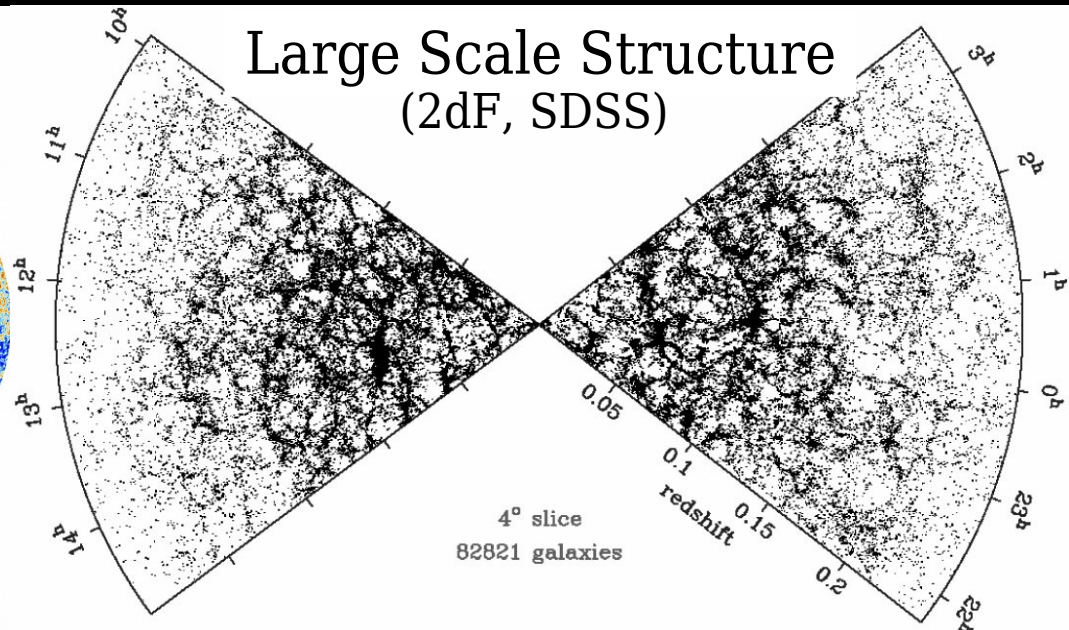


## Standard Gravity + SMOPP = Don't Work

Cosmic Microwave Background  
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# MOND = Modified Newtonian Dynamics or Milgromian Dynamics



Proposed by **Mordehai Milgrom** (1983a, 1983b, 1983c).

MOND is a **general paradigm** that includes several different non-relativistic theories as well as relativistic extensions.

# MOND postulates (at the non-relativistic level)

1) **New constant of Physics:**  $a_0$  ( $\sim 10^{-10}$  m/s<sup>2</sup>)

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Circular orbit at large radii Flat rotation curve at large radii

# A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES<sup>1</sup>



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Received 1982 February 4; accepted 1982 December 28

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⑤ The value of the acceleration constant,  $a_0$ , determined in a few independent ways is approximately  $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$ , which is of the order of  $CH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$ .

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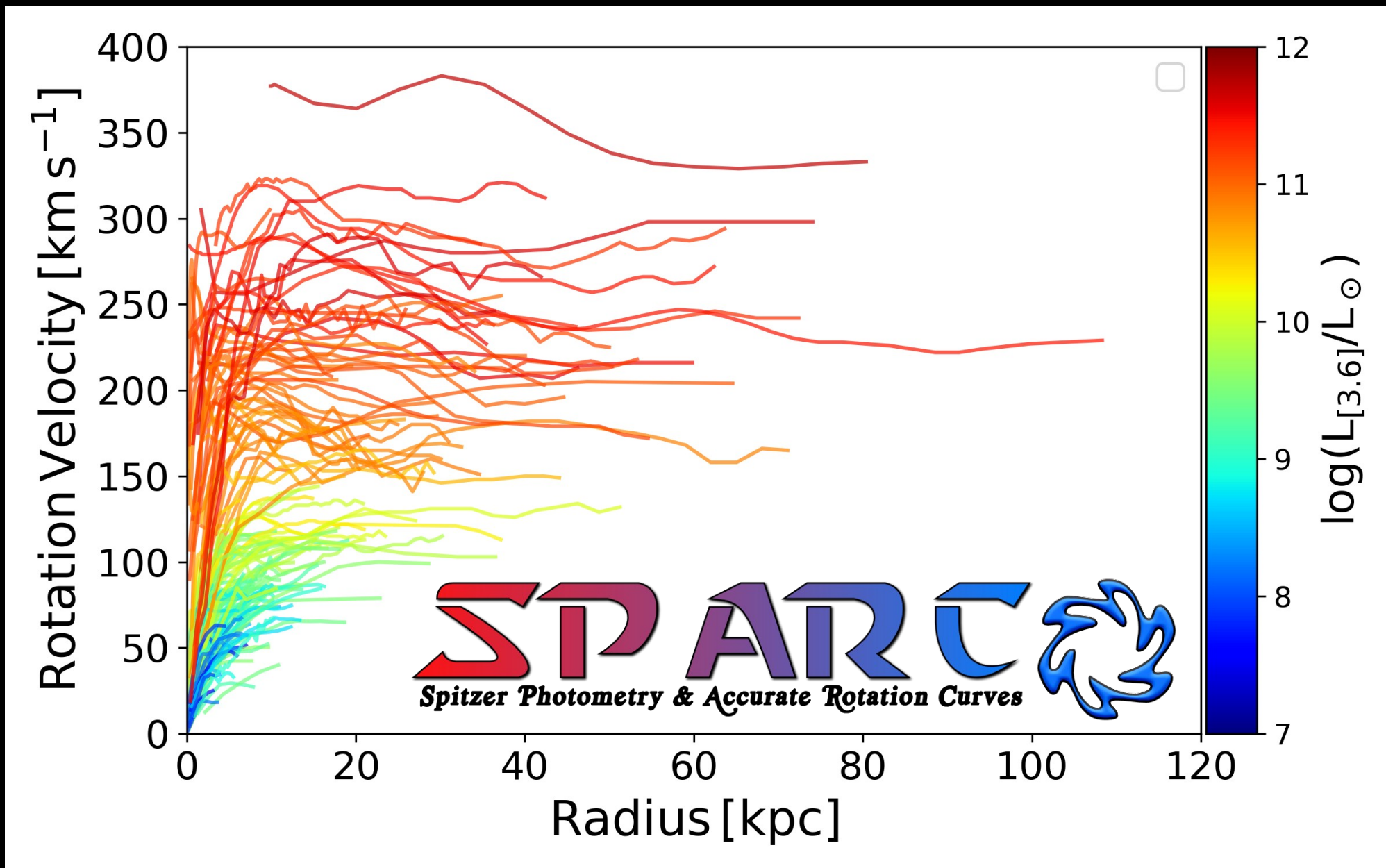
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# Rotation curves stay flat out to large radii



Lelli, McGaugh & Schombert (2016). All data: <http://astroweb.cwru.edu/SPARC/>

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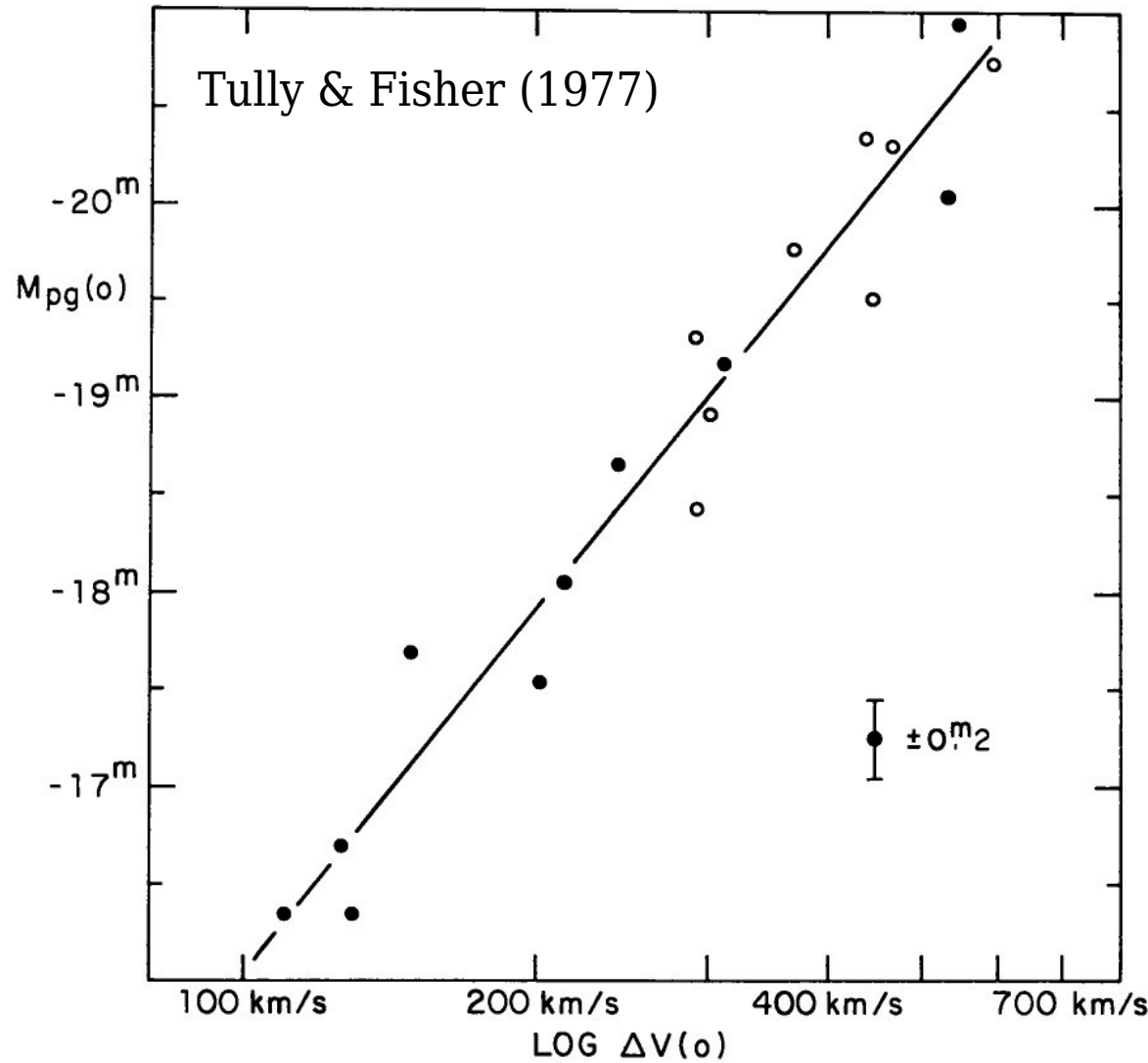
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# The classic Tully-Fisher relation in 1983

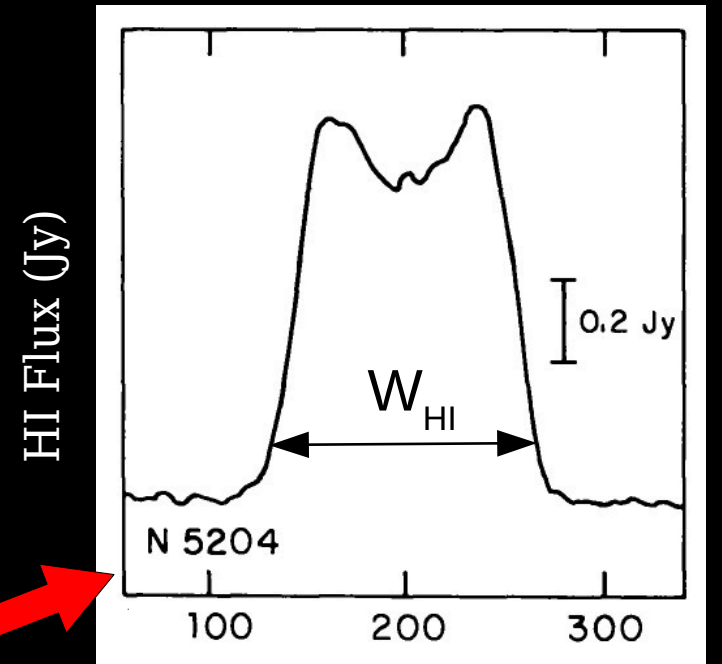
Galaxy Luminosity ( $\sim$ Stellar Mass)



HI Linewidth ( $\sim 2$  Rotation Velocity)

TF slope constrained between  $\sim 2$  and  $\sim 5$   
(Aaronson+1979; Bottinelli+1980; Rubin+1980; Visvanathan 1981; de Vaucouleurs 1982)

Global HI line profile



"The  $V_\infty^4 = a_0 GM_b$  relation should hold exactly"

(Milgrom 1983)

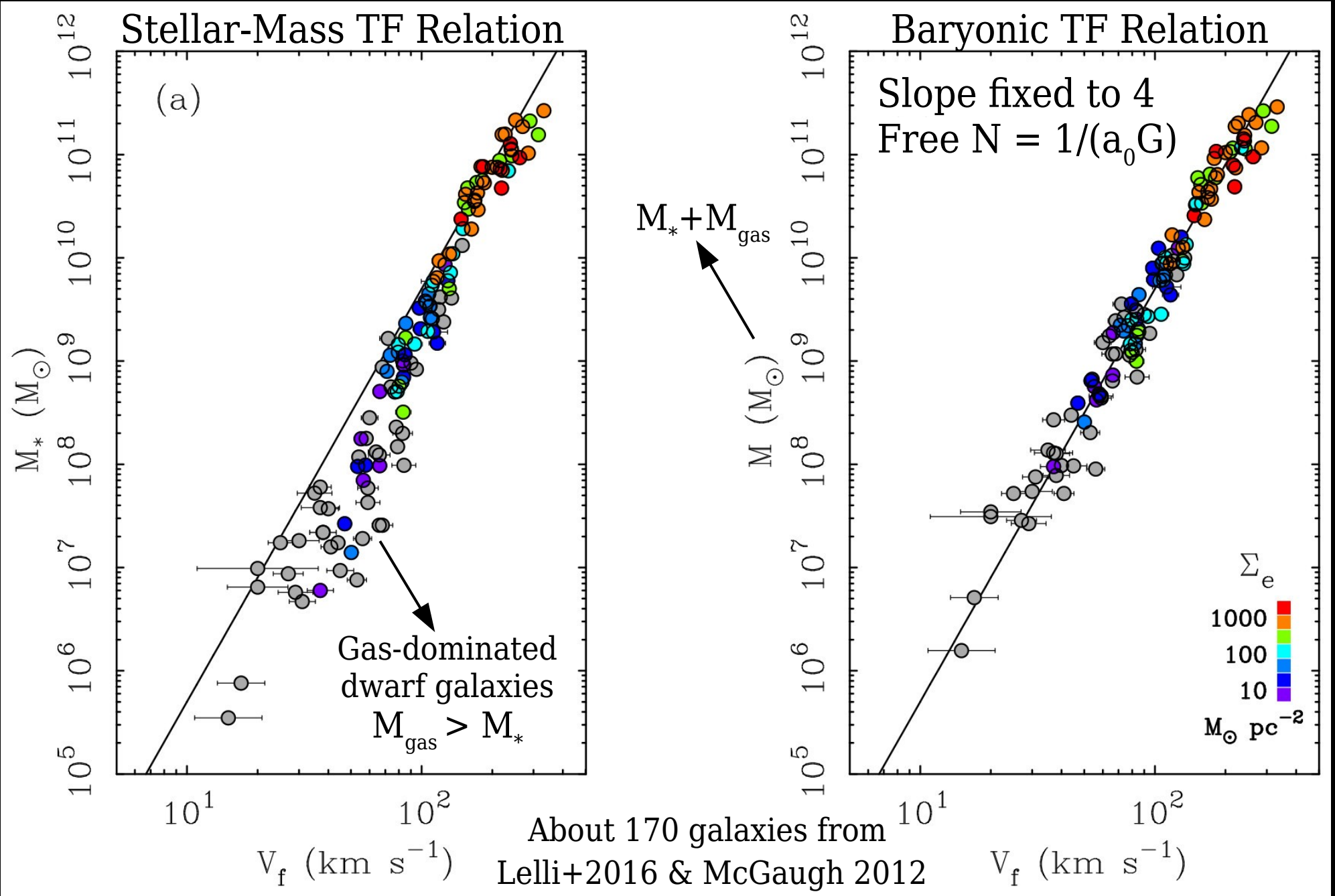
This contains **four independent predictions**:

- 1) Luminosity in TFR  $\rightarrow$  baryonic mass (stars+gas)
- 2) HI linewidth in TFR  $\rightarrow V_\infty \simeq V_f$  (mean  $V$  along the flat part)
- 3) Slope must be equal to 4
- 4) No dependence on galaxy size or surface density  $\Sigma_b$

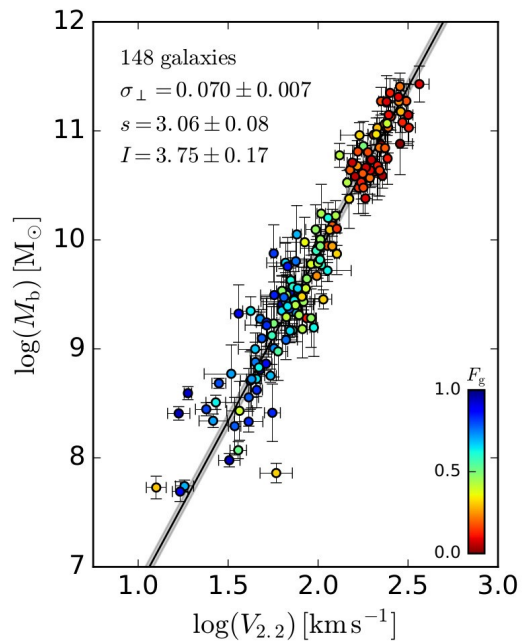
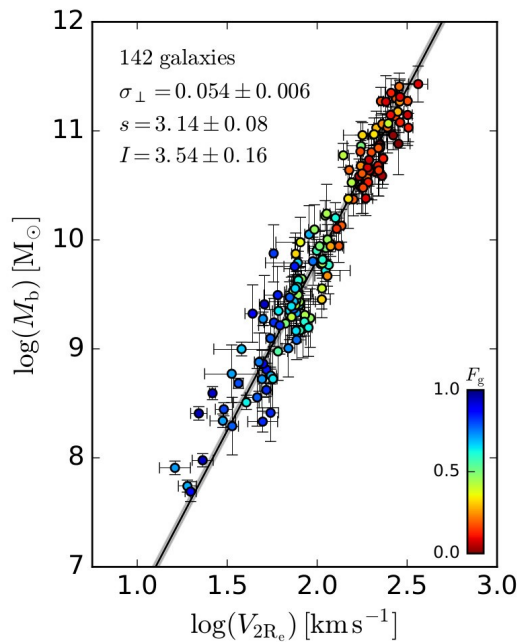
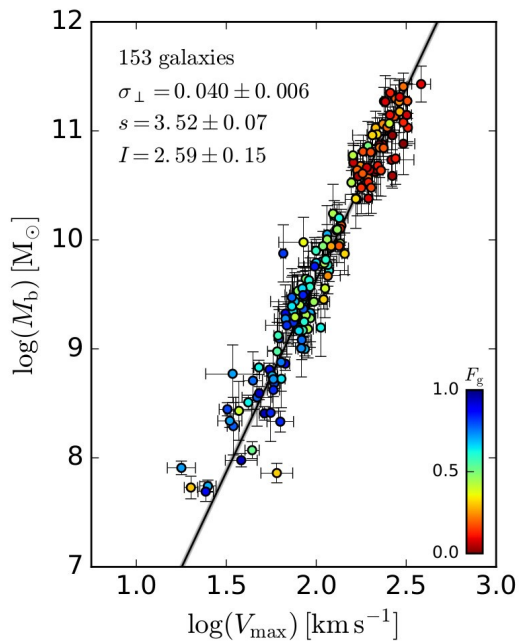
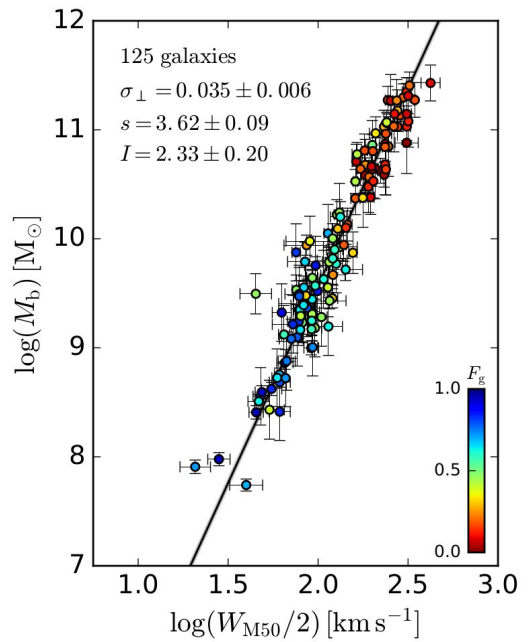
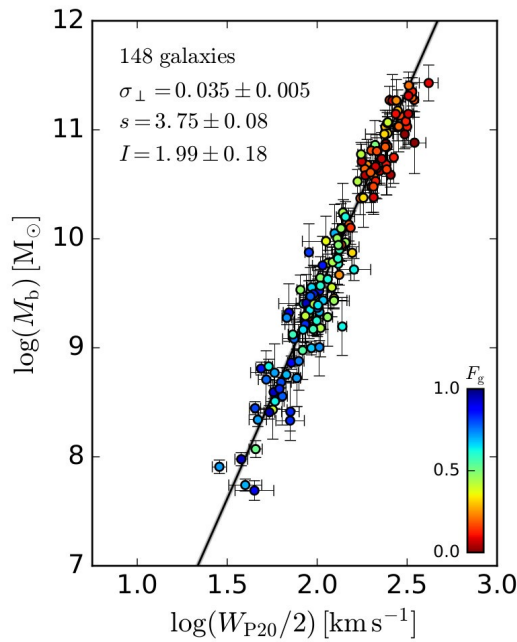
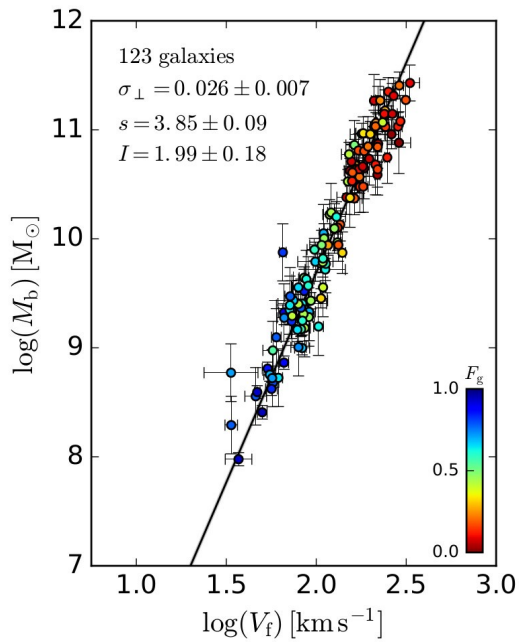
Newton says  $V^2/R = GM_b/R^2 \rightarrow V^4 = G^2 \Sigma_b M_b$



