Numerical Information Field Theory
a NIFTy tutorial

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MPI for Astrophysics

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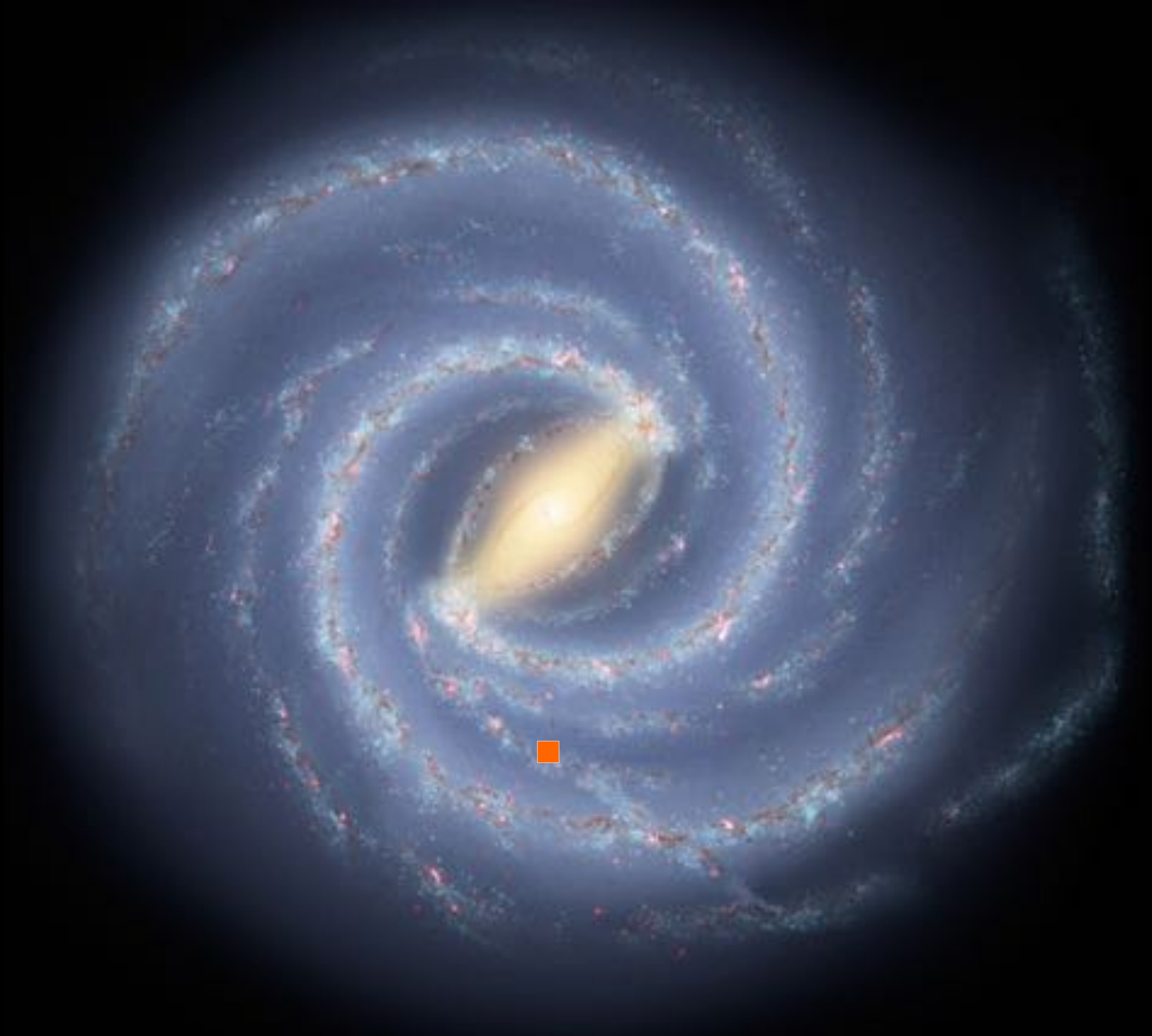


Galactic Tomography

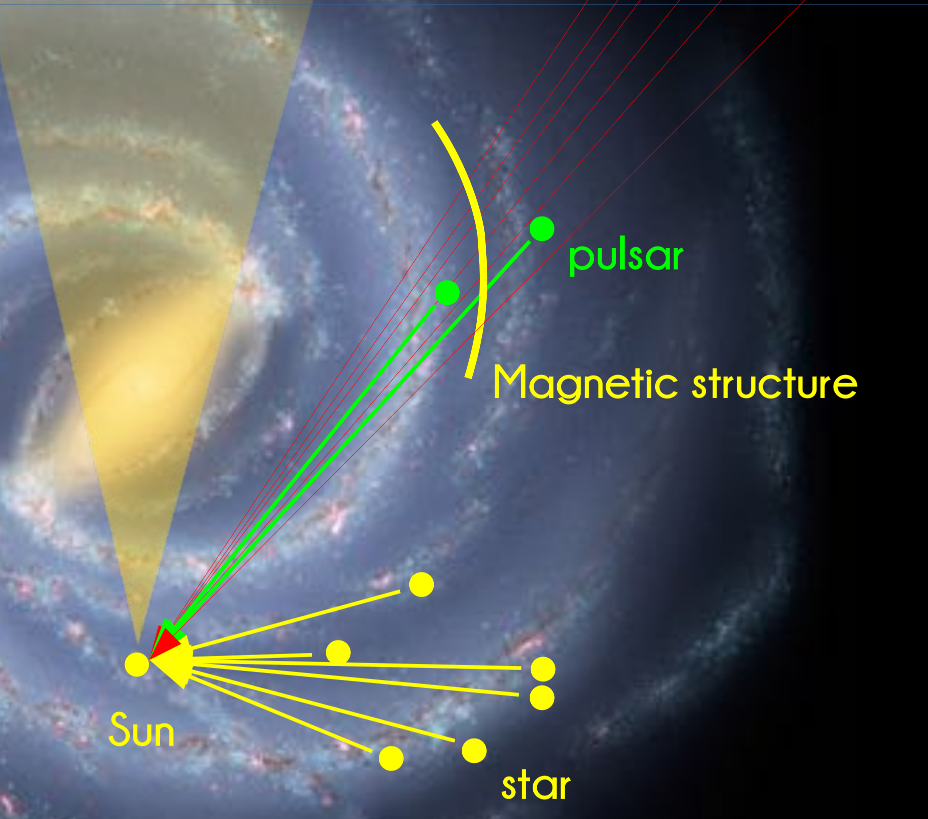
Pulsars:
Dispersion Measure → electron density
Rotation Measure → magnetic field x el. dens.
Scintillation Measure → el. dens. x turbulence