# 37 years after the MOND prediction

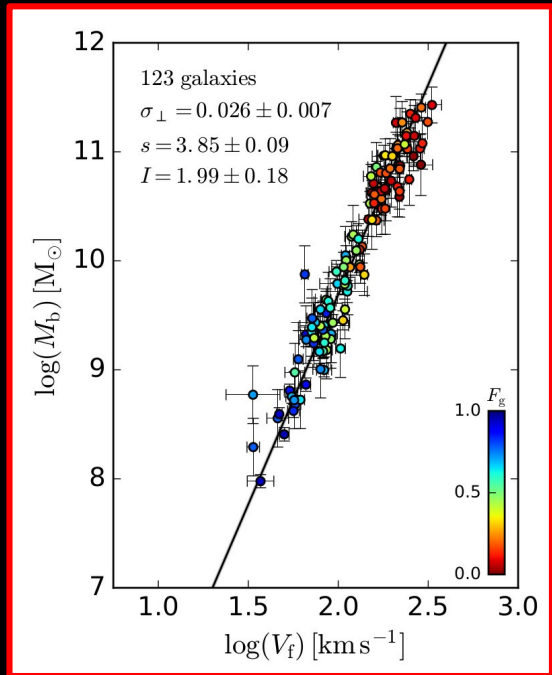


# Which velocity best correlates with $M_b$ ?



Lelli+2019

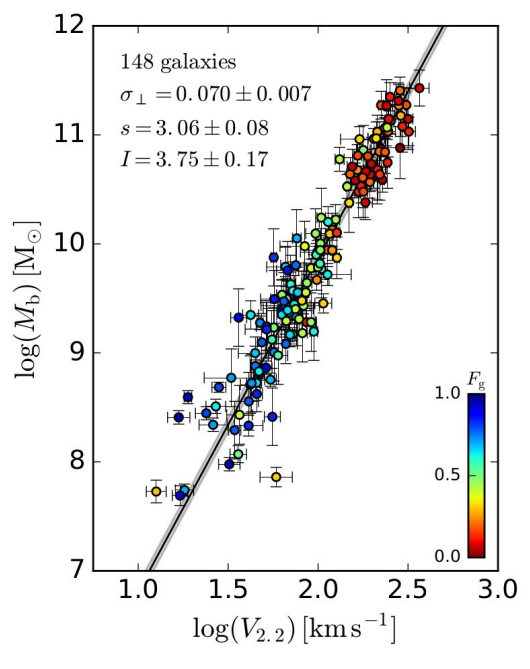
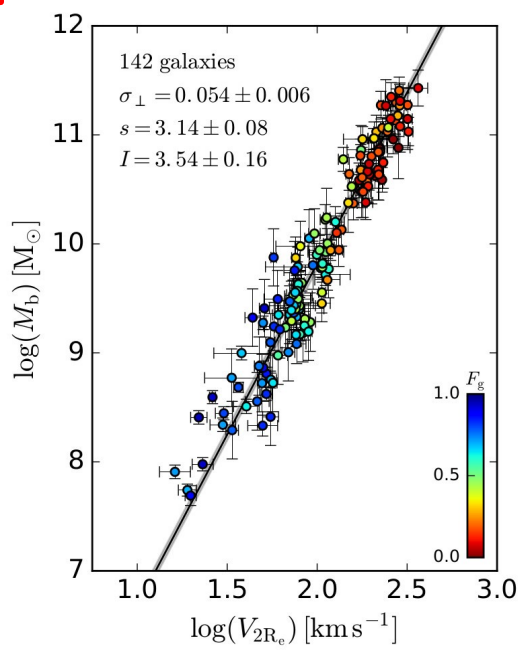
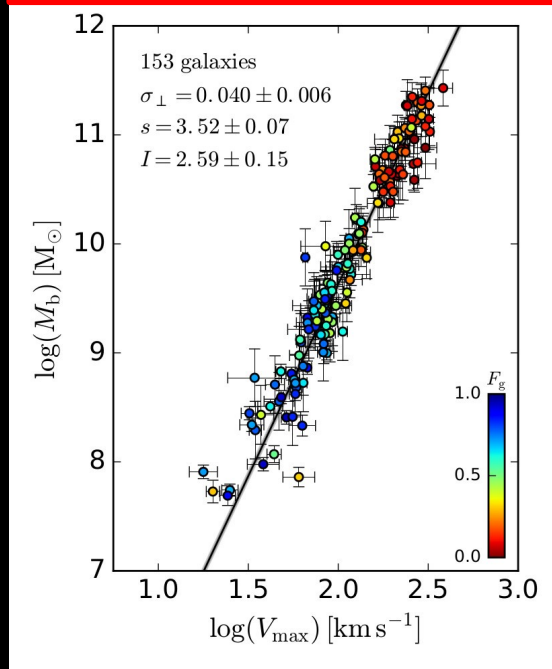
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$V_f$  gives the tightest BTFR (Verheijen 2001; McGaugh 2005; Noordermeer & Verheijen 2007)

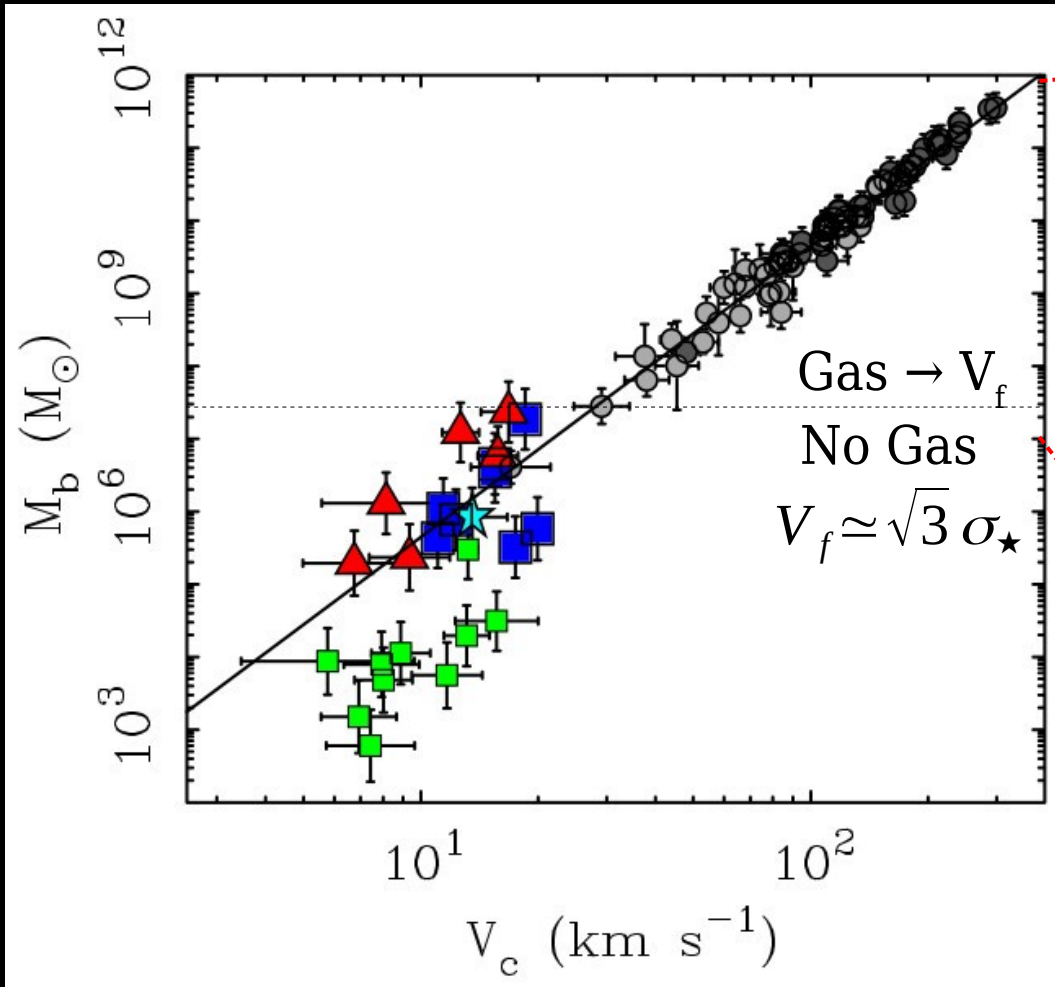
**Counter-intuitive result!**

Baryons are important near the center, but  $M_b$  best correlates with  $V$  at large radii where DM should dominate!

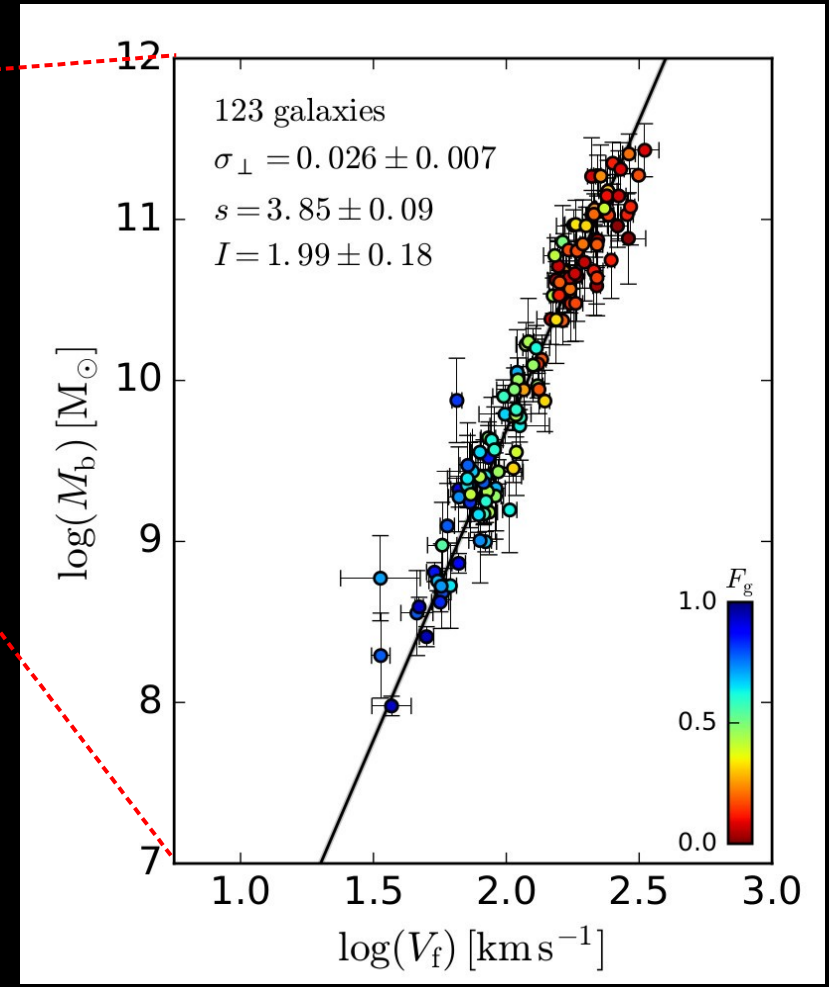


Lelli+2019

# BTFR for MW and M31 Dwarf Satellites

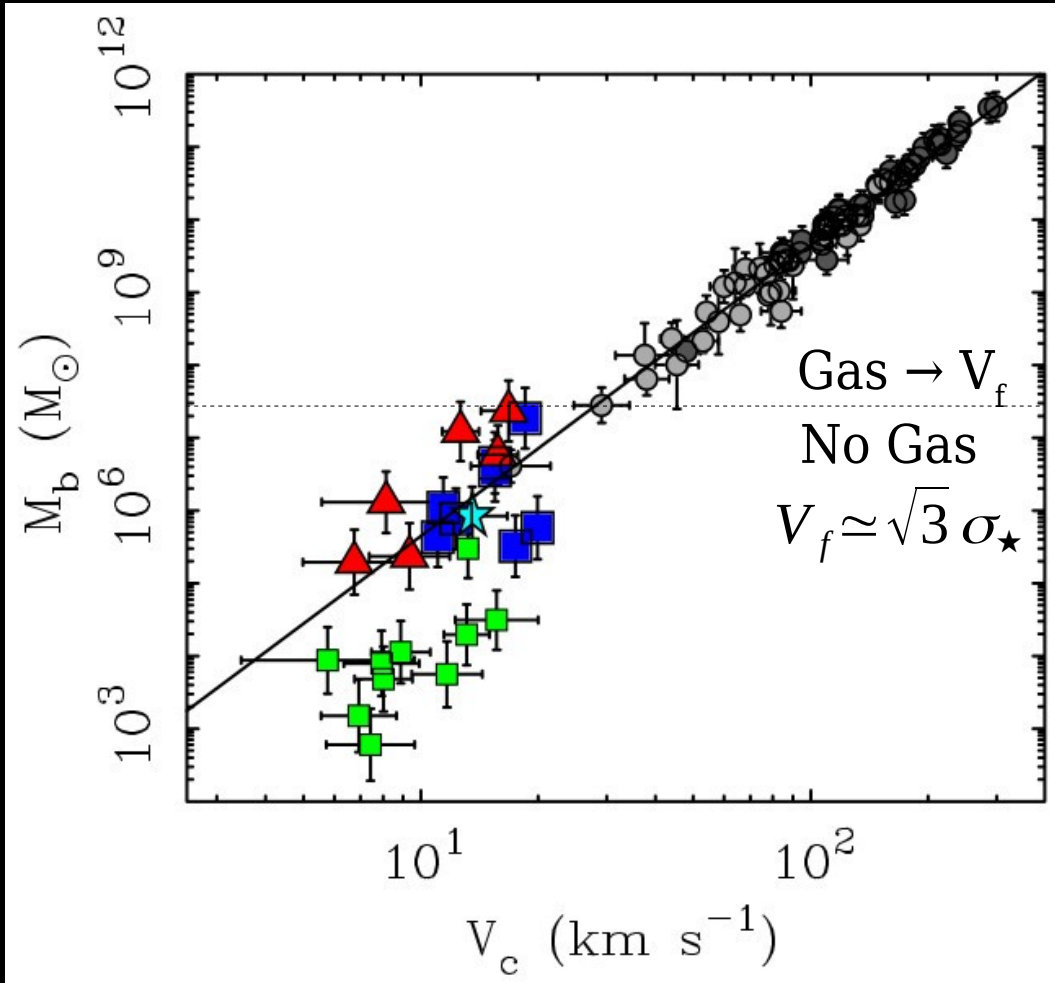


McGaugh & Wolf (2010)



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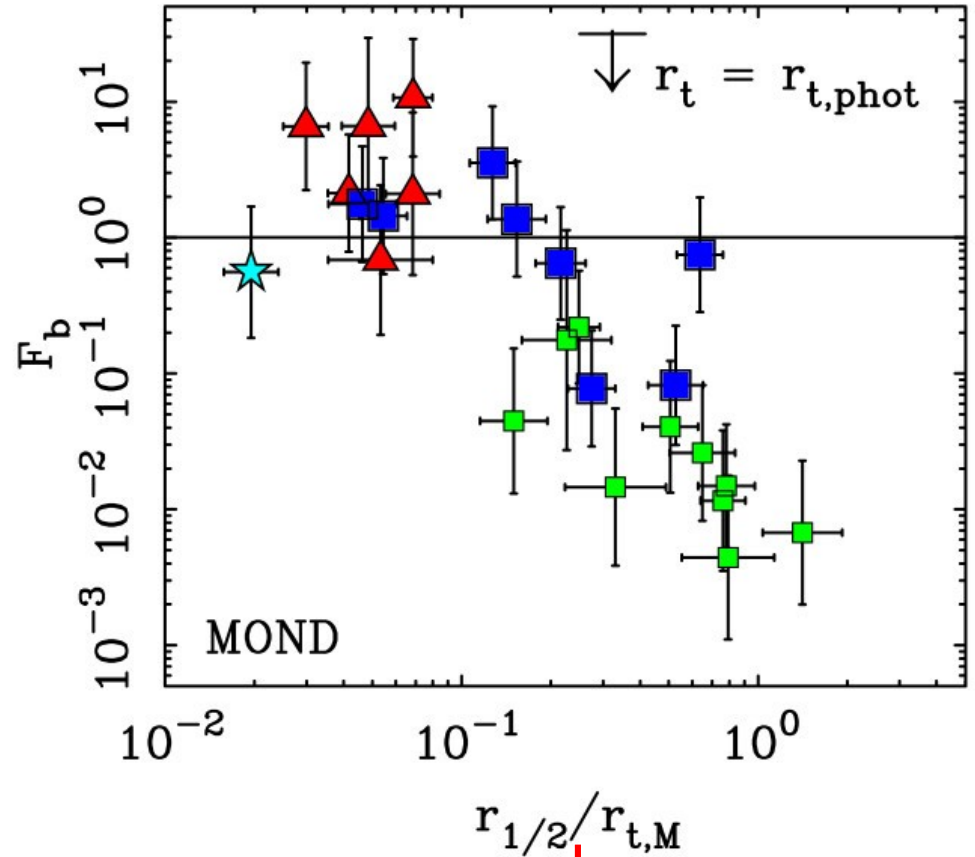


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**BTFR holds for equilibrium systems**

**→ some satellites are out of dynamical equilibrium due to tidal forces**

Strength of Deviations vs Tides



Half-Light Radius / Tidal Radius

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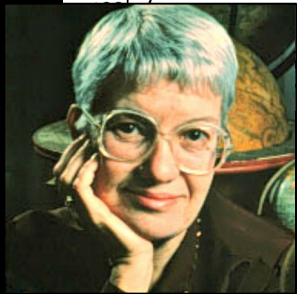
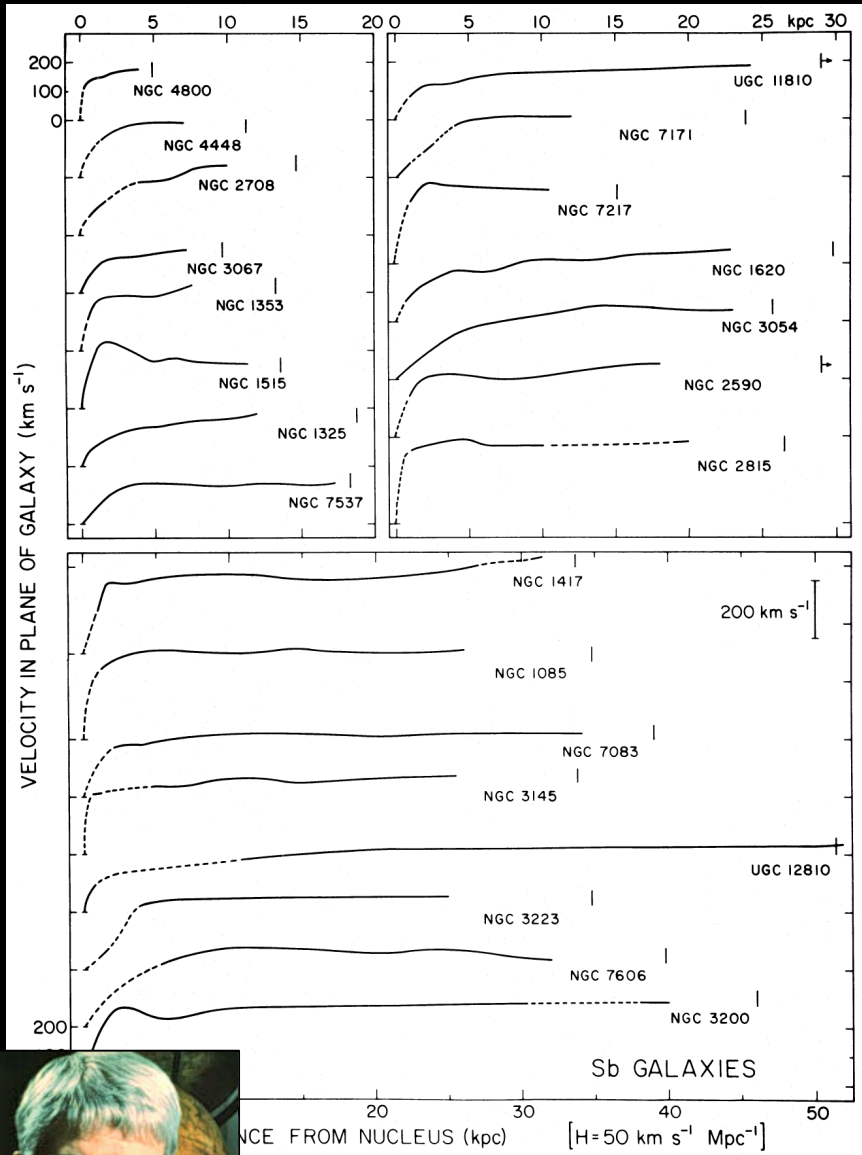
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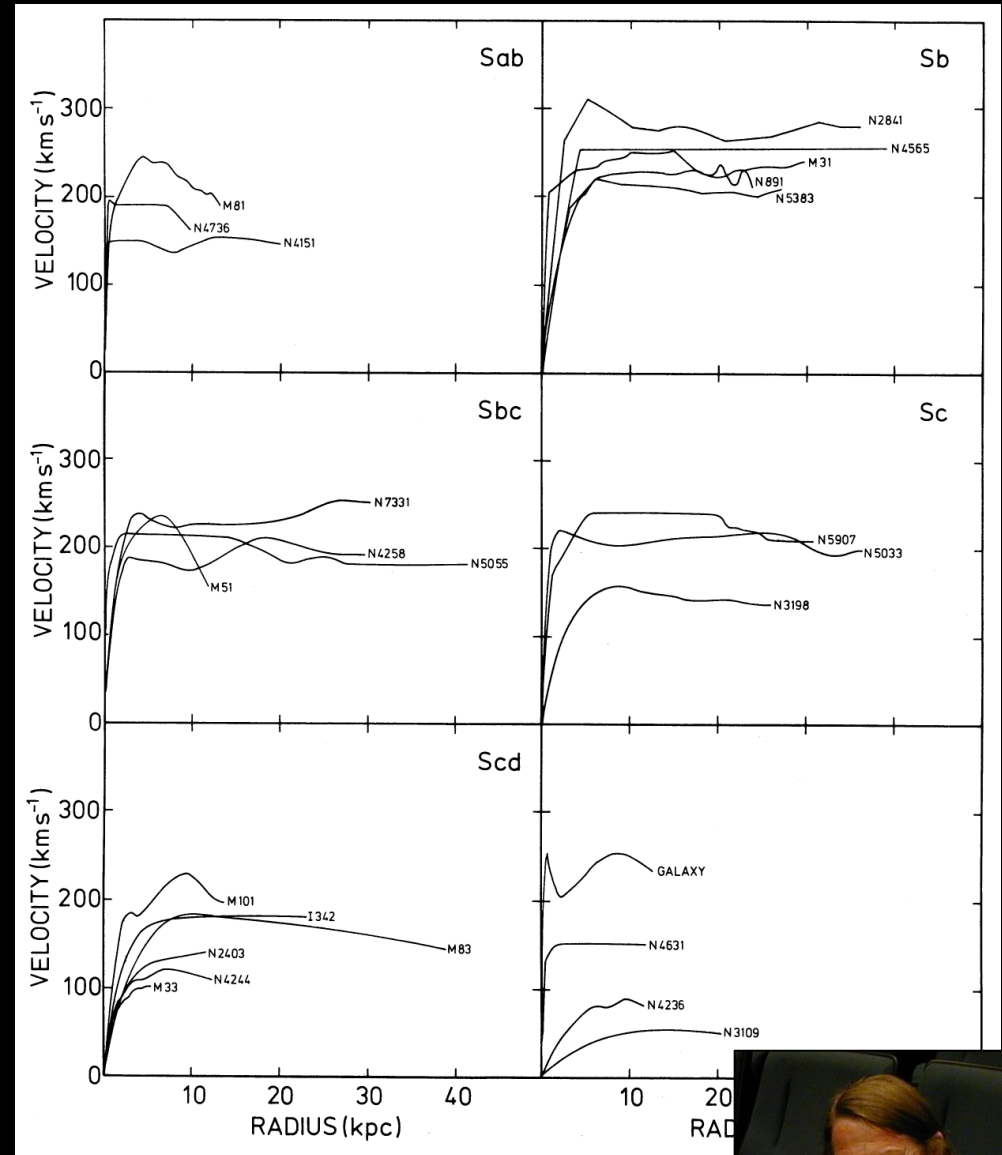
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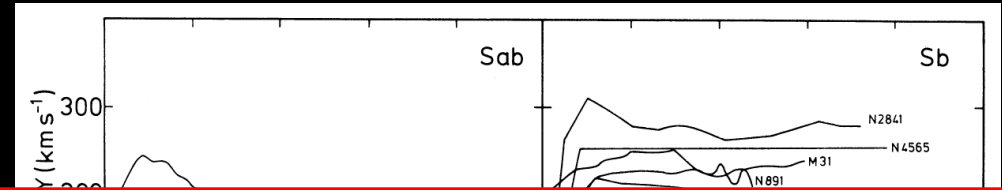
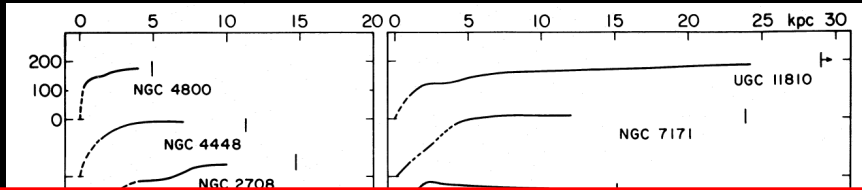
Optical - Ionized Gas ( $H\alpha$ )  
V. Rubin et al. (1981)



Radio - Atomic Gas (HI)  
A. Bosma (1981)

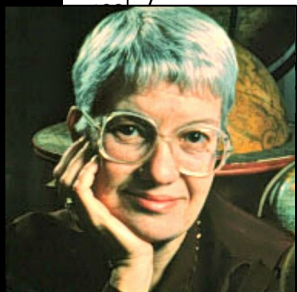
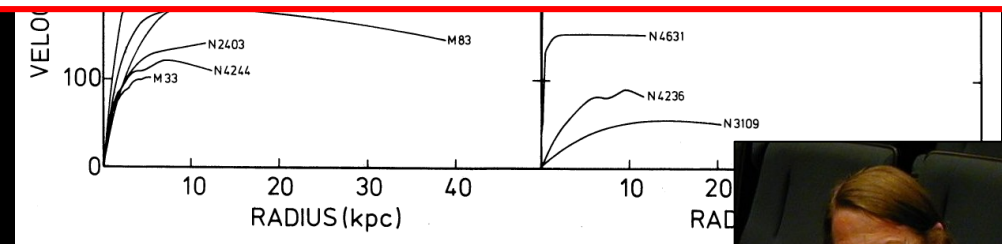
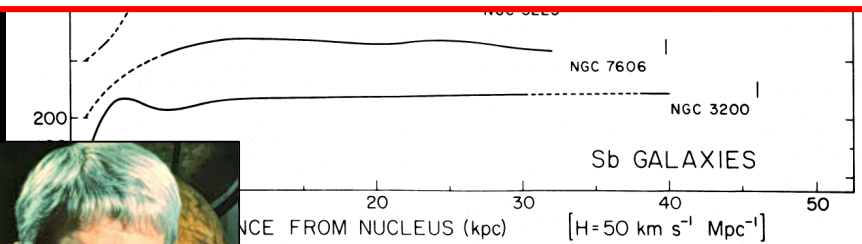


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These are mostly **high-surface-brightness** (high density) galaxies. **Low-surface-brightness** galaxies were poorly understood in the 80s (most of them haven't even been discovered yet...).

**Low-surface-brightness galaxies** are a key test for MOND because  $a < a_0$  at all radii, so they are entirely in the MOND regime.

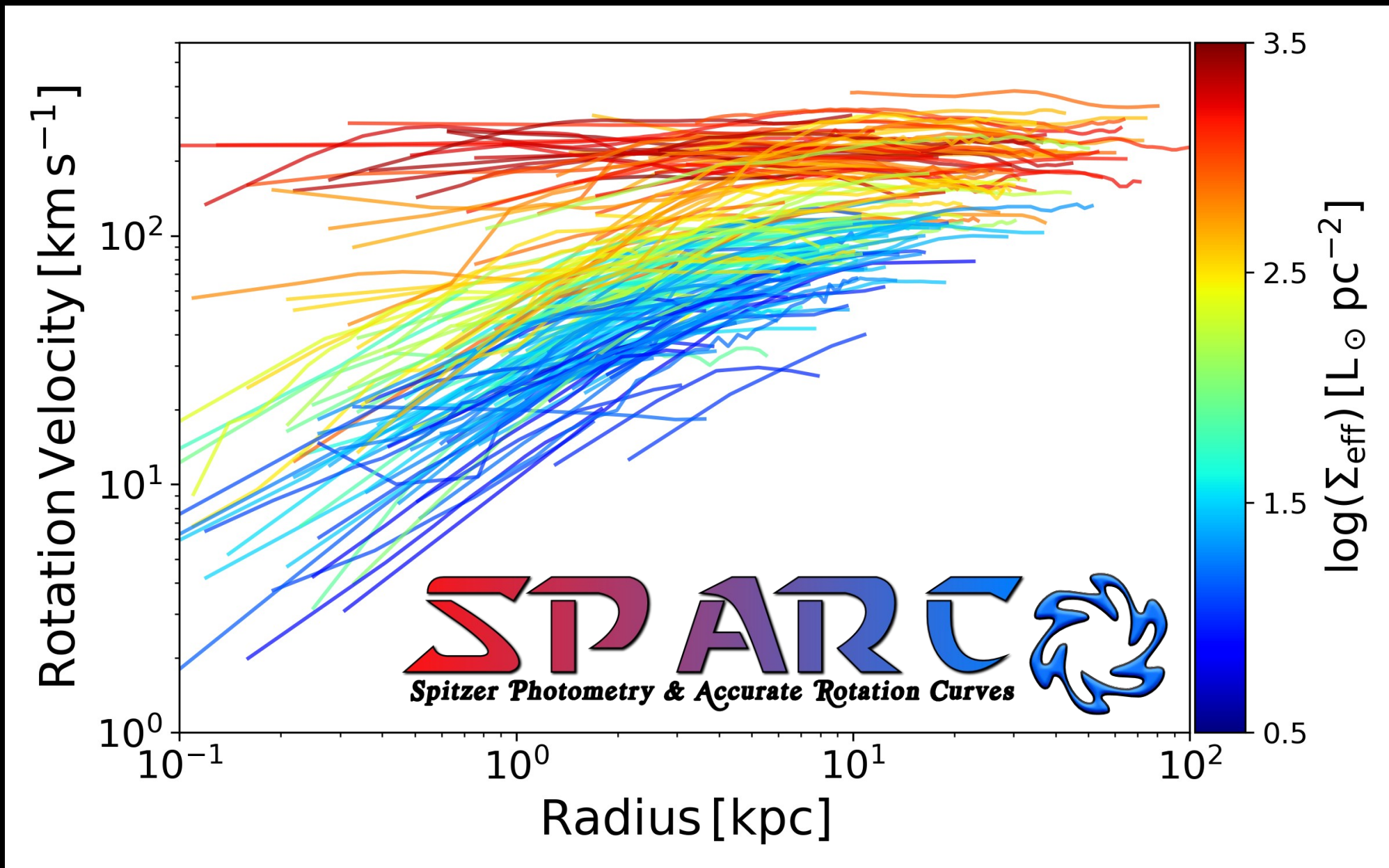


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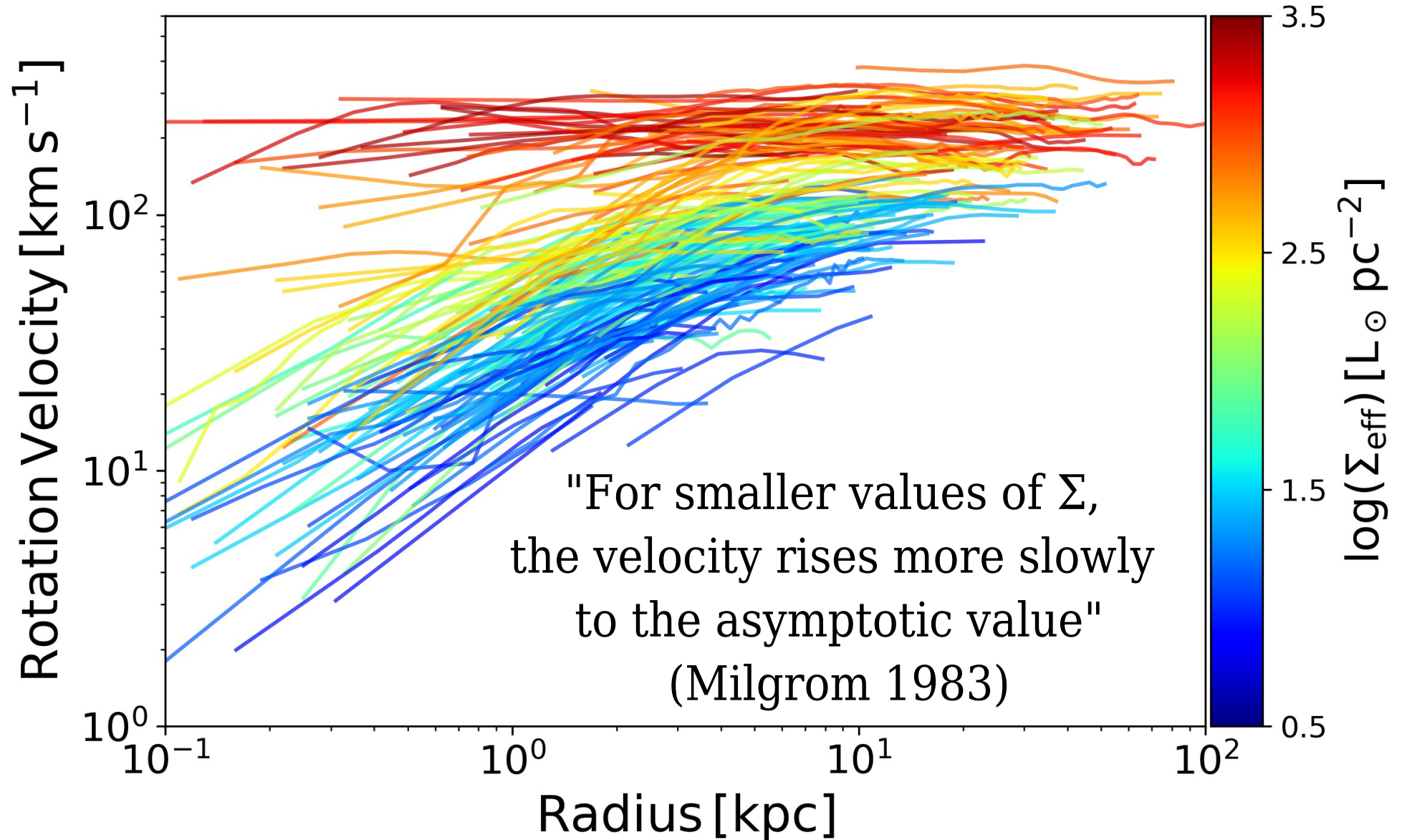
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$$a_0 = (1.2 \pm 0.2) 10^{-8} \text{ cm/s}^2 = (cH_0)/(2\pi)$$

The main predictions are:

1. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

② The  $V_\infty^4 = a_0 GM$  relation should hold exactly.

③ An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing  $r$  in a predictable way.

Milgrom 1983, ApJ, 270, 371

# A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES<sup>1</sup>



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Received 1982 February 4; accepted 1982 December 28

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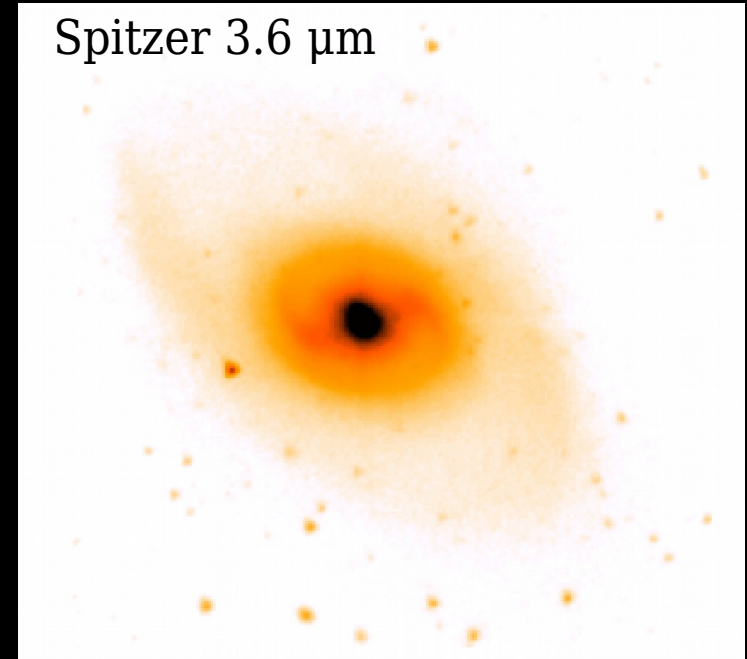
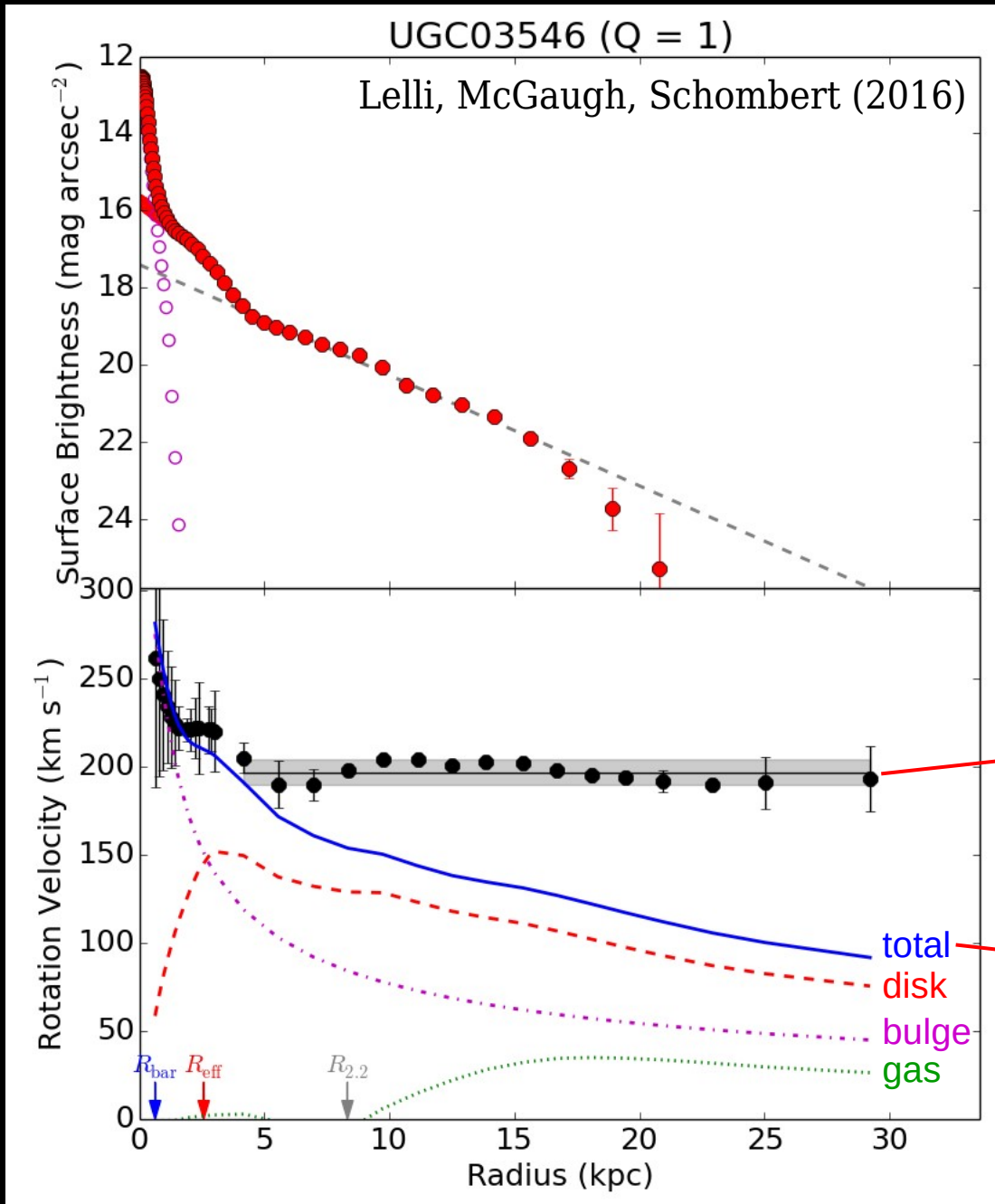
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# Mass Models of Disk Galaxies