Extragalactic sources:
Rotation Measure → magnetic field x el. dens.
Ultra High Energy Cosmic Rays → mag. fields

Stars:
Dust reddening → dust density & properties
Positions → stellar density & radiation field
Kinematics → gravitational potential

Emission Processes:
Dust emission → dust density & radiation field
Synchrotron → relativistic el. x mag. Fields
Bremsstrahlung → thermal, rel. el. x gas density
Inverse Compton → rel. el. x radiation field
Hadronic interactions → rel. nuclei x gas density
Lines (21 cm, CO, ...) → gas density & kinematics

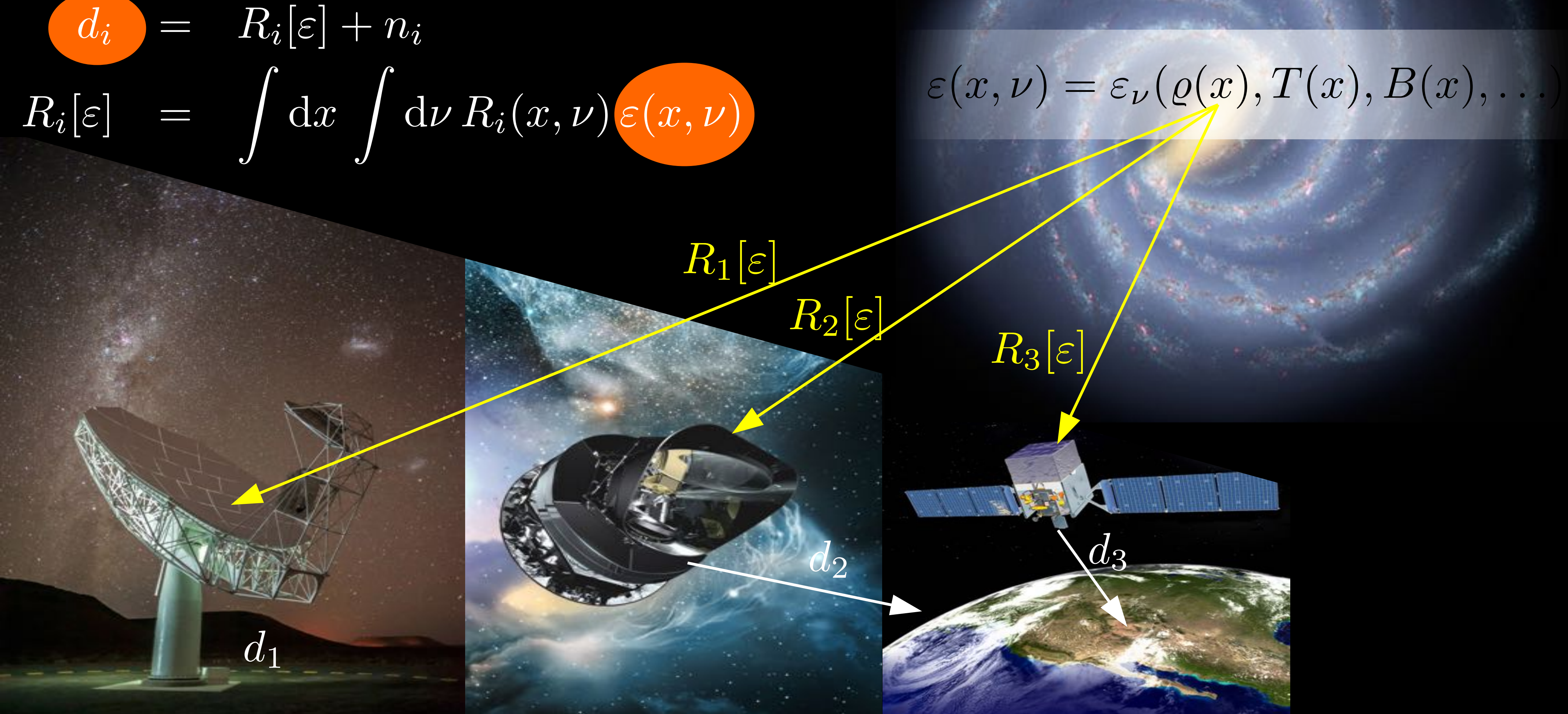
Other information sources:
Correlation structures (auto- & cross-correlations)
Approximate symmetries
Physical laws
Empirical laws, ...



Data Fusion

$$d_i = R_i[\varepsilon] + n_i$$

$$R_i[\varepsilon] = \int \mathrm{d}x \int \mathrm{d}\nu \, R_i(x,\nu)\, \varepsilon(x,\nu)$$



Information theory

$$
\begin{aligned}
\mathcal{P}(s|d) &= \frac{\mathcal{P}(d,s)}{\mathcal{P}(d)} = \frac{e^{-\mathcal{H}(d,s)}}{\mathcal{Z}(d)} \\
\mathcal{H}(d,s) &= -\log \mathcal{P}(d,s) \\
\mathcal{Z}(d) &= \mathcal{P}(d) \\
&= \int \mathcal{D}s\, \mathcal{P}(d,s) \\
\mathcal{P}(d,s) &= \mathcal{P}(d|s)\, \mathcal{P}(s) \\
\mathcal{H}(d_1, d_2, s) &= \mathcal{H}(d_1|s) + \mathcal{H}(d_2|s) + \mathcal{H}(s)
\end{aligned}
$$