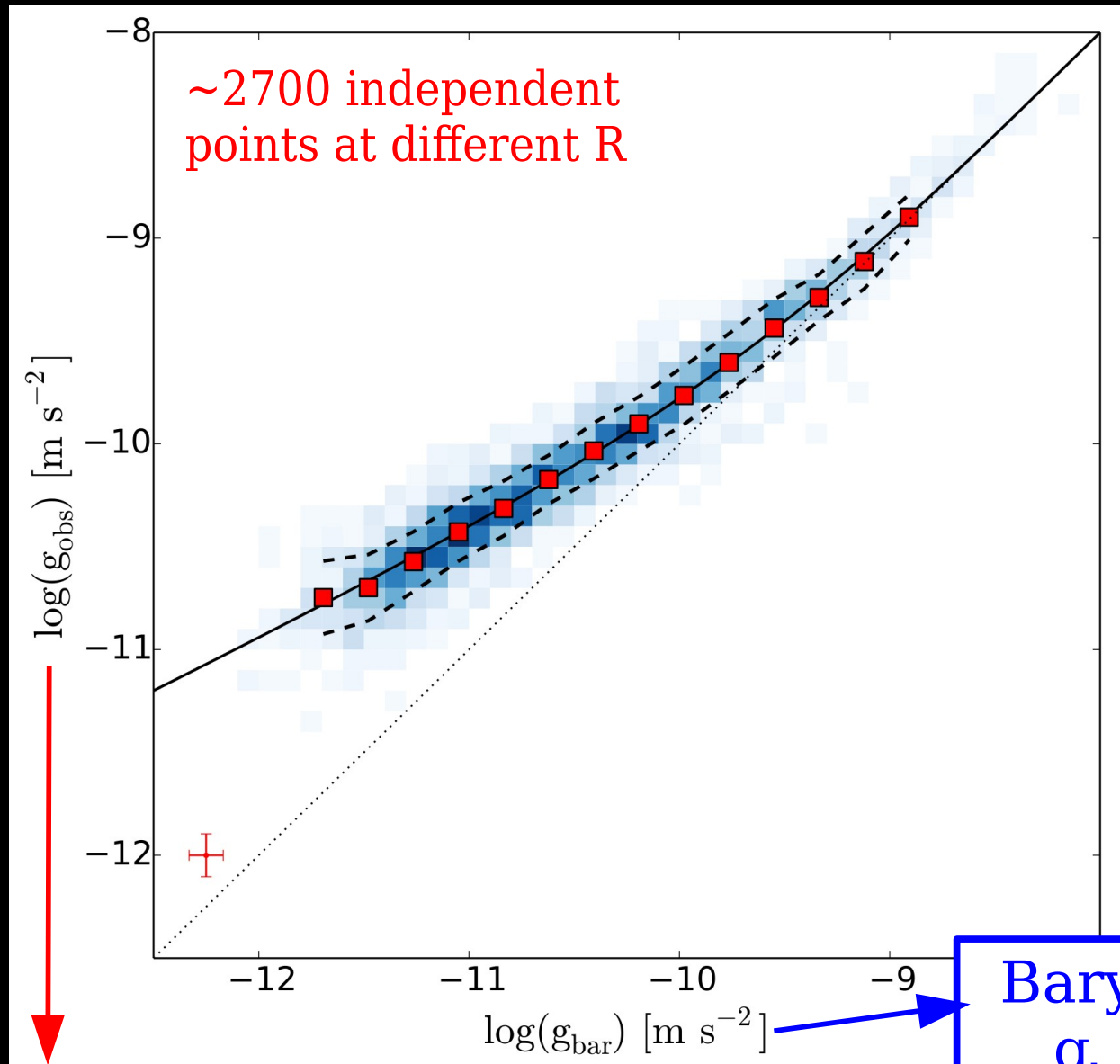


$$g_{\text{obs}} = \frac{V_{\text{obs}}^2}{R} = -\nabla \Phi_{\text{tot}}$$

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# Radial Acceleration Relation



McGaugh+2016  
Lelli+2017  
Li, Lelli+2018

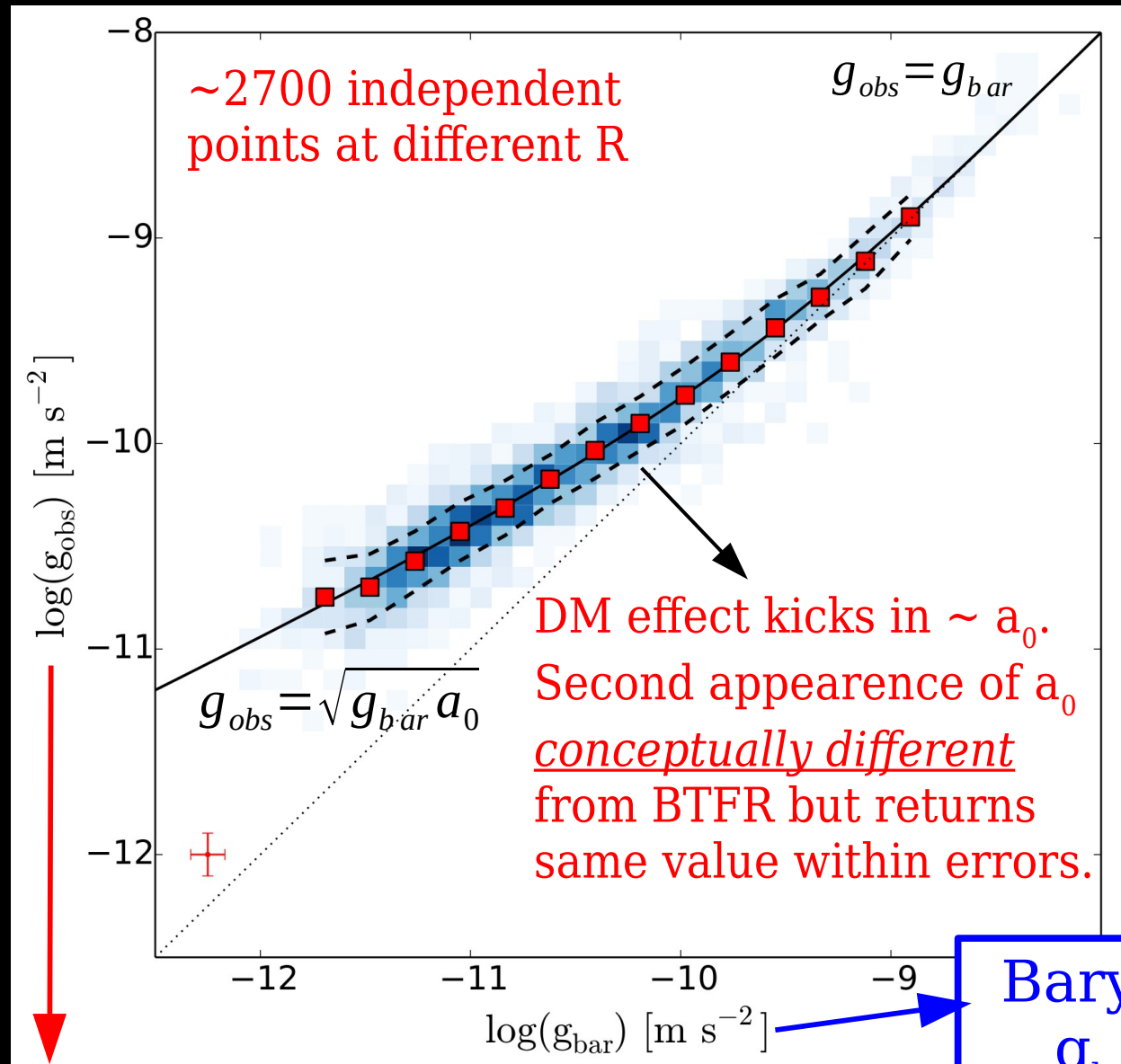
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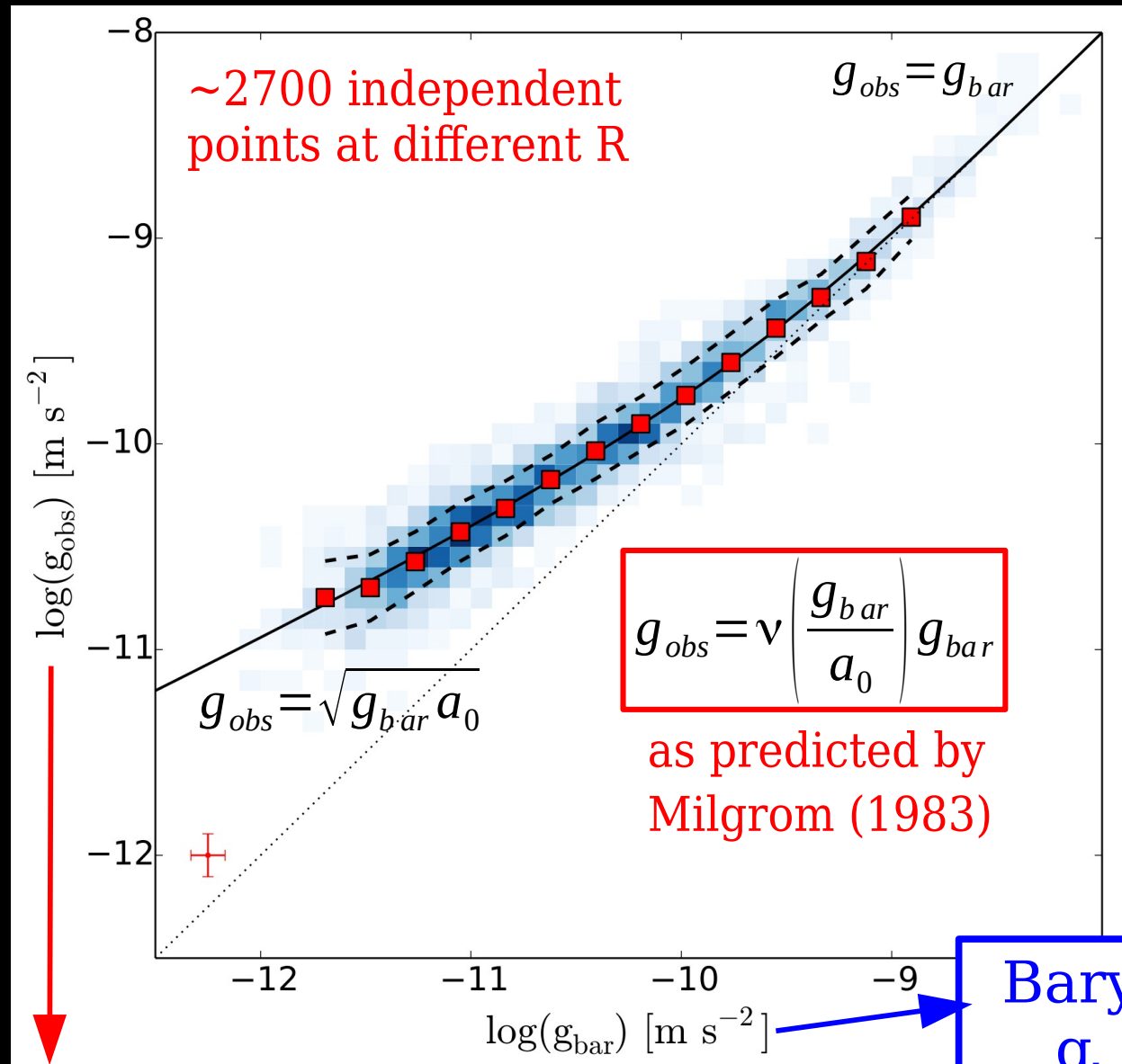
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I consider the implications of a modification of the Newtonian dynamics to galaxy systems. Masses and mass-to-light ratios are rederived, on the basis of existing data, for binary galaxies, small groups, clusters of galaxies, and the Virgo Supercluster. For each type of galaxy system, the average  $M/L$  values come out to be a few solar units. These results eliminate the need to assume large amounts of hidden mass in galaxy systems, if the modified dynamics applies.

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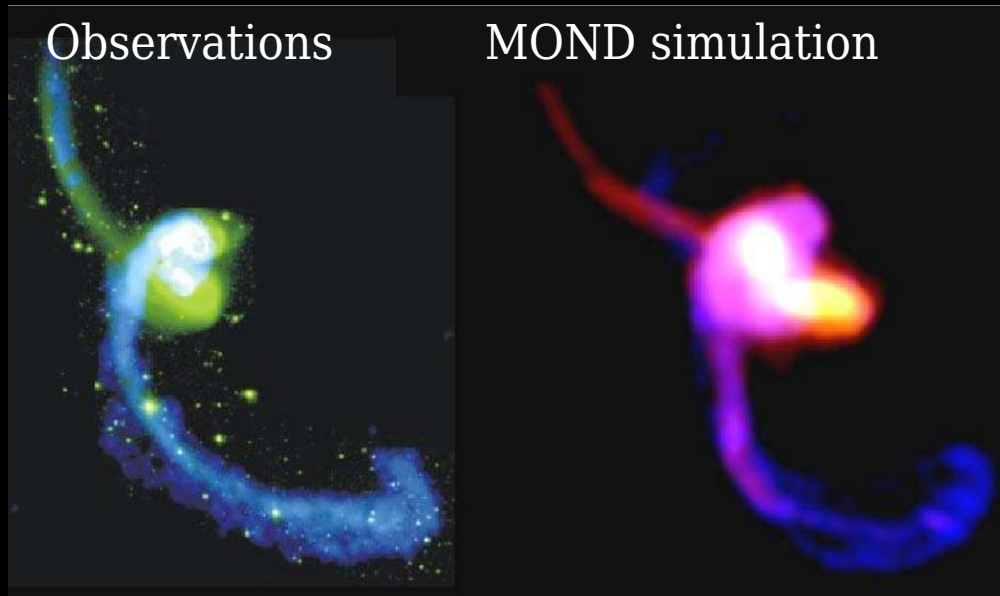
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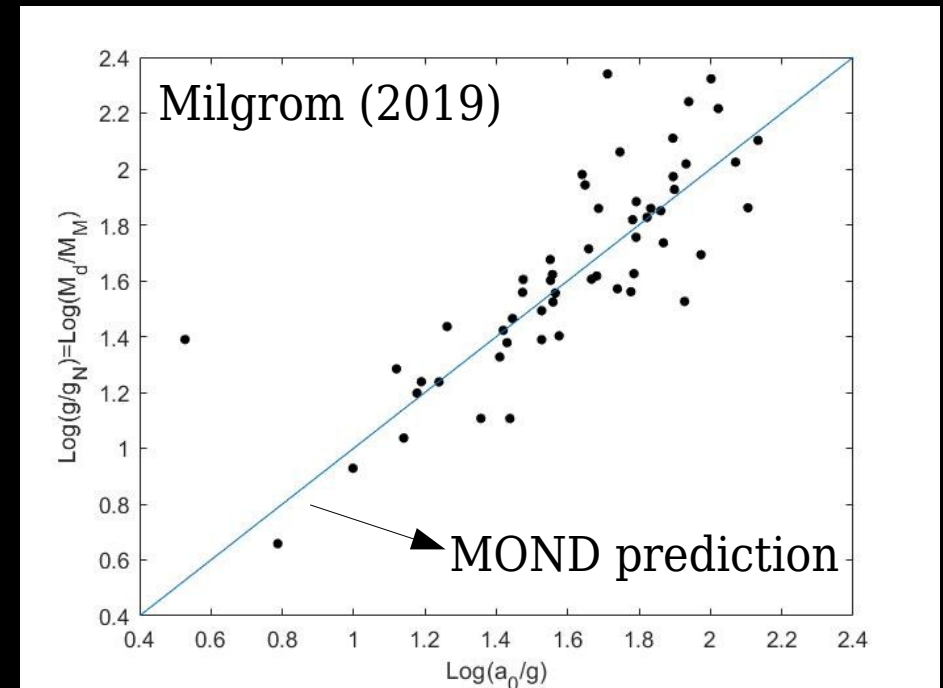
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Binary Galaxies / Galaxy Interactions ✓

Galaxy Groups ✓



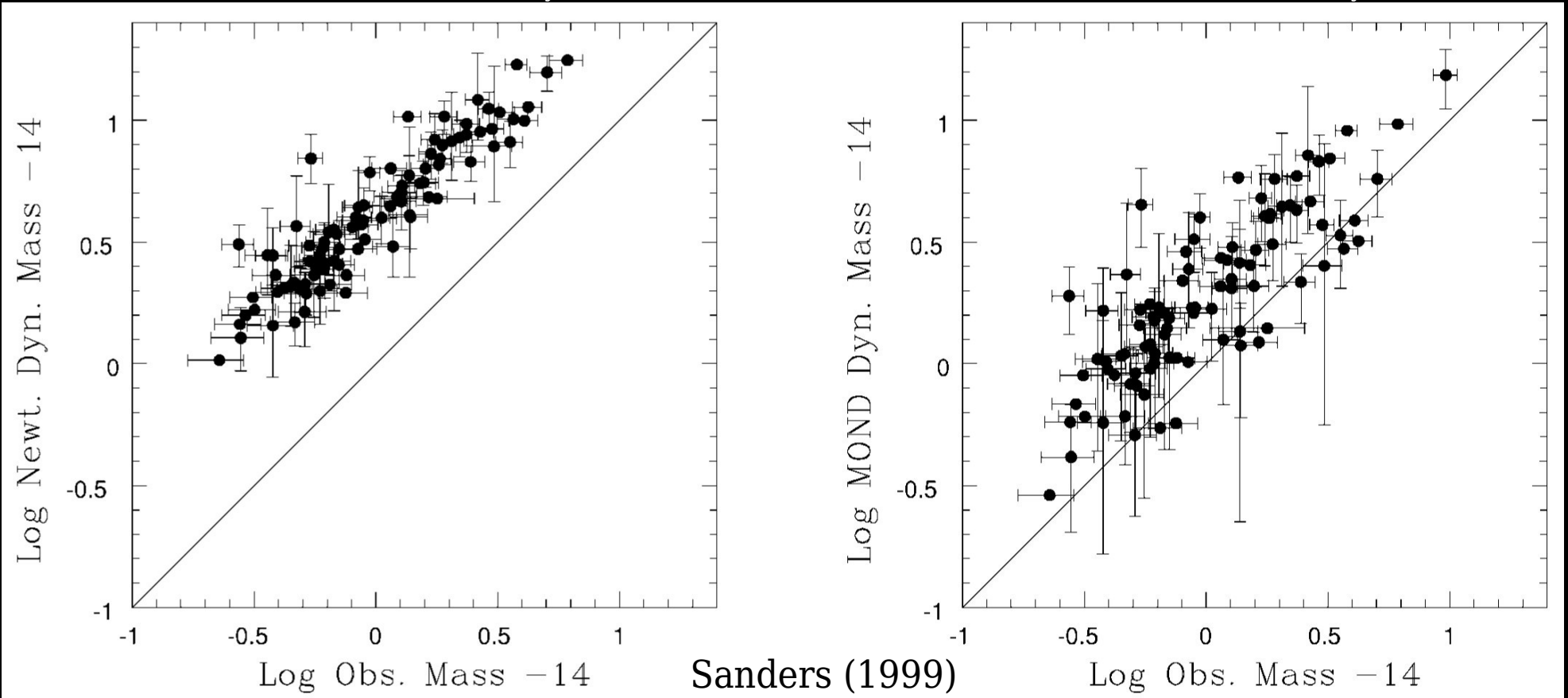
The Antennae - Tiret & Combes (2008)



# Key Problem for MOND: Galaxy Clusters

Newtonian analysis:  $M_{\text{dyn}}/M_{\text{bar}} \sim 4$

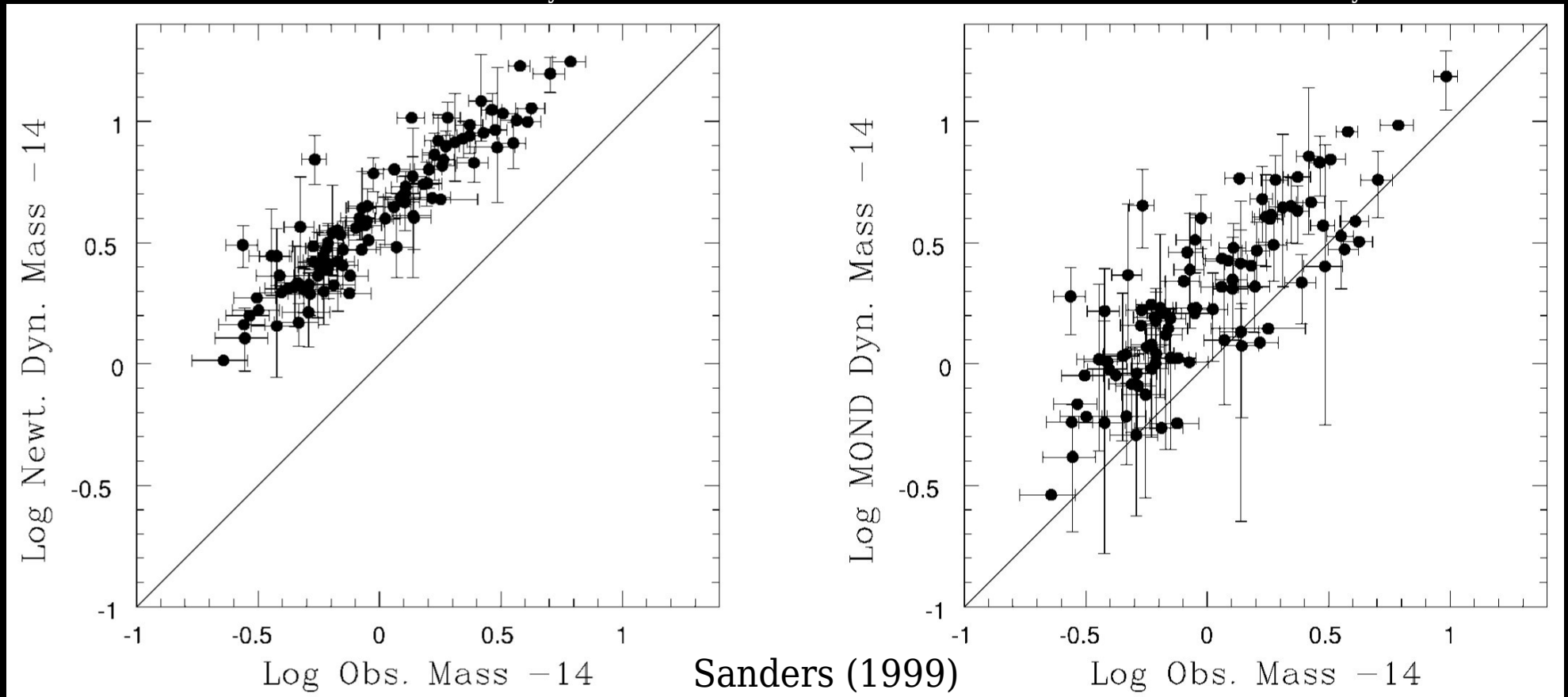
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## Proposed solutions:

- 1) Undetected baryons (Milgrom 2008)  $\rightarrow$  BBN implies  $\sim 30\%$  missing baryons
- 2) Sterile neutrinos with  $m \sim 10$  eV (Angus 2008)  $\rightarrow$   $\nu$  oscillations and masses
- 3) Extended MOND:  $a_0 \propto \Phi$  (Zhao & Famaey 2012)  $\rightarrow$  deeper theory?

**Why did MOND get any prediction right?**

**What is the deeper meaning of MOND?**

# MOND paradigm

```
graph TD; A[MOND paradigm] --> B["Modified Gravity  
(→ Poisson's eq.)"]; A --> C["Modified Inertia  
(→ F = ma)"];
```

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**Other ideas:**  
Entropic Gravity (Verlinde 2017)  
Bipolar DM (Blanchet & LeTiec 2008)  
Superfluid DM (Khoury 2015)

# MOND vs $\Lambda$ CDM

Small scales (galaxies)      Large scales (cosmology)

How do we weight the different evidence? (Kroupa 2012, McGaugh 2015)

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A-posteriori reactions

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What would make us change our mind about the existence of DM?

*"The only relevant evidence is the evidence anticipated by a theory"*

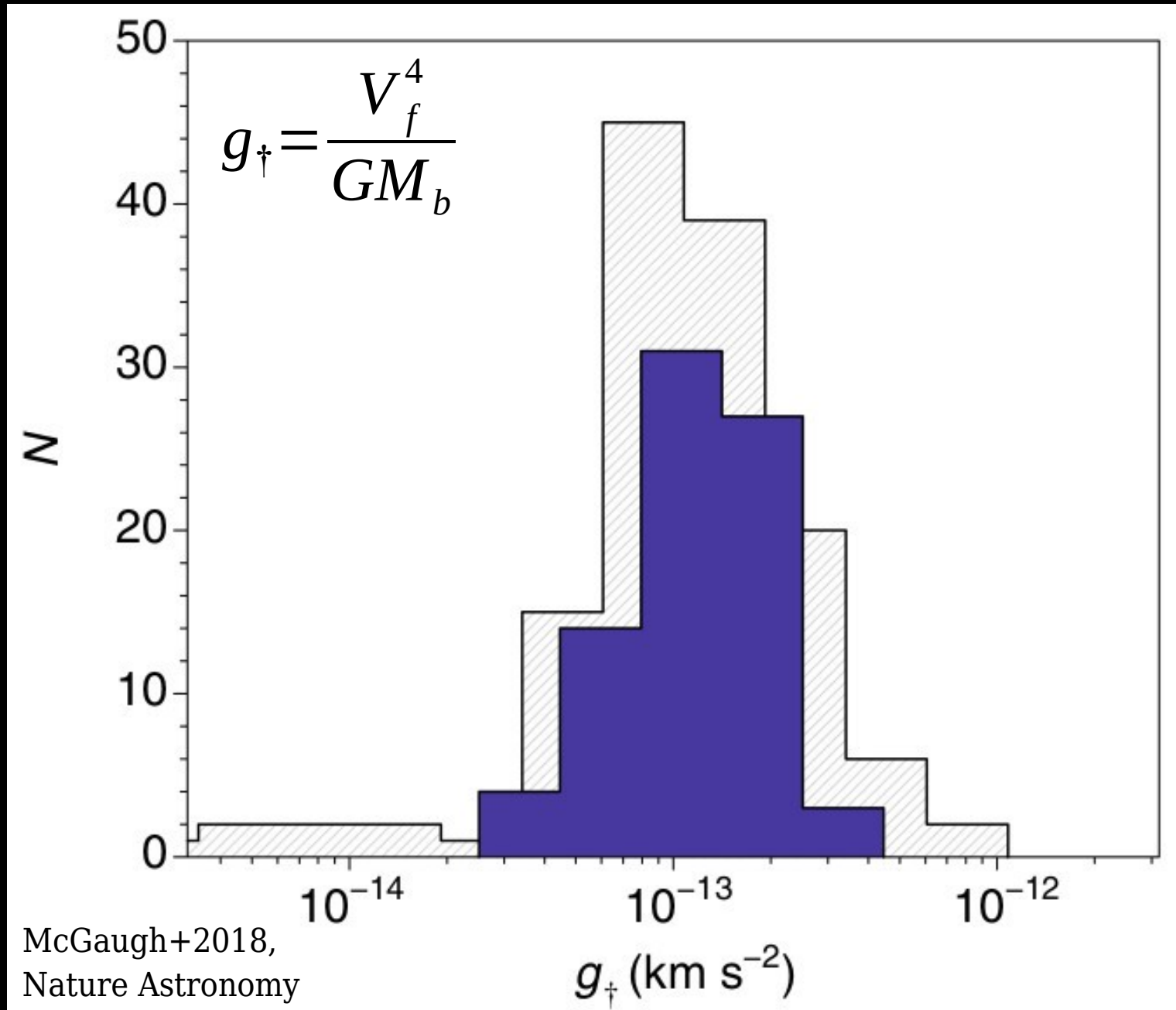
in Methodology of Scientific Research  
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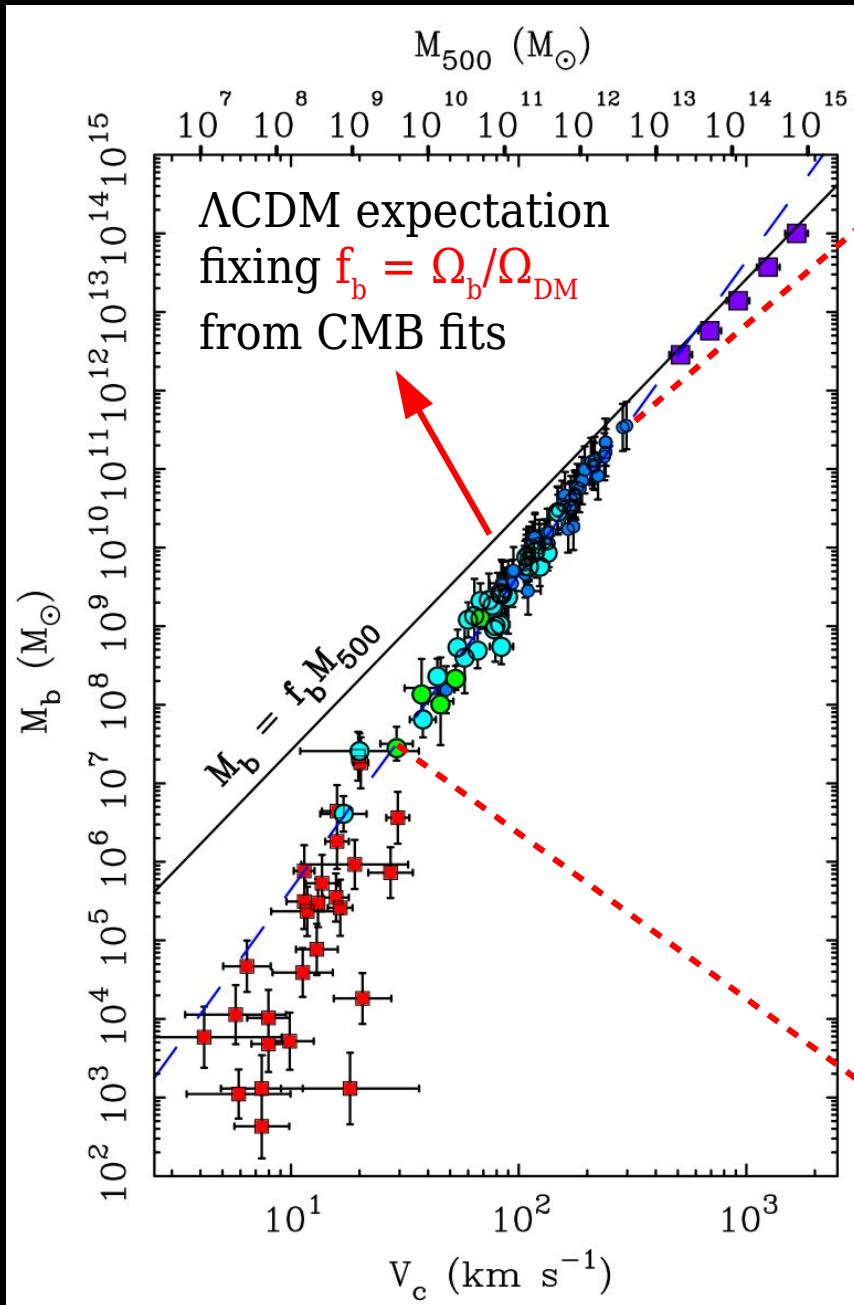
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**Thank you!**

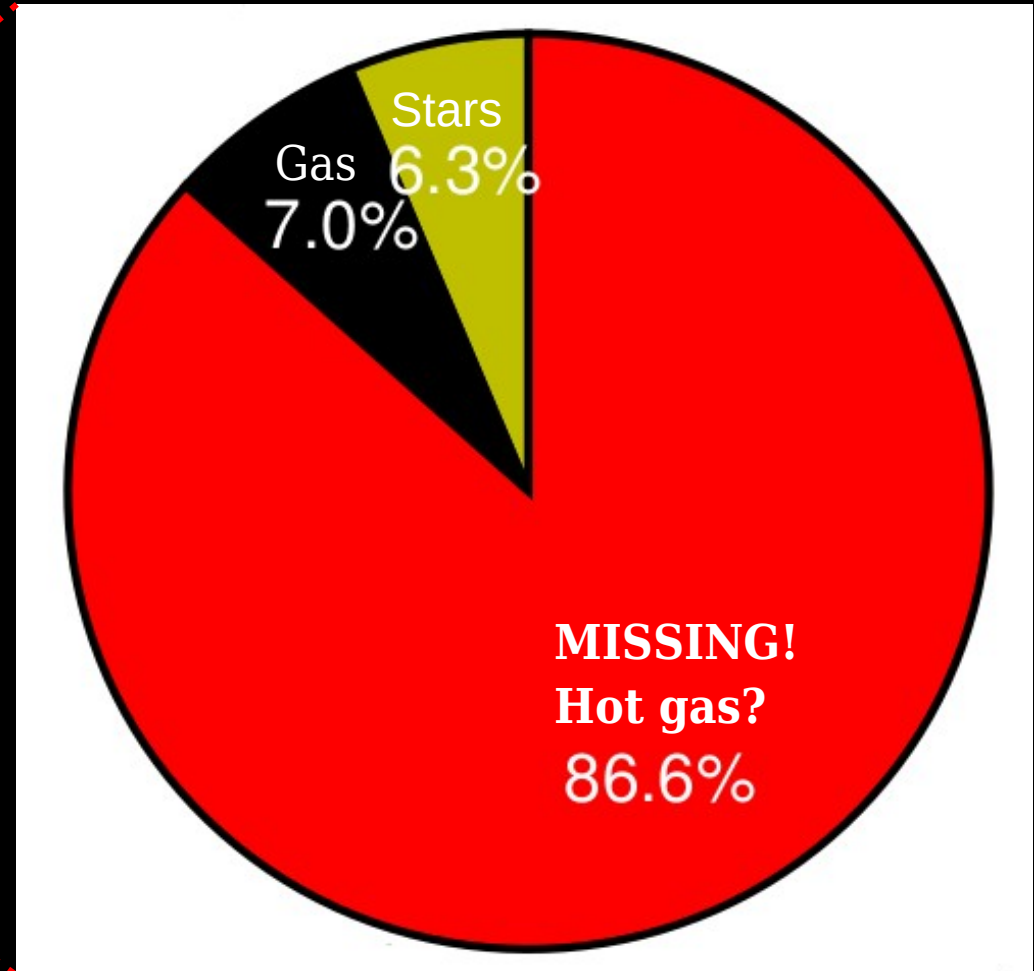
# Independently of MOND, $a_0$ is in the data



# Baryon Content of Cosmic Structures



Mean Budget in Galaxies in a  $\Lambda\text{CDM}$  context:

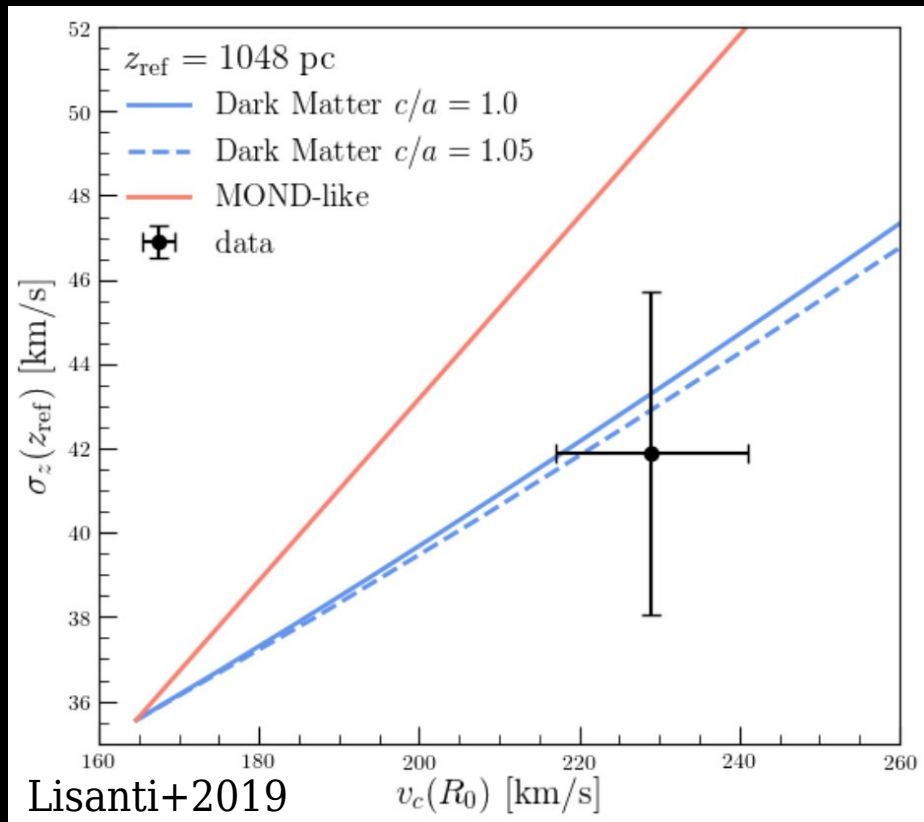


Katz, Desmond, Lelli et al. 2018

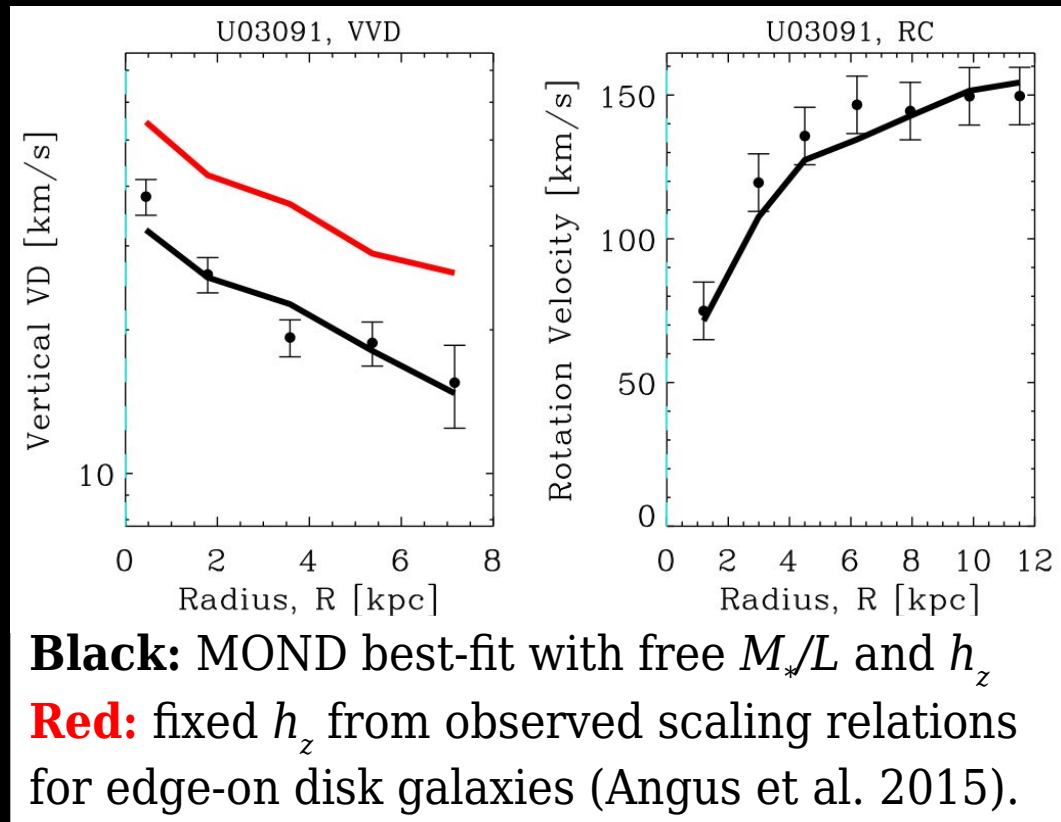
McGaugh+2012

# Vertical Disk Dynamics: Oort's Problem

Milky Way data near the Sun's location



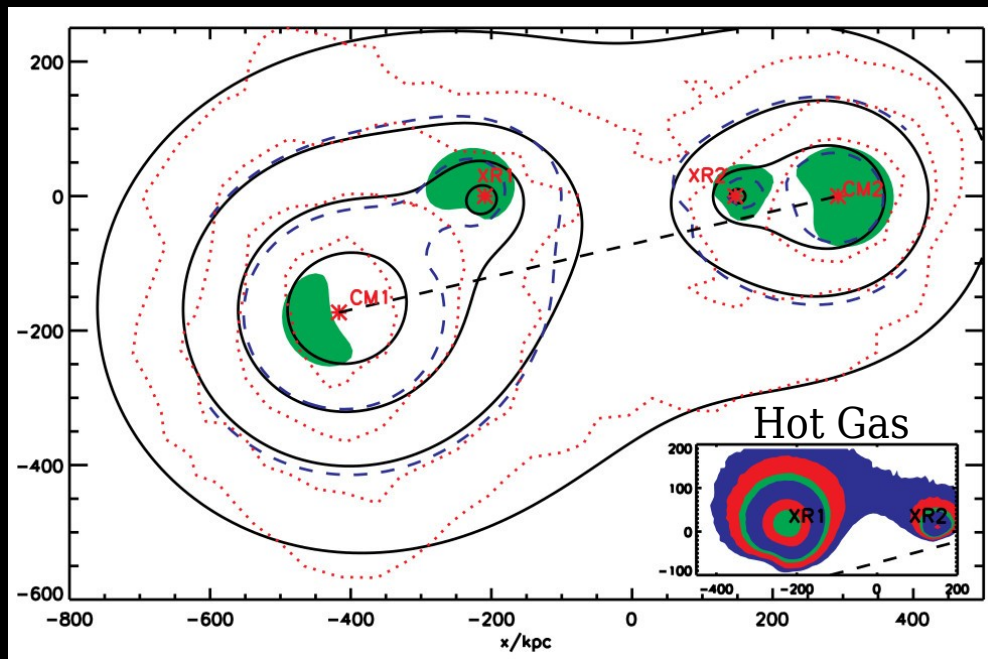
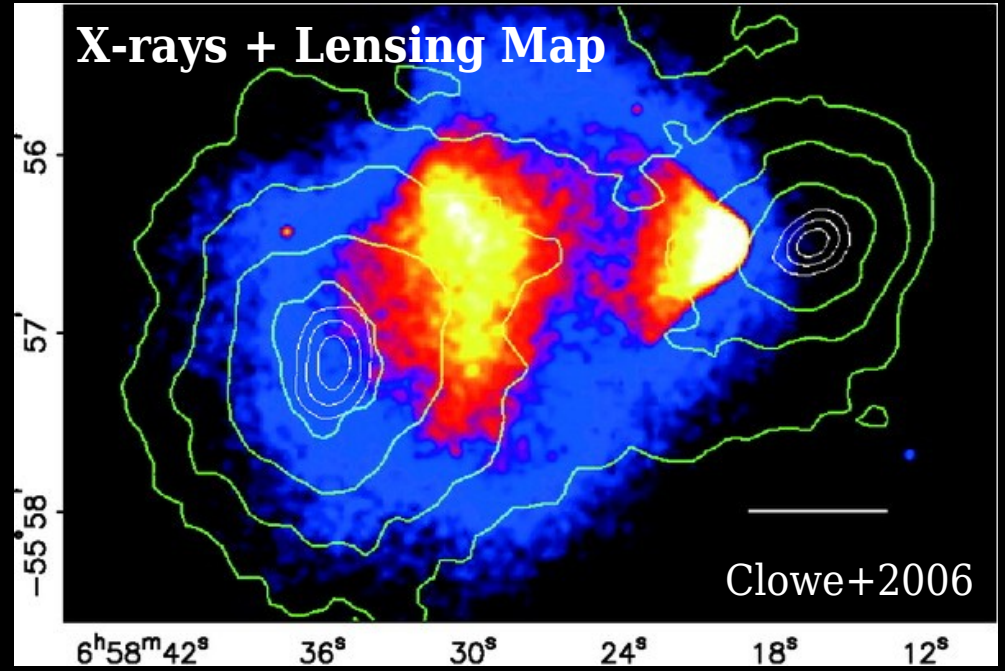
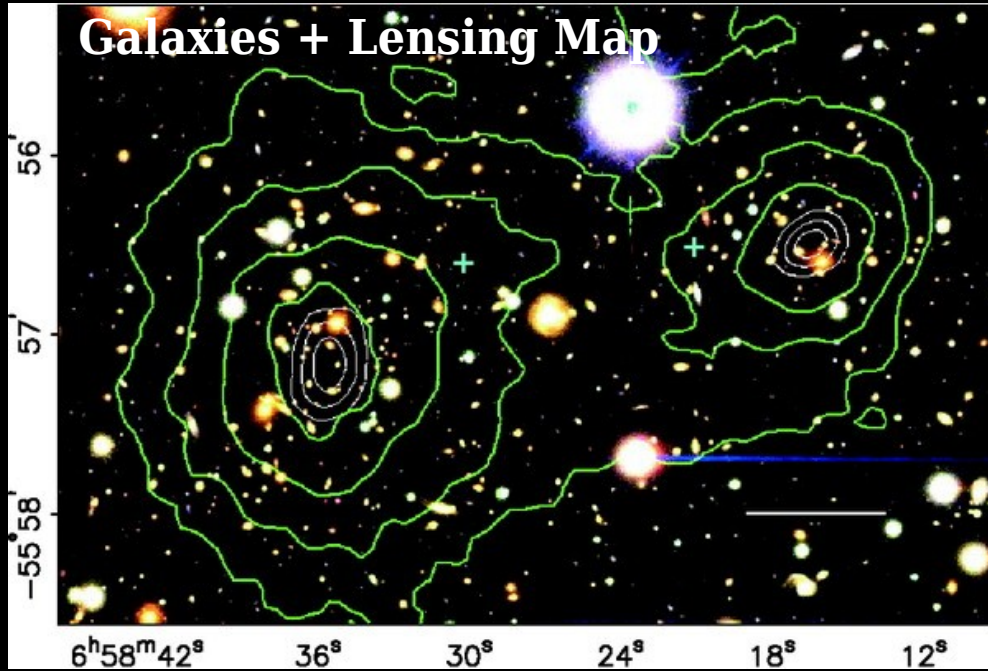
30 *face-on* galaxies from the DiskMass Survey



**Caution:** Gaia DR2 shows that the MW is *not* in vertical equilibrium  
 → Interaction with Sagittarius?  
 (Antoja+2018; Morgan & Bovy 2019; Carrillo+2019; Bland-Hawthorn+2019)

**Caution:**  $h_z \rightarrow$  old stars (NIR images)  
 $\sigma_z \rightarrow$  light-weighted for a mixed pop. of stars of different ages (optical IFUs)  
 (Aniyan+2016,2018; Milgrom 2016, 2018)

# Bullet Cluster is nothing special in MOND

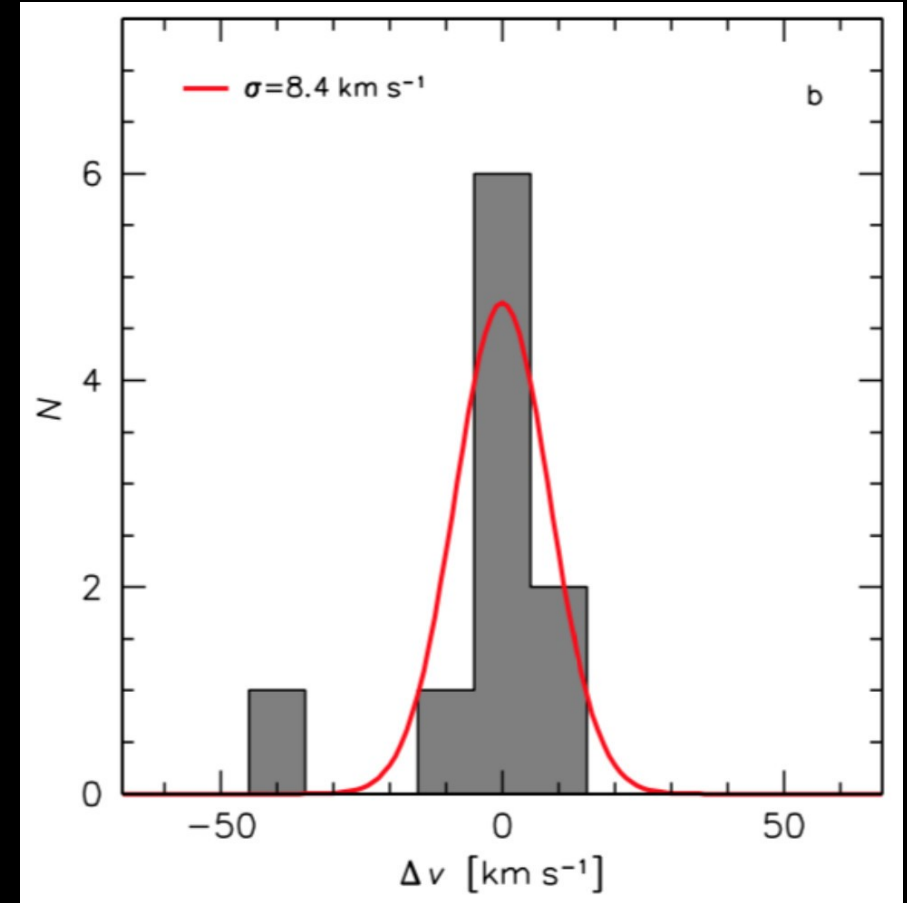
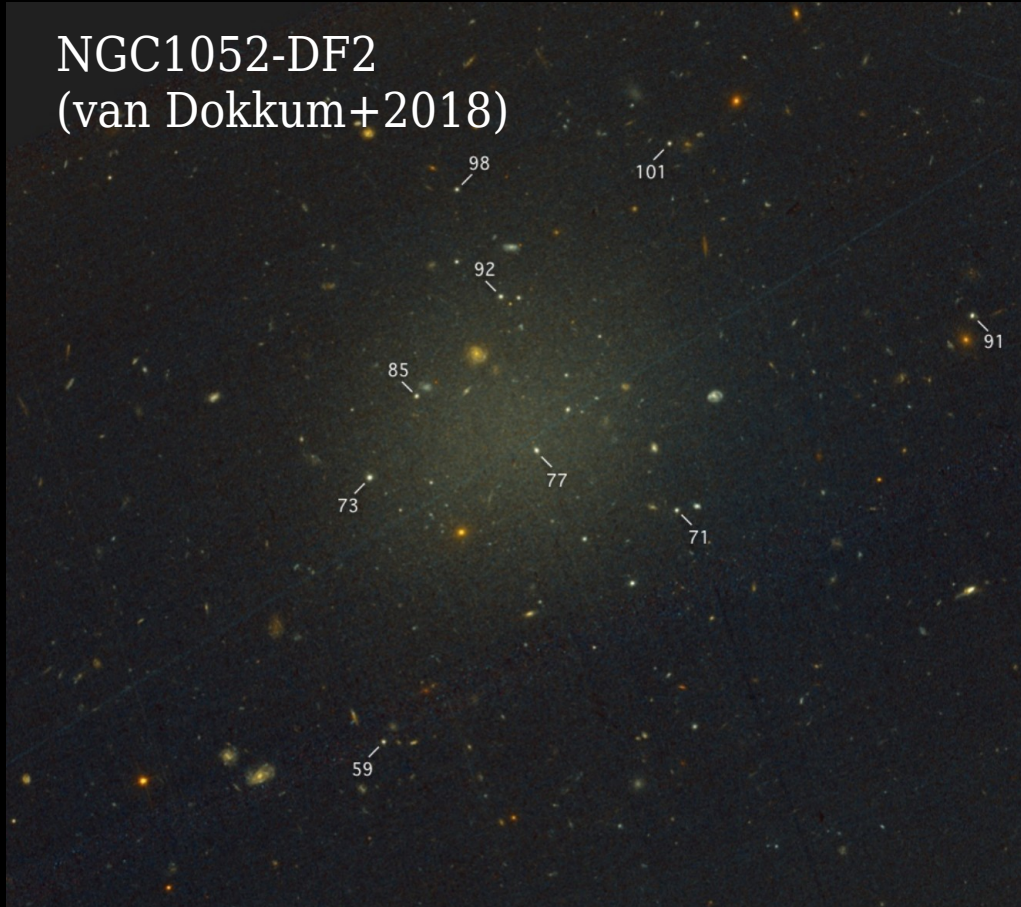


**MOND model with 2eV  $\nu$**  (Angus+2007):  
**Red:** Observed lensing convergence map  
**Black:** best-fit MOND+ $\nu$  convergence map  
**Blue:** total surface densities (baryons+ $\nu$ )  
**Green:** peak surface densities (neutrinos)

**High collision speed** ( $\sim 4500$  km/s) is rare in  $\Lambda$ CDM but natural in MOND (Hayashi & White 2006; Farrar & Rosen 2006; Angus+2007; Angus & McGaugh 2008).

# DM-deficient Ultra-Diffuse Galaxies?

NGC1052-DF2  
(van Dokkum+2018)



Initially, **low  $\sigma_v$**  from **10 globular clusters** suggesting no mass discrepancy **but**:

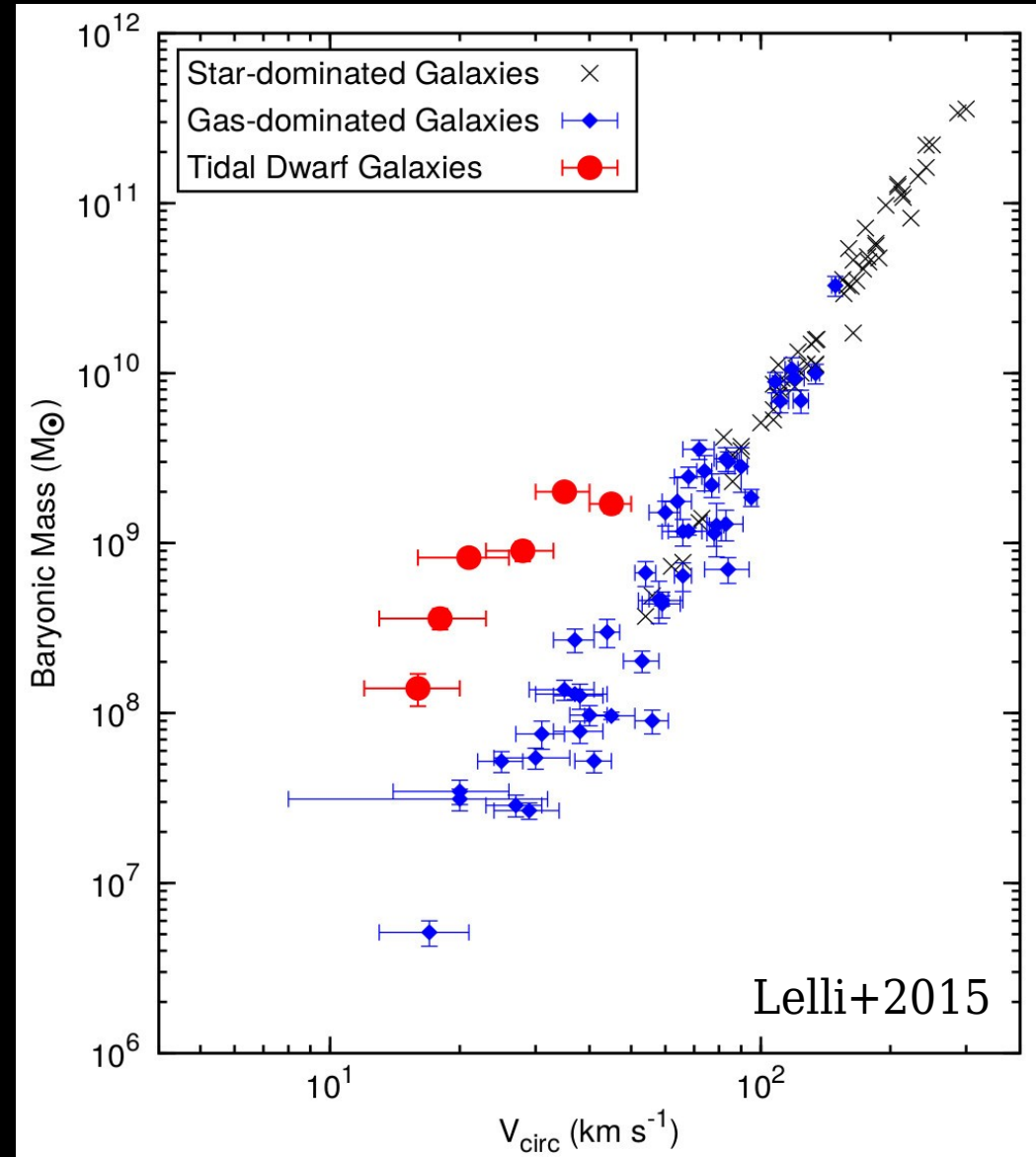
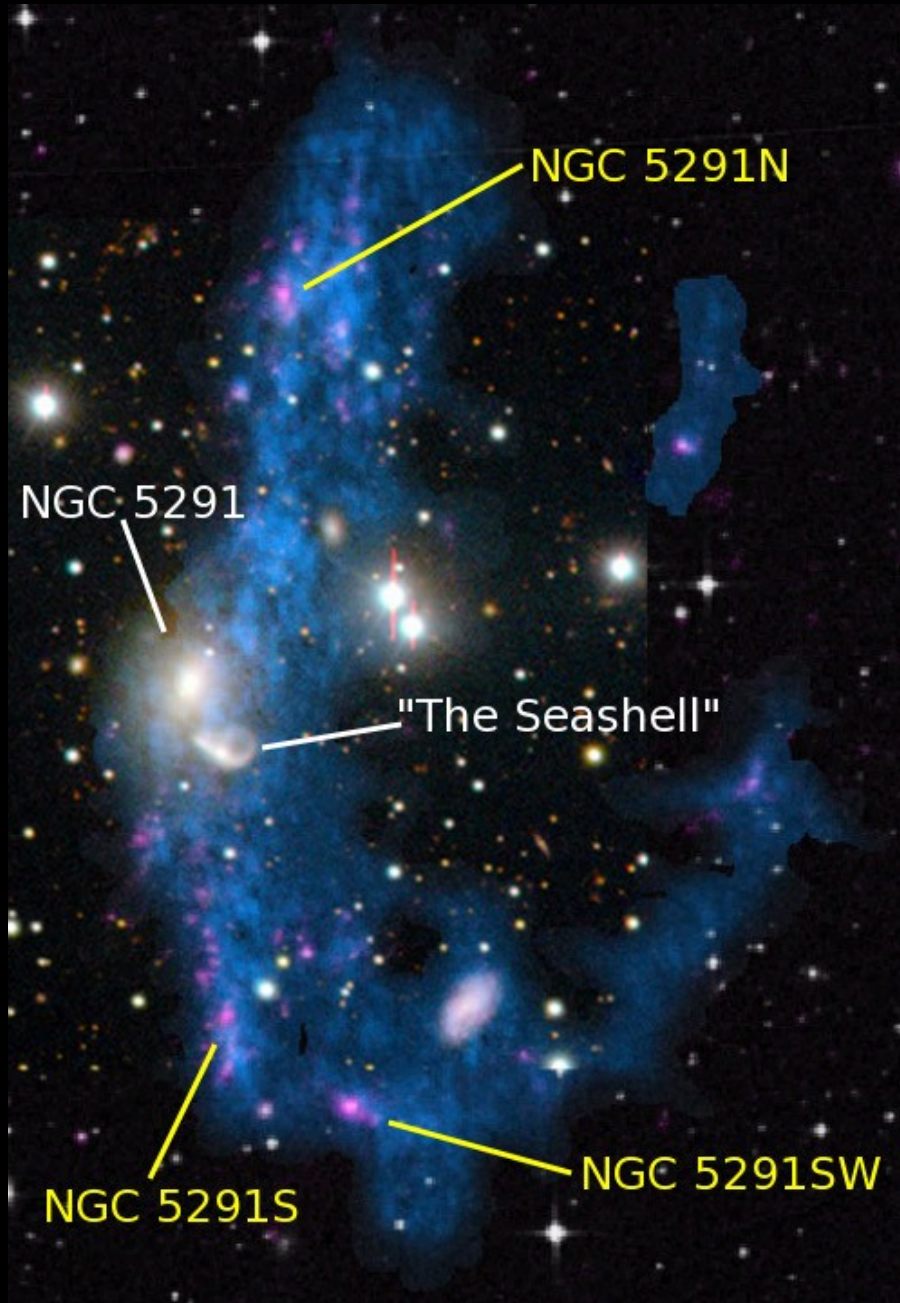
- Small number statistics (Martin+2018; Laporte+2019)
- Debated distance:  **$D \sim 20$  Mpc** (van Dokkum+2018) vs **13 Mpc** (Trujillo+2019)

IF at 20 Mpc, MOND predicts  **$\sigma_v \sim 13 \pm 4$  km/s** considering the EFE (Famaey+2018).

Consistent with **stellar  $\sigma_v \sim 11 \pm 4$**  (Emsellem+2019) and  **$8.5 \pm 3.1$  km/s** (Danieli+2019).