Information

is additive

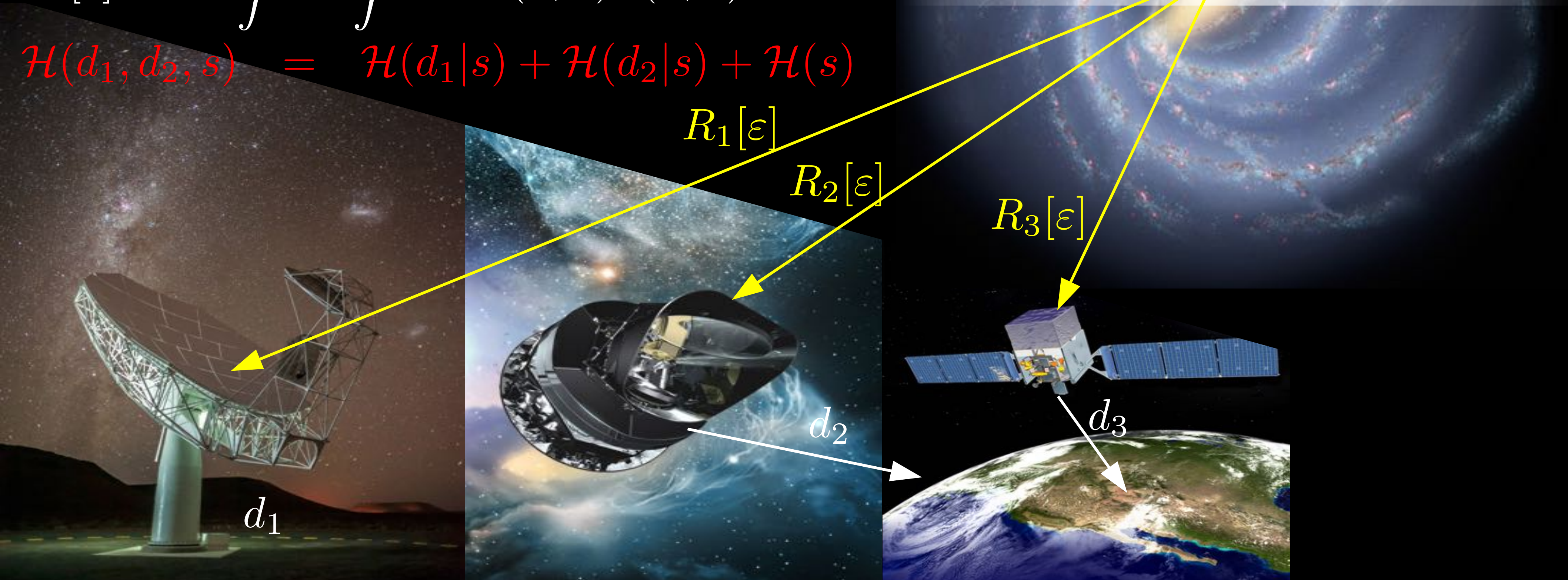
Data Fusion

$$d_i = R_i[\varepsilon] + n_i$$

$$R_i[\varepsilon] = \int \mathrm{d}x \int \mathrm{d}\nu \, R_i(x,\nu)\,\varepsilon(x,\nu)$$

$$\mathcal{H}(d_1, d_2, s) = \mathcal{H}(d_1|s) + \mathcal{H}(d_2|s) + \mathcal{H}(s)$$

$$\varepsilon(x,\nu) = \varepsilon_\nu(\varrho(x), T(x), B(x), \ldots)$$



Probability & Information

$$\mathcal{P}(s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{s^2}{2\sigma^2}}$$

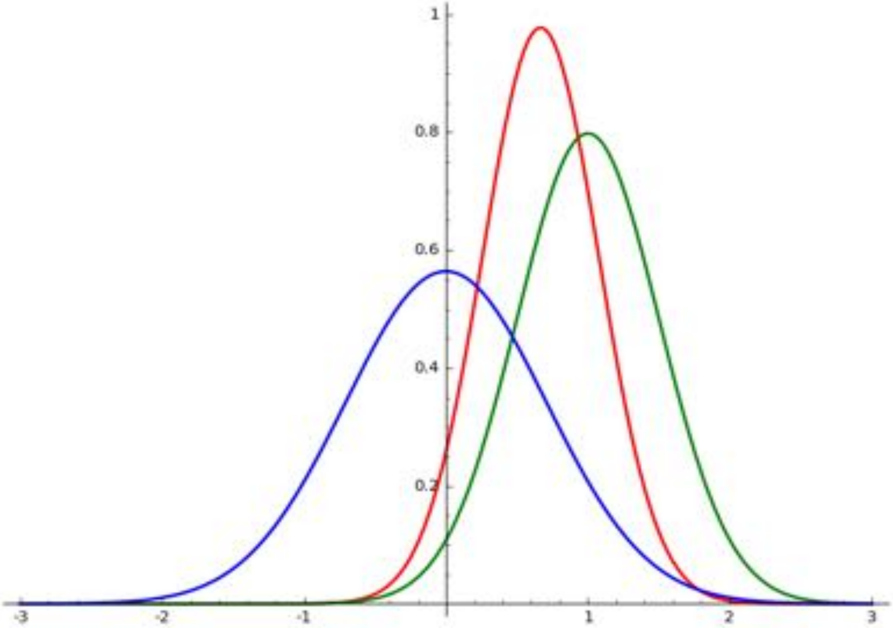
$$\mathcal{P}(d|s) \propto e^{-\frac{(s-d)^2}{2\sigma'^2}}$$

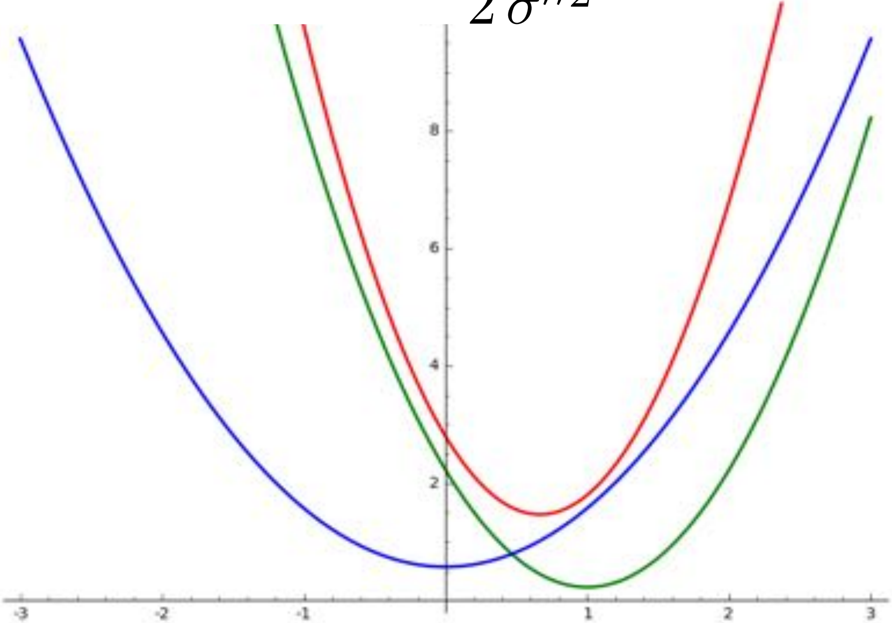
$$\mathcal{P}(s|d) \propto e^{-\frac{(s-m)^2}{2\sigma''^2}}$$