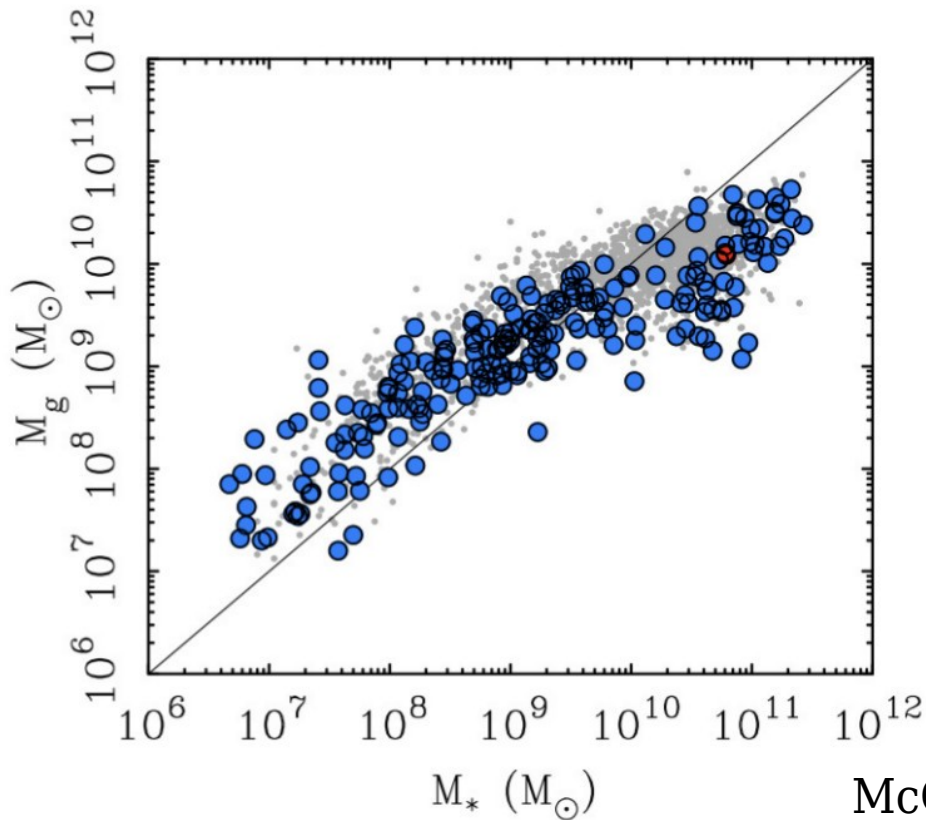


# Tidal Dwarf Galaxies: Off the BTFR?

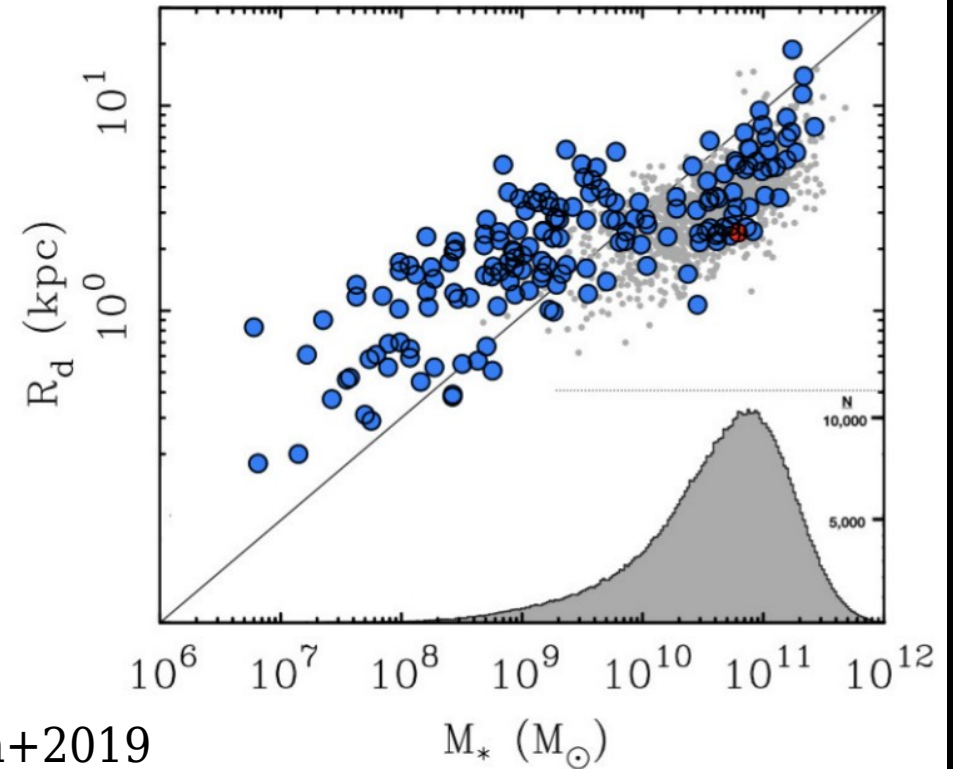


**Caution:** TDGs didn't have time to make a single circular orbit: out of equilibrium?

# SPARC vs larger "complete" samples



McGaugh+2019

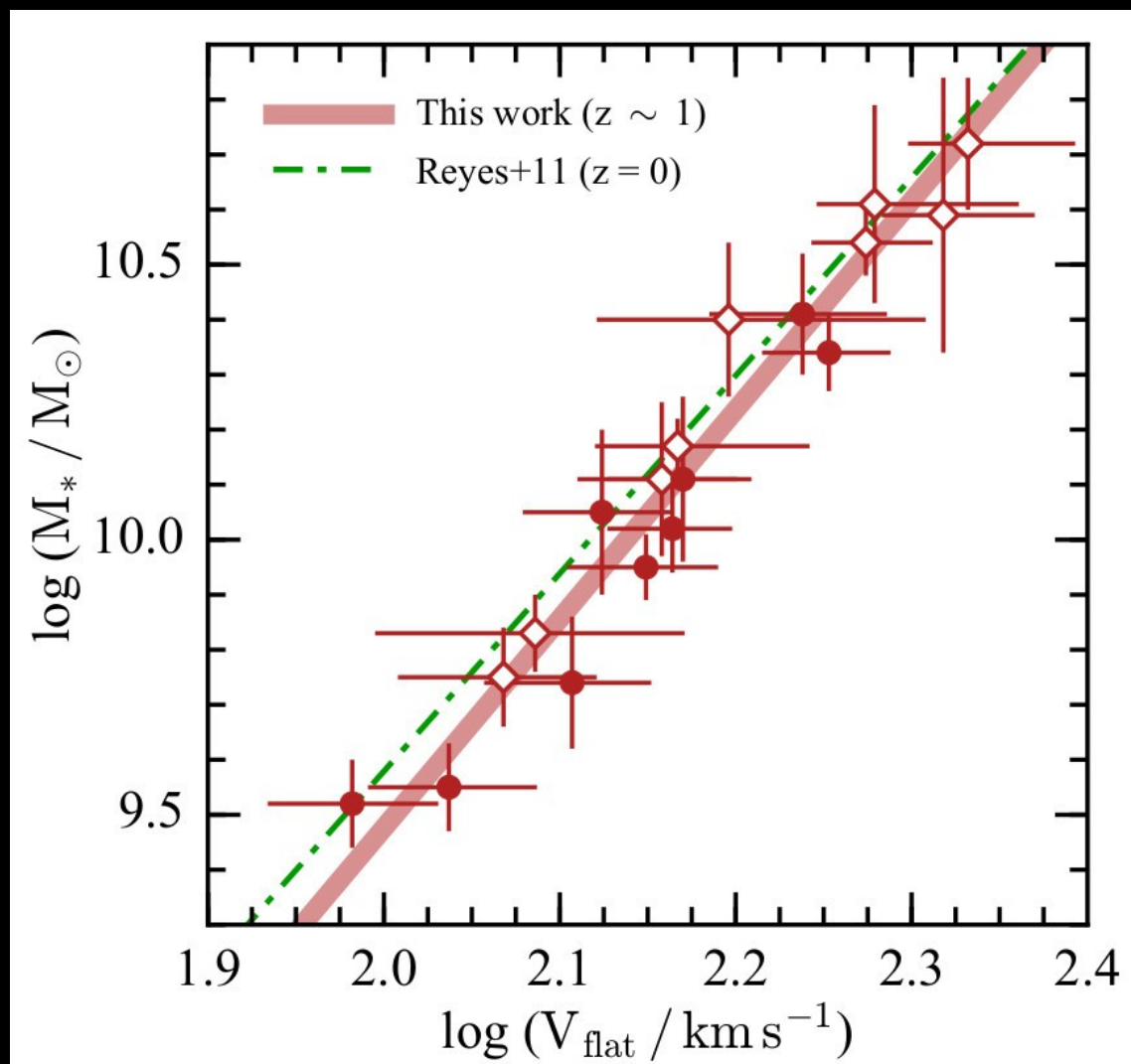
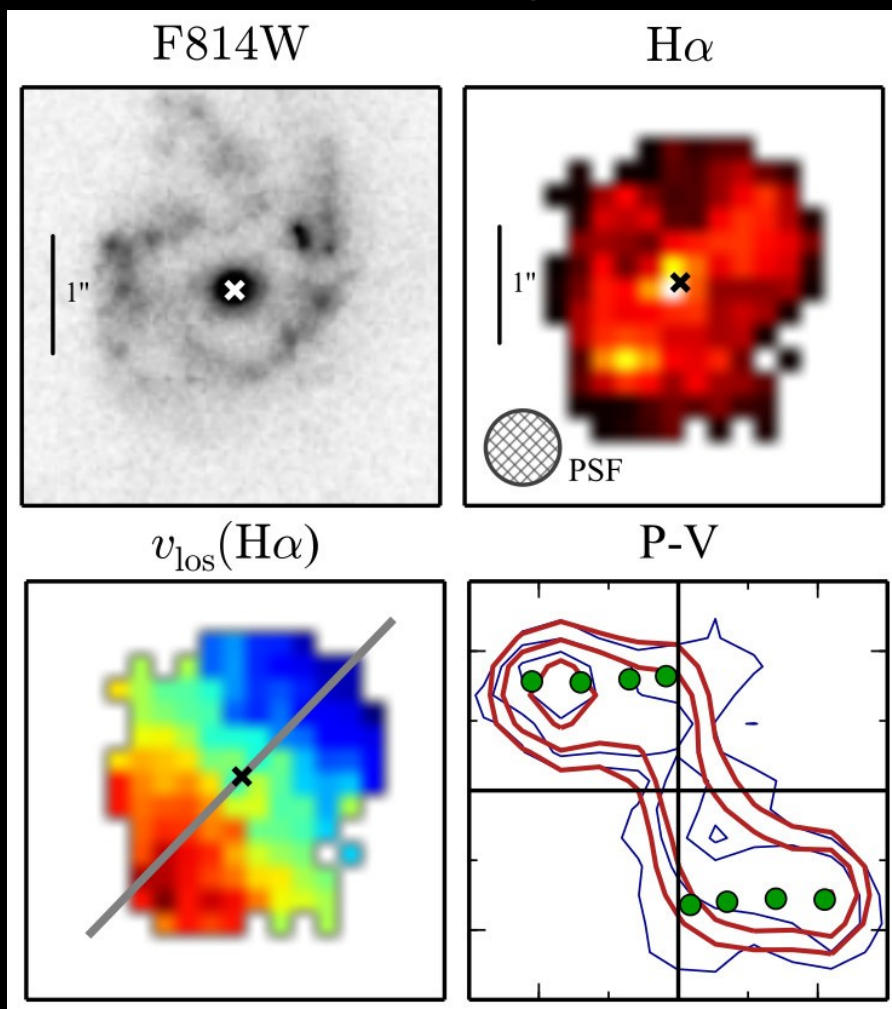


Grey dots: HI-selected data from Bradford+2015 (single-dish survey)

Grey dots:  $H\alpha$ -selected galaxies from Courteau+2007 (long-slit surveys)  
Histogram: all galaxies in the SDSS DR7.

# Does the TF relation evolve with $z$ ?

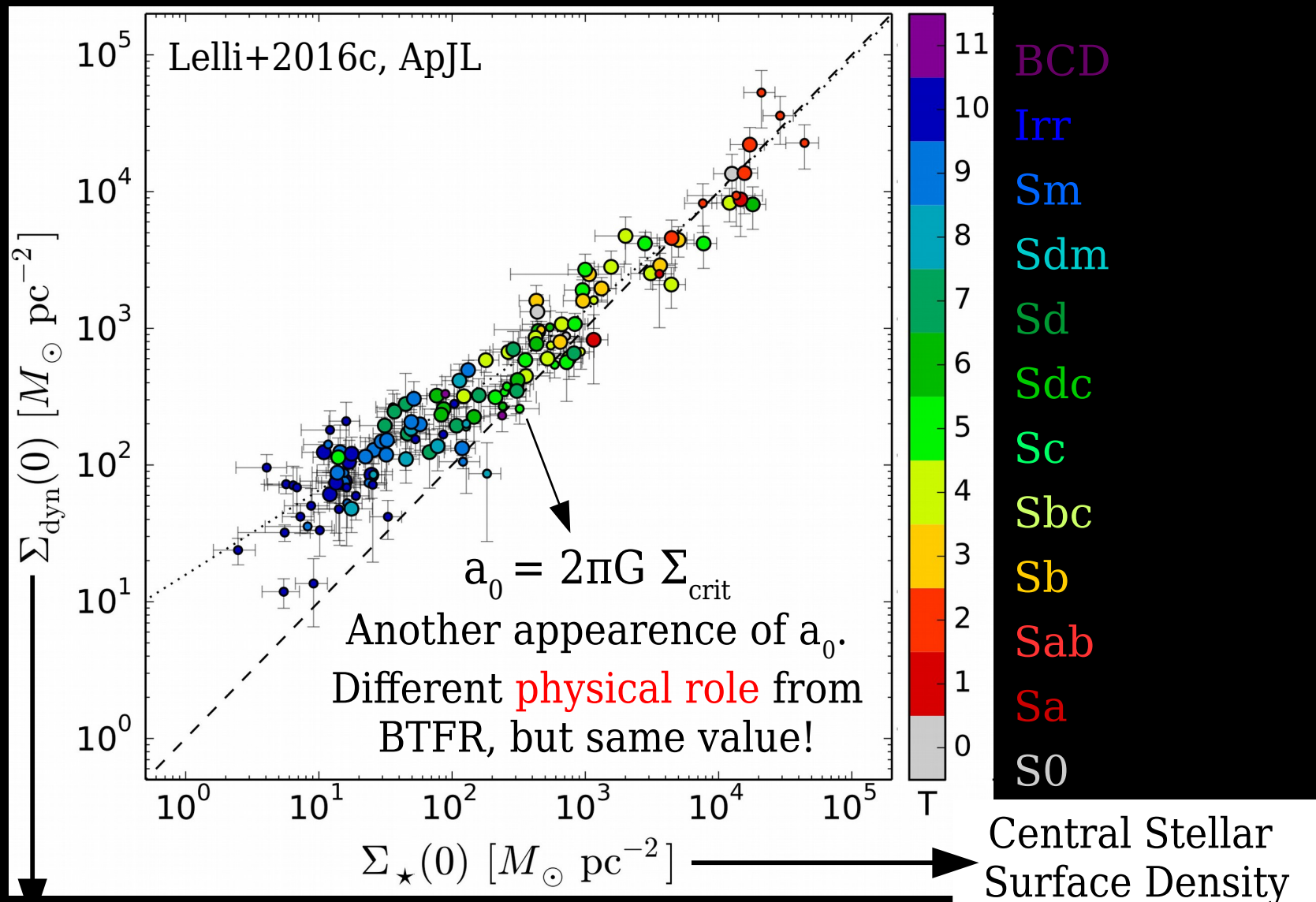
HST + KMOS@VLT data



Very hard and debated measurements but  
consistent with no TFR evolution up to  $z \sim 1$

Di Teodoro+2016

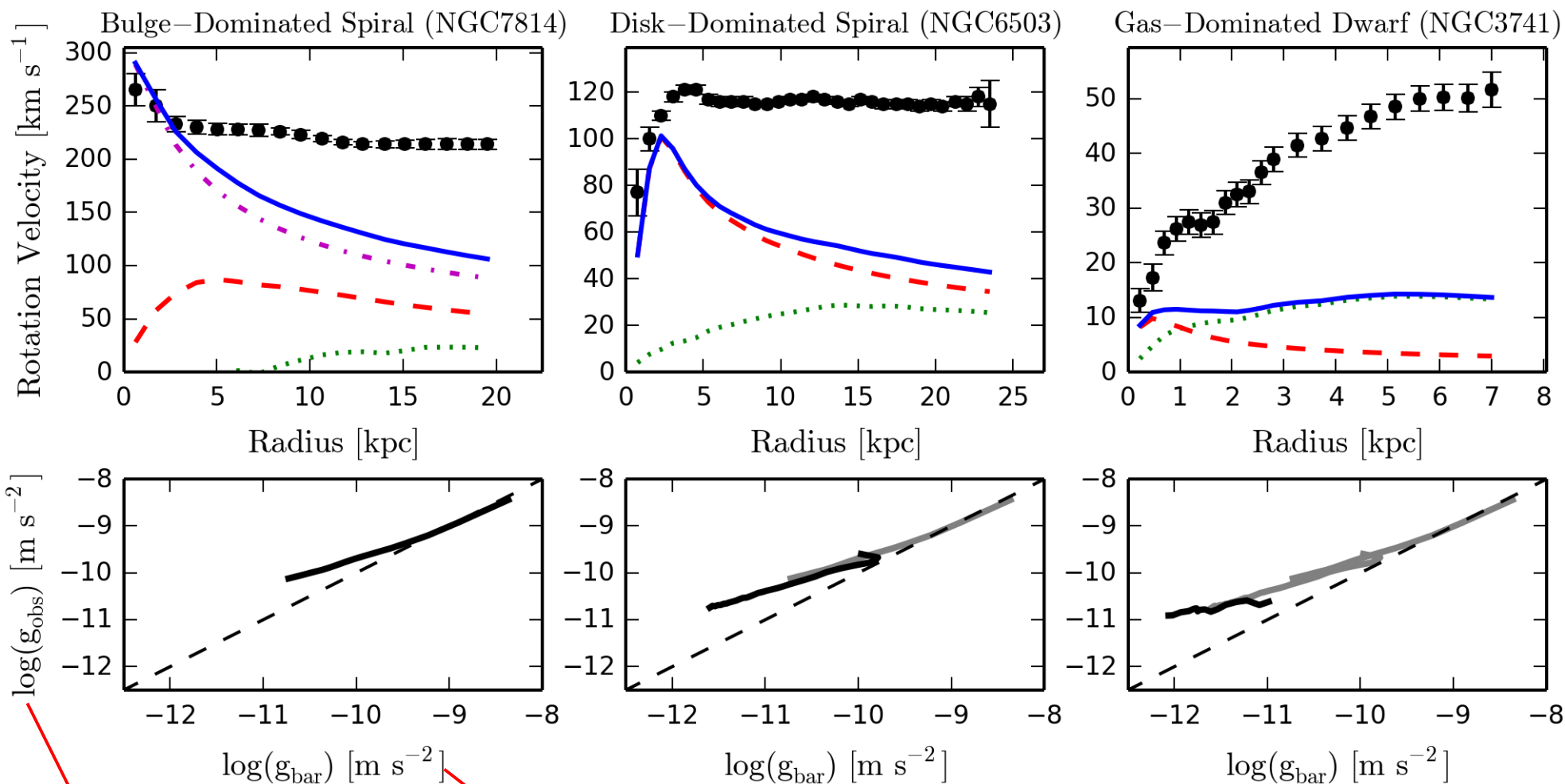
# Central Surface Density Relation



$$\Sigma_{\text{dyn}}(0) = \frac{1}{2\pi G} \int_0^{\infty} \frac{V^2(R)}{R^2} dR,$$

**Toomre (1963):** central dynamical surface density  
→ How fast a rotation curve reaches the flat part

# Very different galaxies but same relation



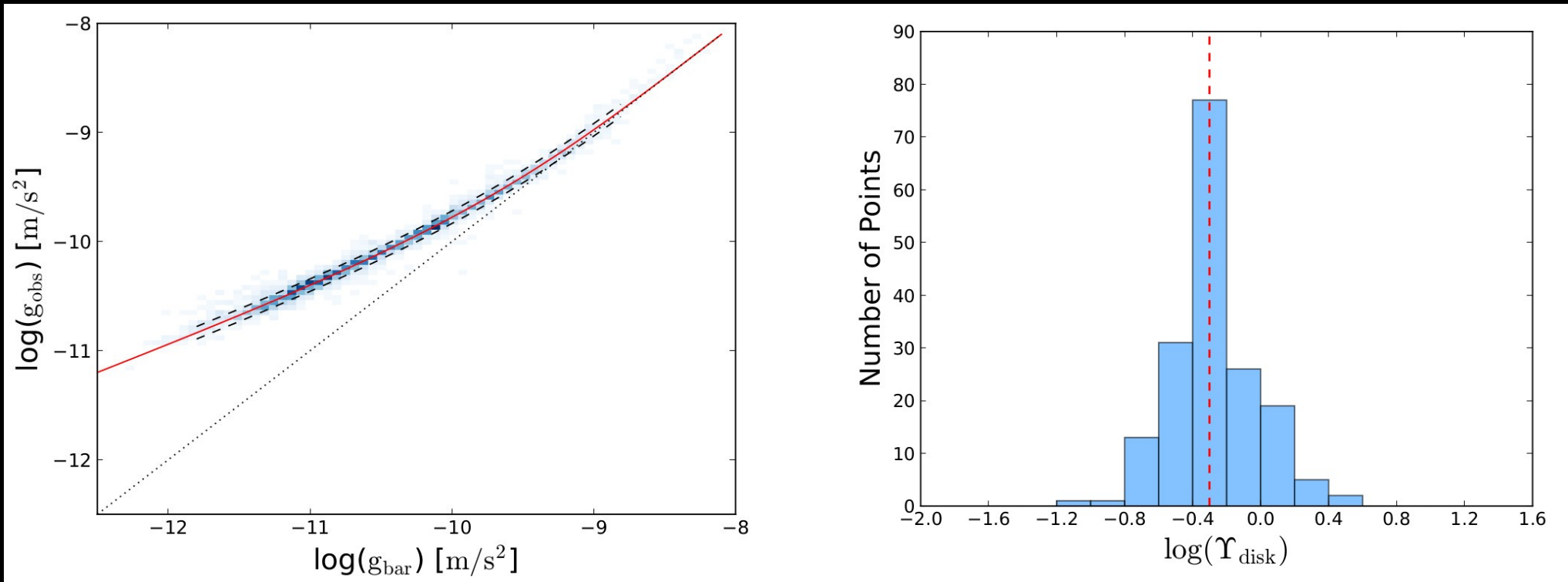
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McGaugh, Lelli, Schombert (2016)