$$\mathcal{H}(s) \widehat{=} \frac{s^2}{2\,\sigma^2}$$

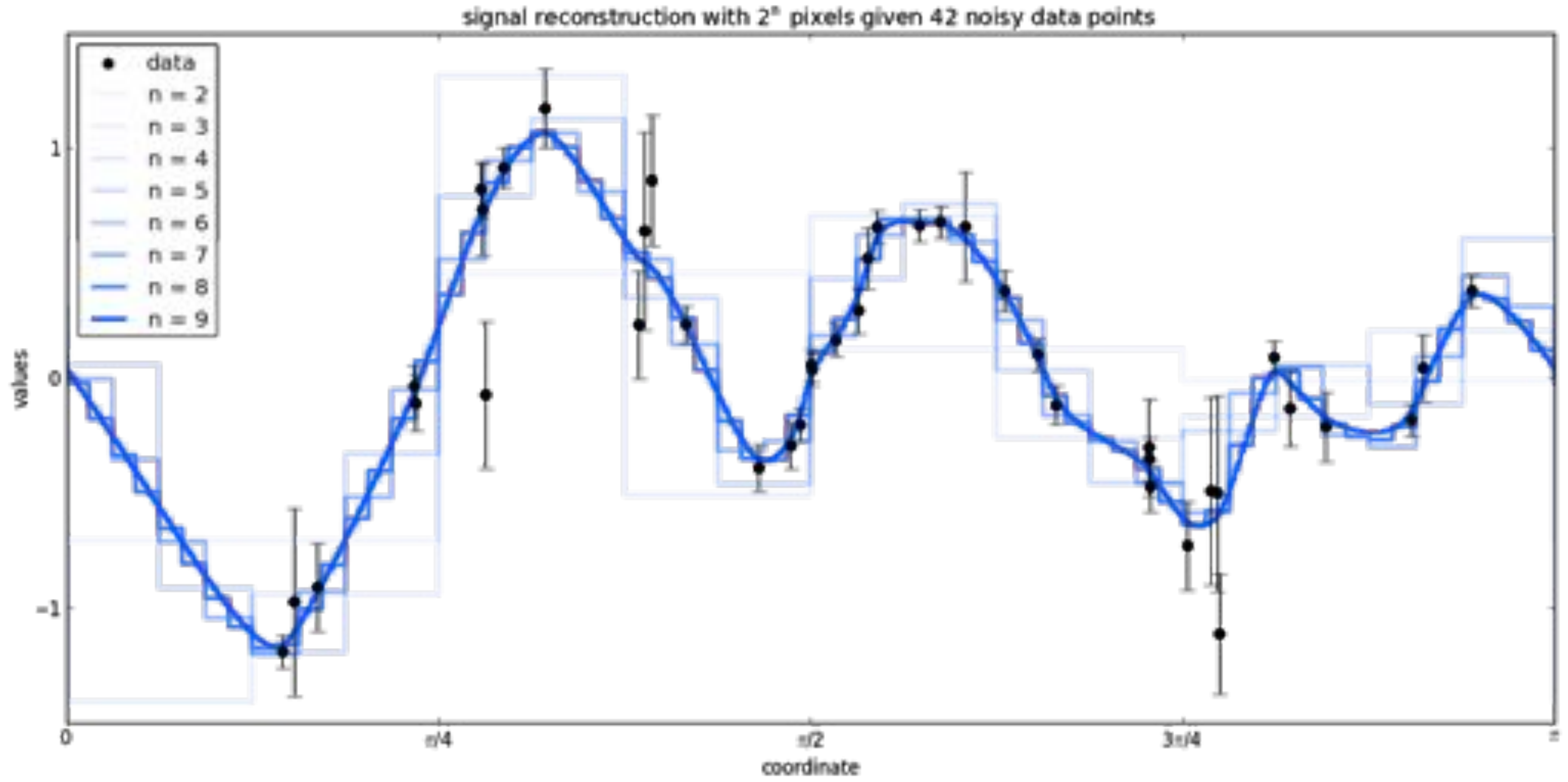
$$\mathcal{H}(d|s) \widehat{=} \frac{(s-d)^2}{2\,\sigma'^2}$$

$$\mathcal{H}(d,s) \widehat{=} \frac{(s-m)^2}{2\,\sigma''^2}$$






Correlations

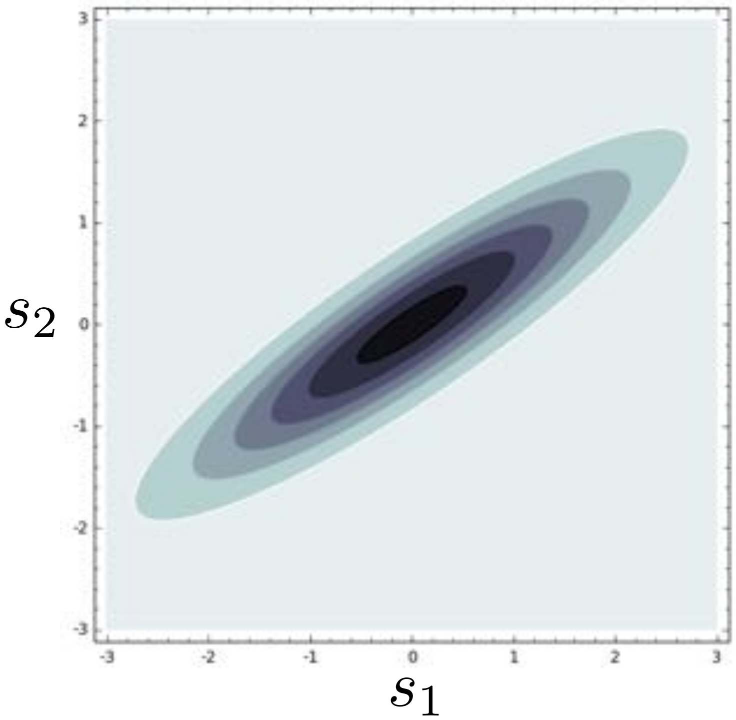


signal reconstruction with 2ⁿ pixels given 42 noisy data points
data
n = 2
n = 3
n = 4
n = 5
n = 6
n = 7
n = 8
n = 9
values
1
0
−1
0
π/4
π/2
3π/4
π
coordinate