# What is driving the RAR scatter?

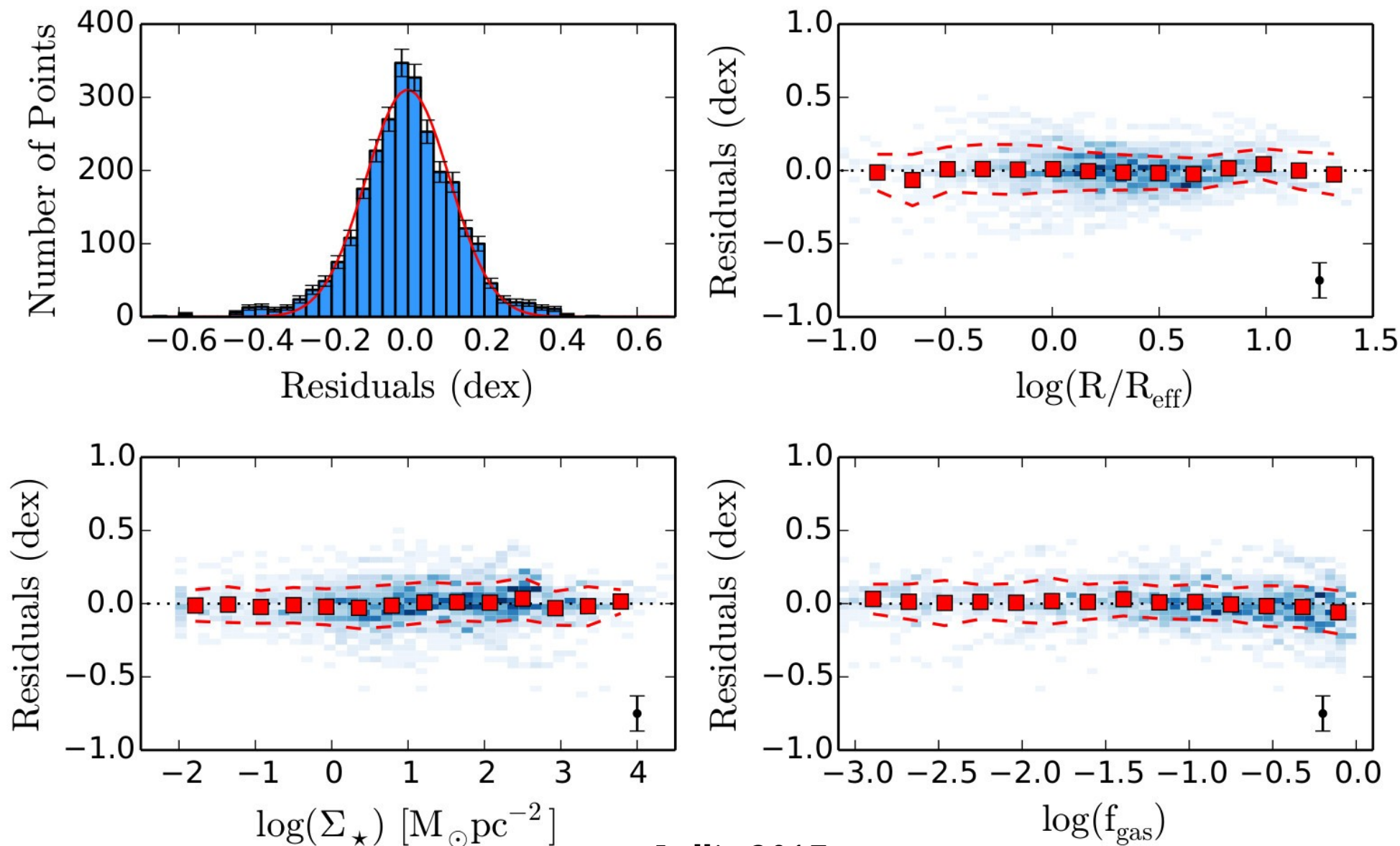


RAR fits to individual galaxies return:

- extremely tight relation (scatter  $\sim 13\%$ )
- sensible distribution of stellar  $M_*/L$
- sensible values of distance and inclination

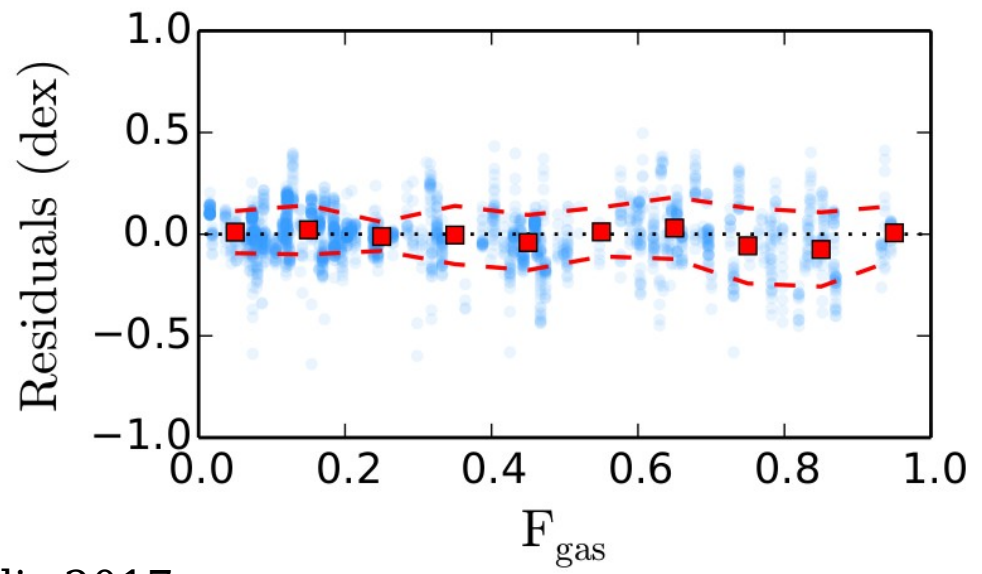
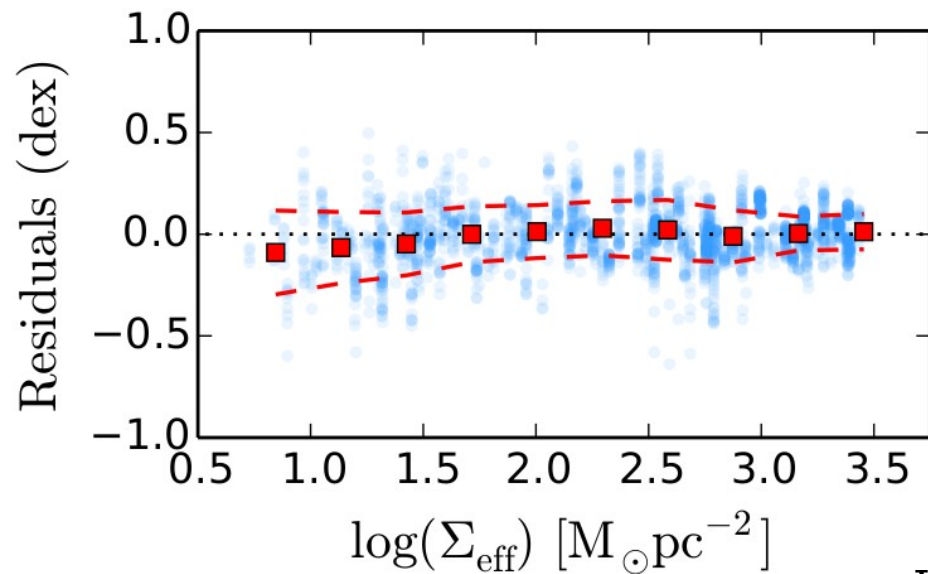
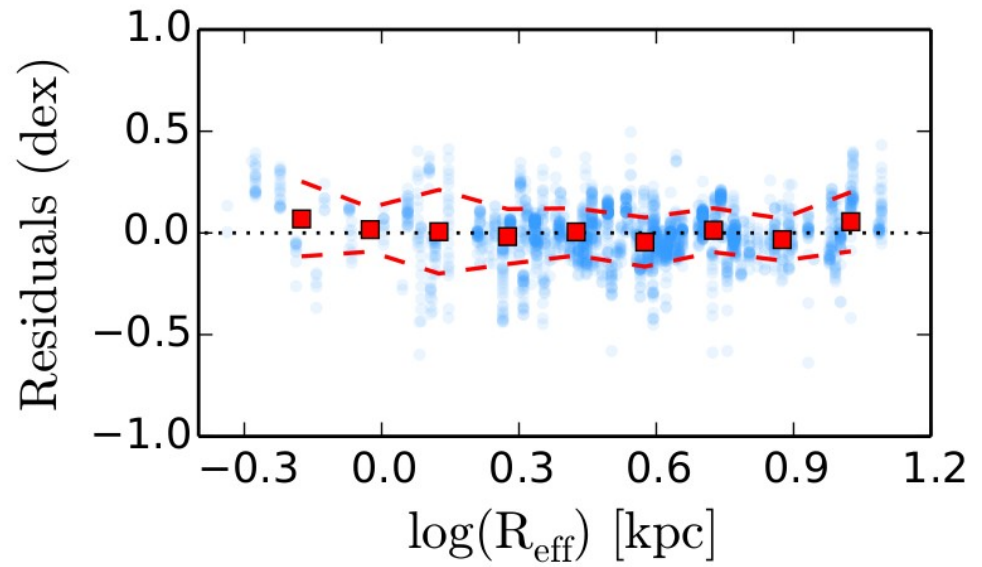
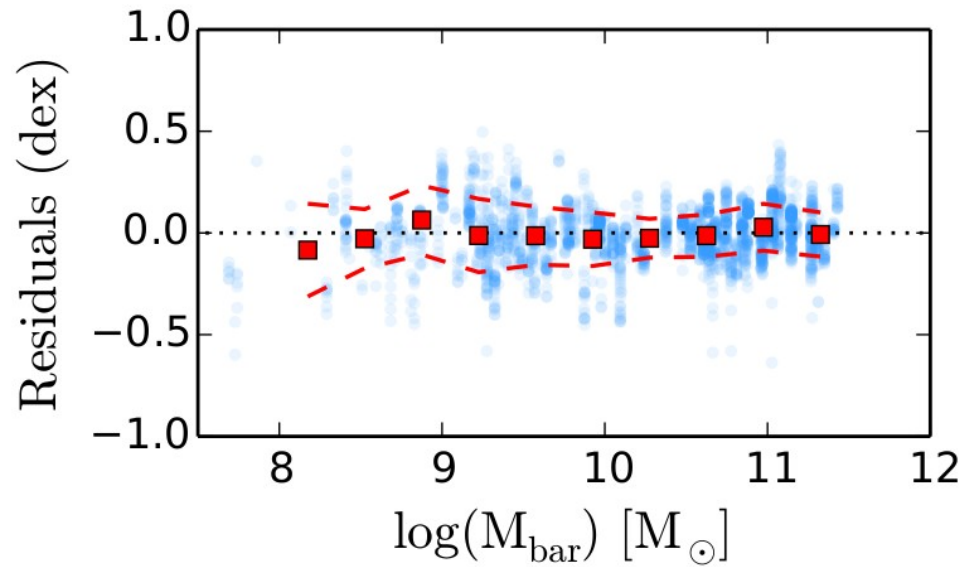
Li, Lelli, McGaugh, Schombert 2018, A&A

# RAR Residuals vs Local Galaxy Properties



Lelli+2017

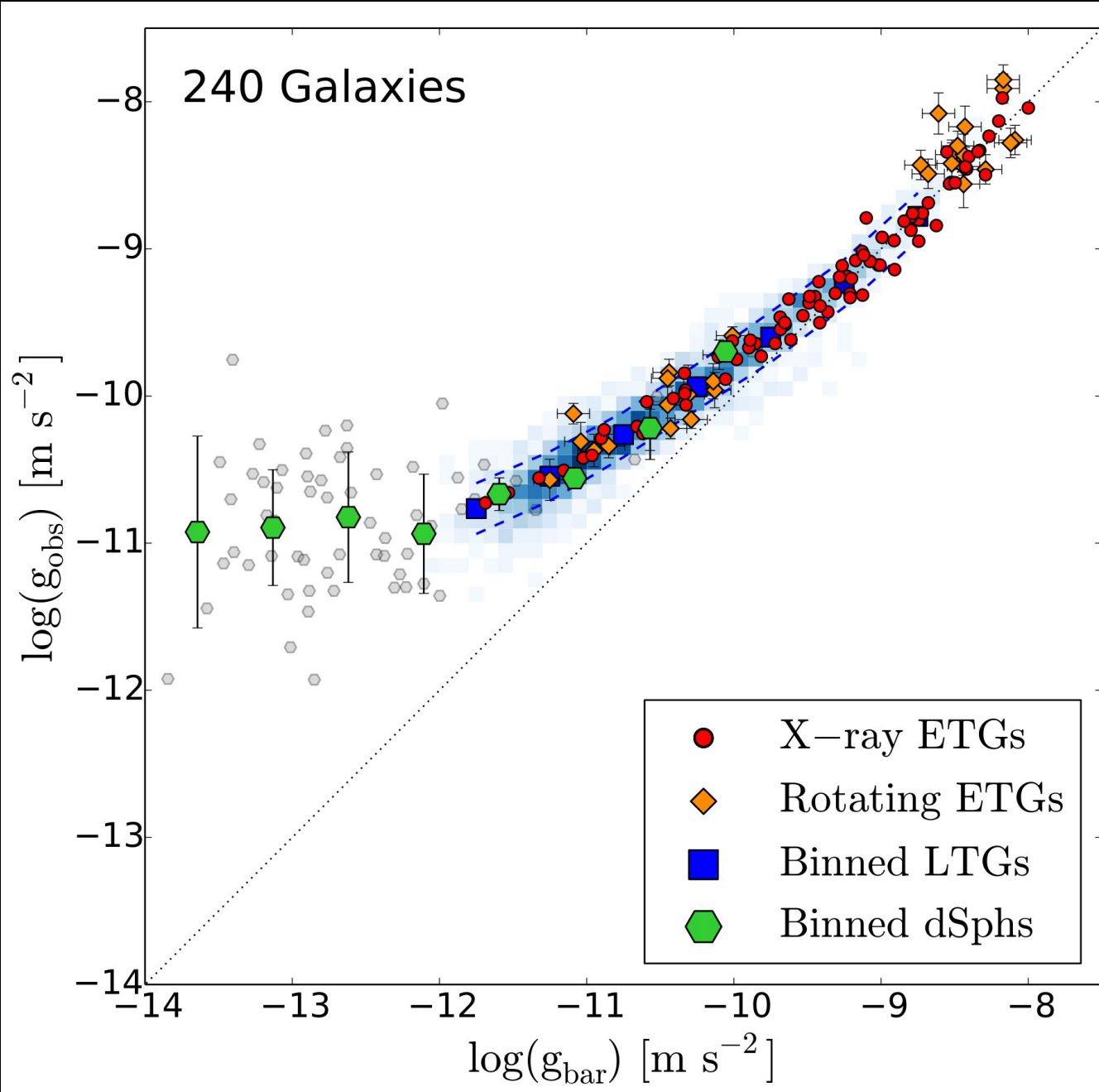
# RAR Residuals vs Global Galaxy Properties



Lelli+2017



# Works for any galaxy type with *good* data



**Giant Ellipticals:**  
with hot X-rays haloes  
in hydrostatic equilibrium  
(Humprey+2006,2009,2012)

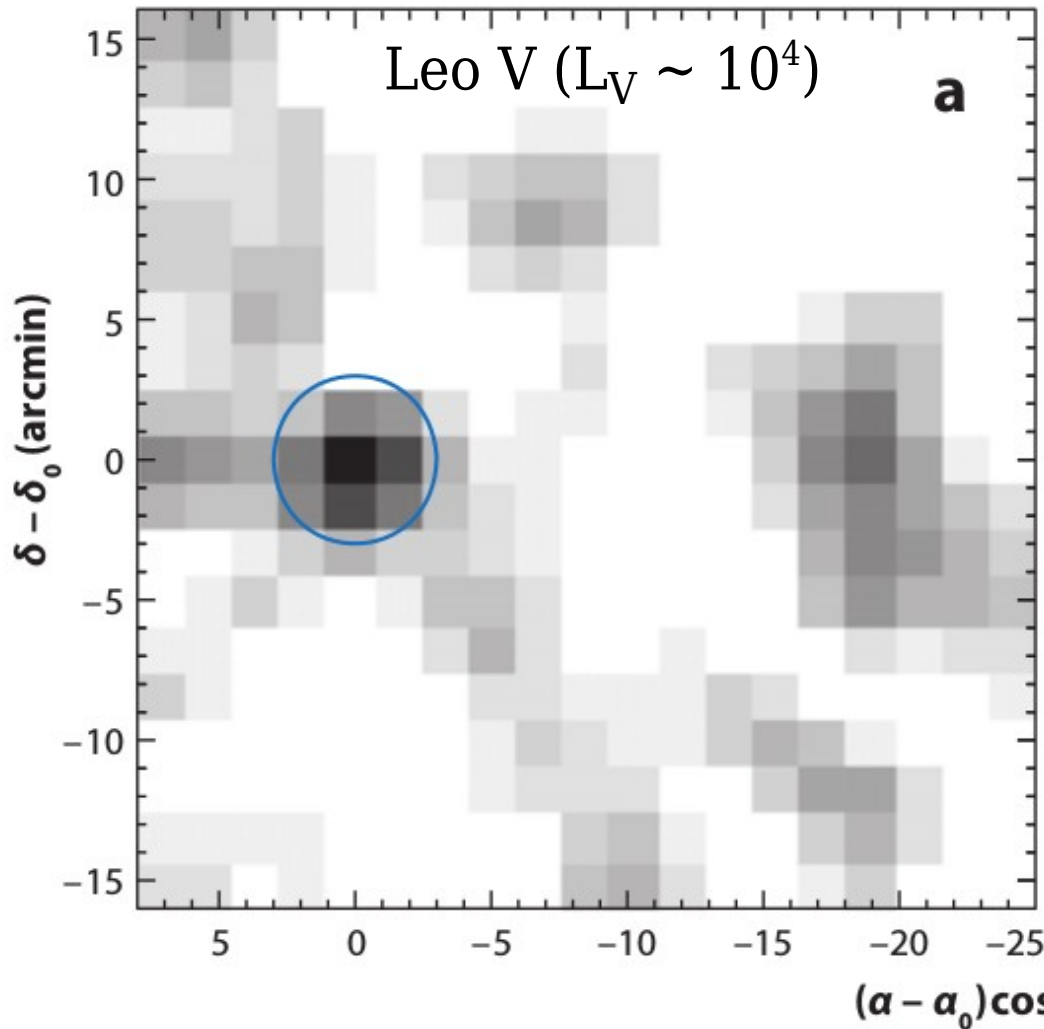
**Disky Es and S0s:**  
with stellar kinematics from  
integral-field spectroscopy  
(Atlas<sup>3D</sup> - Cappellari+2010)

**Dwarf Spheroidals:**  
with high-res spectroscopy  
of individual bright stars  
(many many references...)

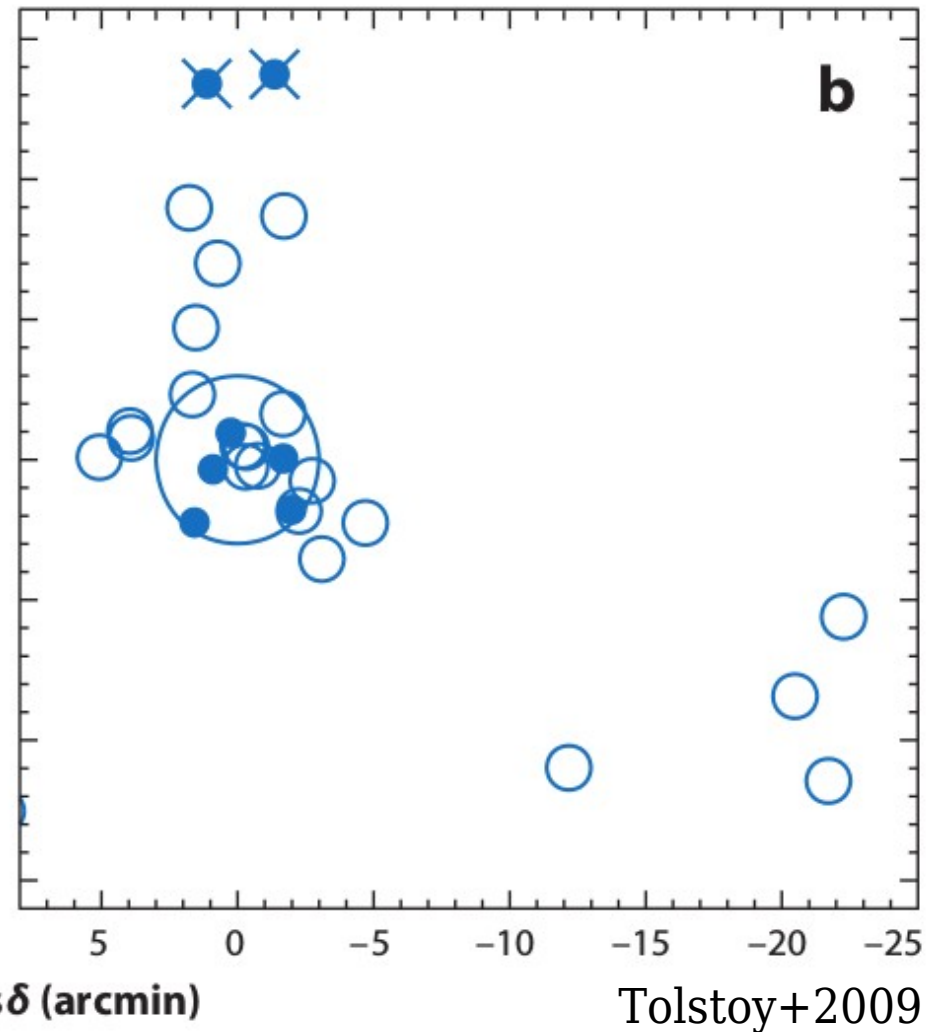
Lelli+2017, ApJ

# Need to be careful with ultra-faint dwarfs

Density of RGB stars



Candidate BHB stars



- $\sigma_v$  often based on  $\sim 5-10$  stars  $\rightarrow$  undetected binaries overestimate  $\sigma_v$
- Tides from MW and M31  $\rightarrow$  out of dynamical equilibrium!