Correlations

$\mathcal{P}(s)$

$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$



Correlations

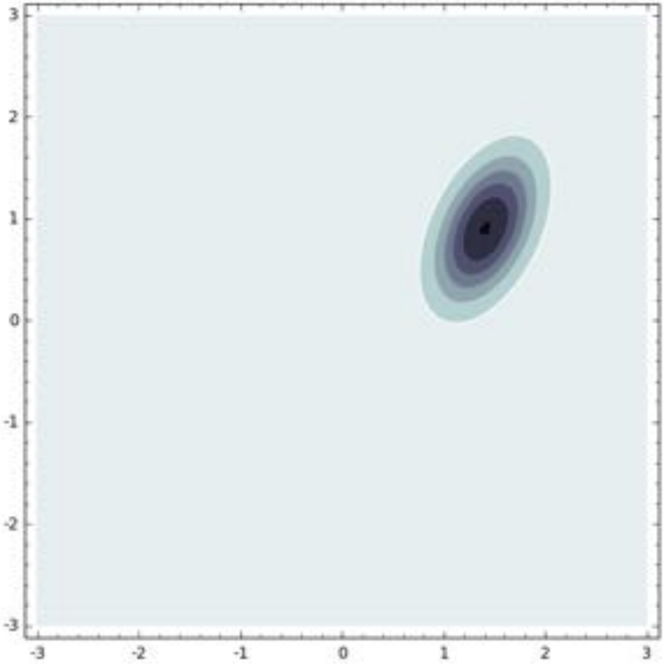
$\mathcal{P}(s|d)$

$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

$\mathcal{P}(d|s)$

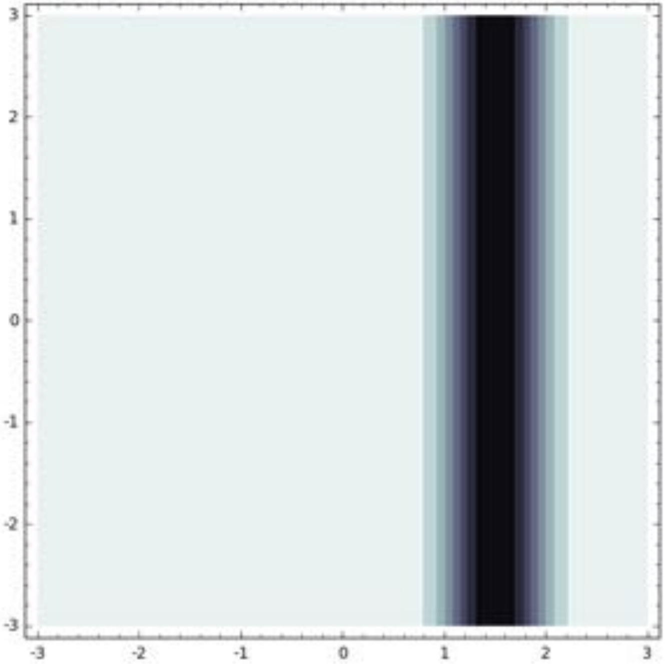
$$d = s_1 + n$$

s_2



s_1

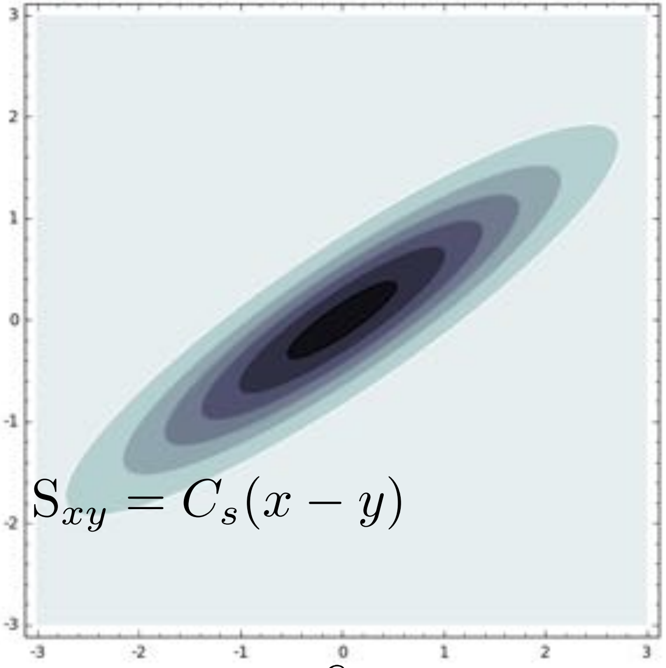
s_2



s_1

Correlations

$$\mathcal{P}(s) \qquad s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

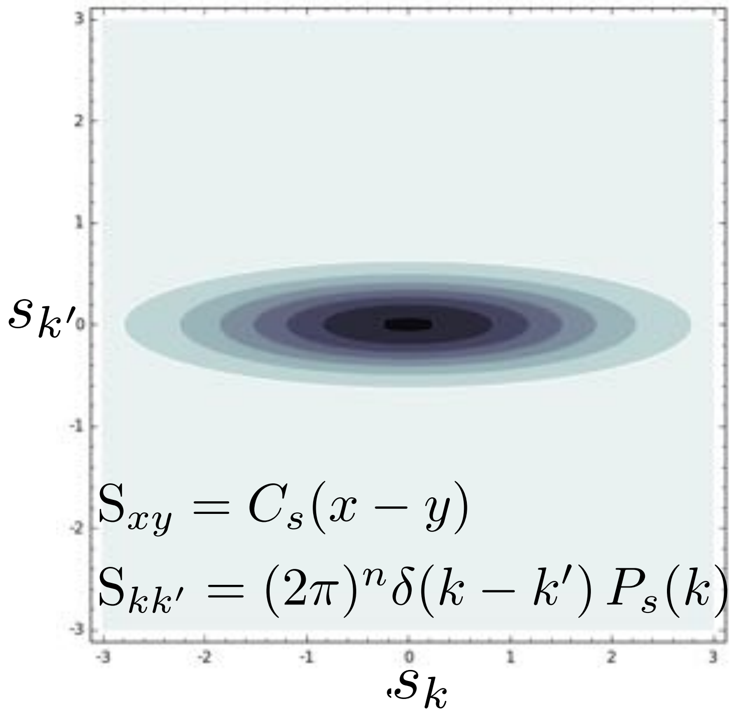


$$\begin{aligned}
\mathcal{P}(s) &= \mathcal{G}(s, S) \\
&= \frac{1}{\sqrt{|2\pi S|}} \exp\left(-\frac{1}{2} s^\dagger S^{-1} s\right) \\
S &= \begin{pmatrix} \langle s_1 s_1 \rangle & \langle s_1 s_2 \rangle \\ \langle s_2 s_1 \rangle & \langle s_2 s_2 \rangle \end{pmatrix} \quad \text{2-dim.} \\
S_{ij} &= \langle s_i s_j \rangle \qquad n\text{-dim.} \\
S_{xy} &= \langle s_x s_y \rangle,\ x \in \mathbb{R}^n \qquad \infty\text{-dim.}
\end{aligned}$$

Correlations

$$\mathcal{P}(s)$$

$$s = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$



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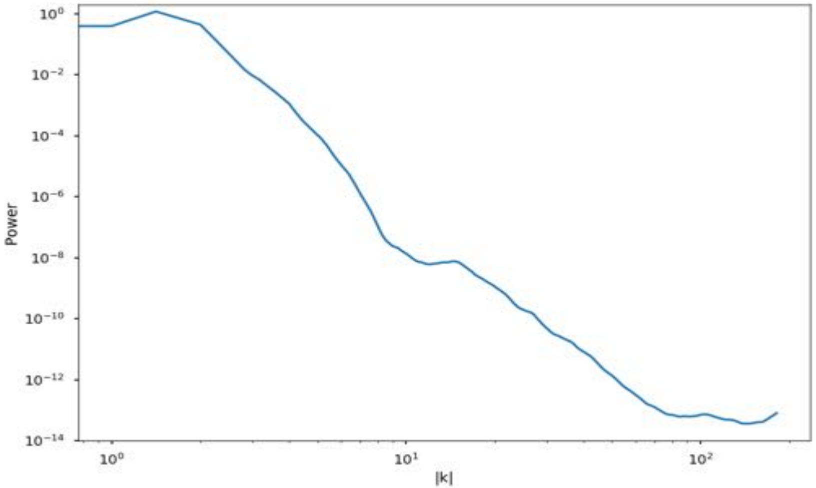
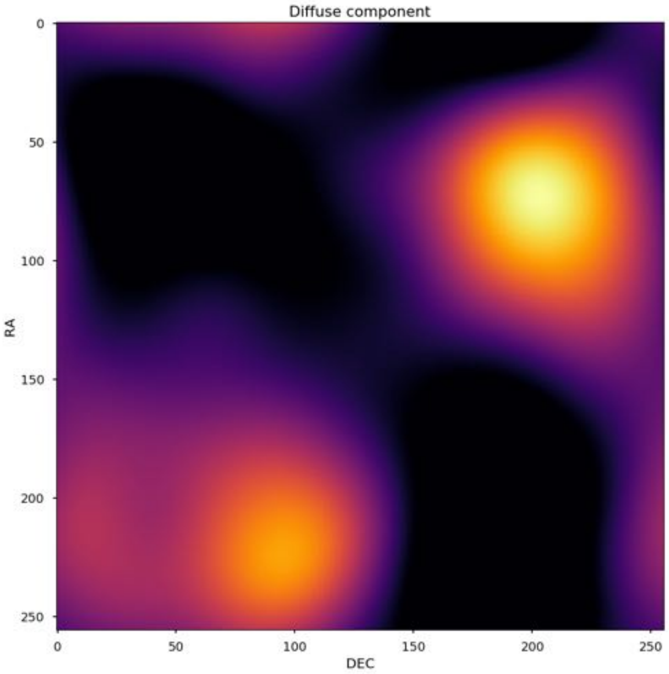
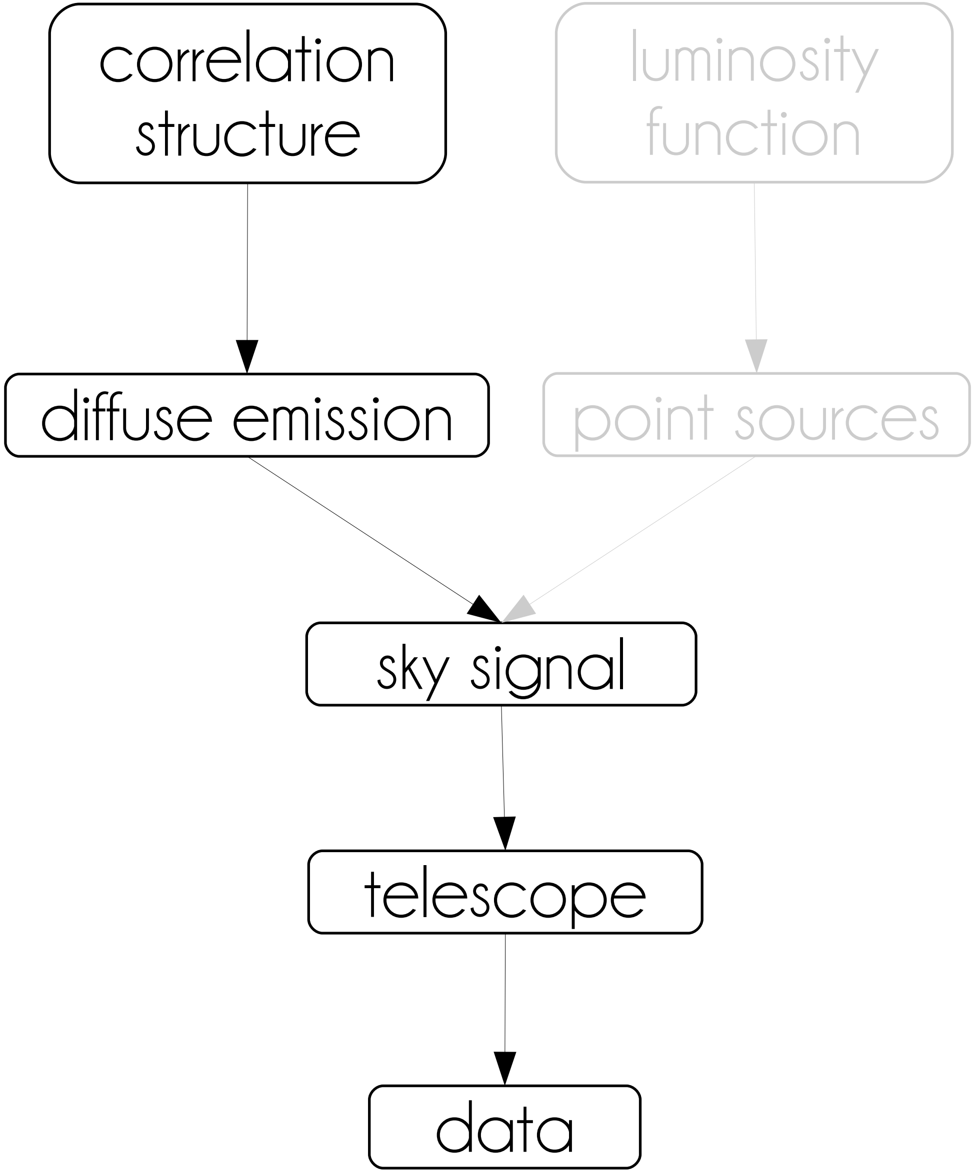
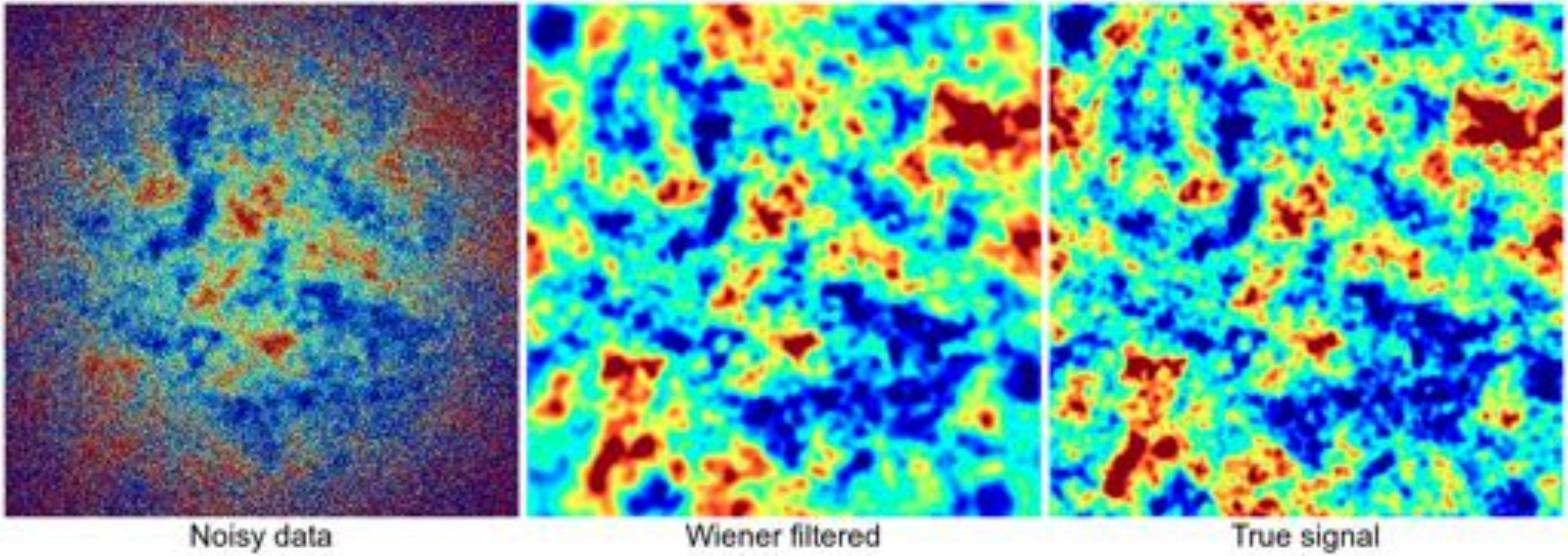

Power
|k|


Diffuse component
RA
DEC


correlation structure
luminosity function
diffuse emission
point sources
sky signal
telescope
data

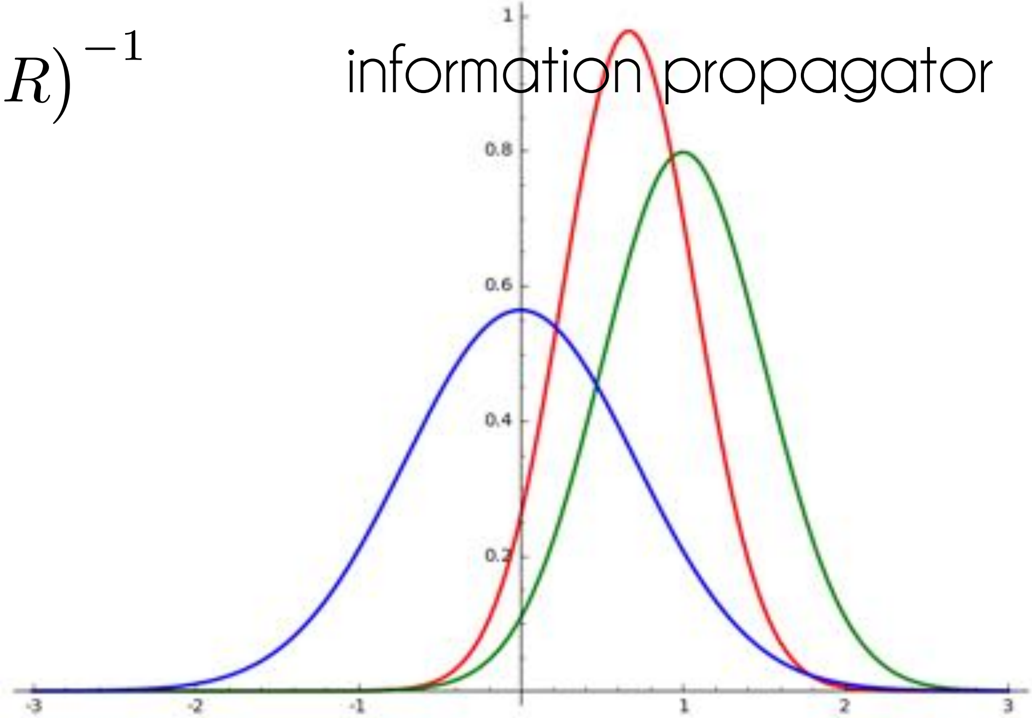
Wiener Filter



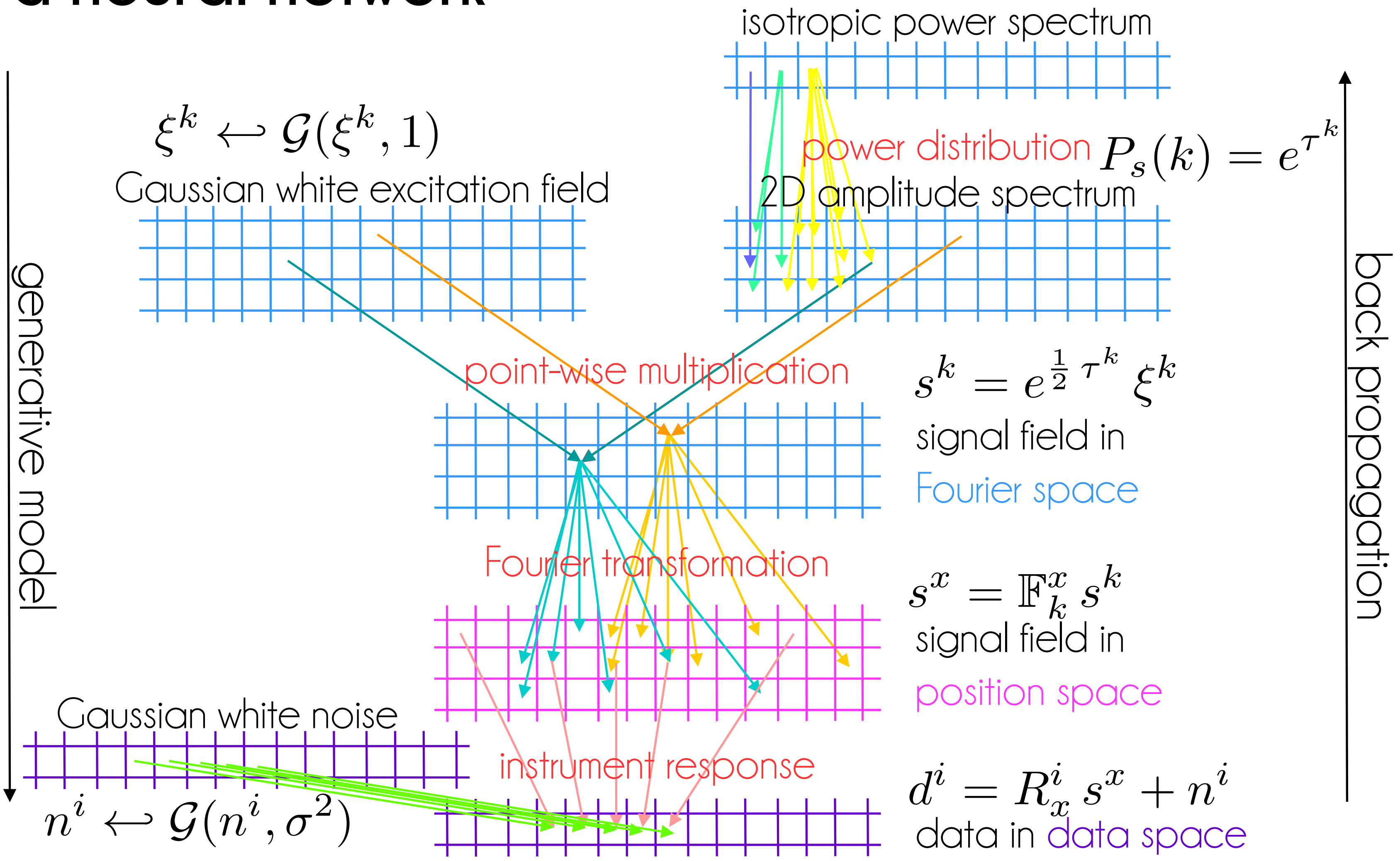
https://en.wikipedia.org/wiki/Generalized_Wiener_filter

$$
\begin{aligned}
d &= R\,s + n && \text{data} \\
\mathcal{P}(d, s|R, S, N) &= \mathcal{G}(s, S)\,\mathcal{G}(d - R\,s, N) && \text{prior \& likelihood} \\
\mathcal{P}(s|d, R, S, N) &= \mathcal{G}(s - m, D) && \text{posterior} \\
\mathcal{H}(d, s|R, S, N) &\widehat{=} \frac{1}{2}\, s^\dagger S^{-1} s + \frac{1}{2}\,(d - R\,s)^\dagger N^{-1}(d - R\,s) \\
&\widehat{=} \frac{1}{2}\,[s^\dagger \underbrace{\left(S^{-1} + R^\dagger N^{-1} R\right)}_{=D^{-1}} s + s\,\underbrace{R^\dagger N^{-1} d}_{=j} + \underbrace{d^\dagger N^{-1} R}_{=j^\dagger}\,s] \\
&= \frac{1}{2}\,[s^\dagger D^{-1} s + s^\dagger j + j^\dagger s] \\
&= \frac{1}{2}\,[s^\dagger D^{-1} s + s^\dagger D^{-1}\,\underbrace{D\,j}_{=m} + j^\dagger D\,D^{-1} s] \\
&\widehat{=} \frac{1}{2}\,[(s - m)^\dagger D^{-1}(s - m)]
\end{aligned}
$$

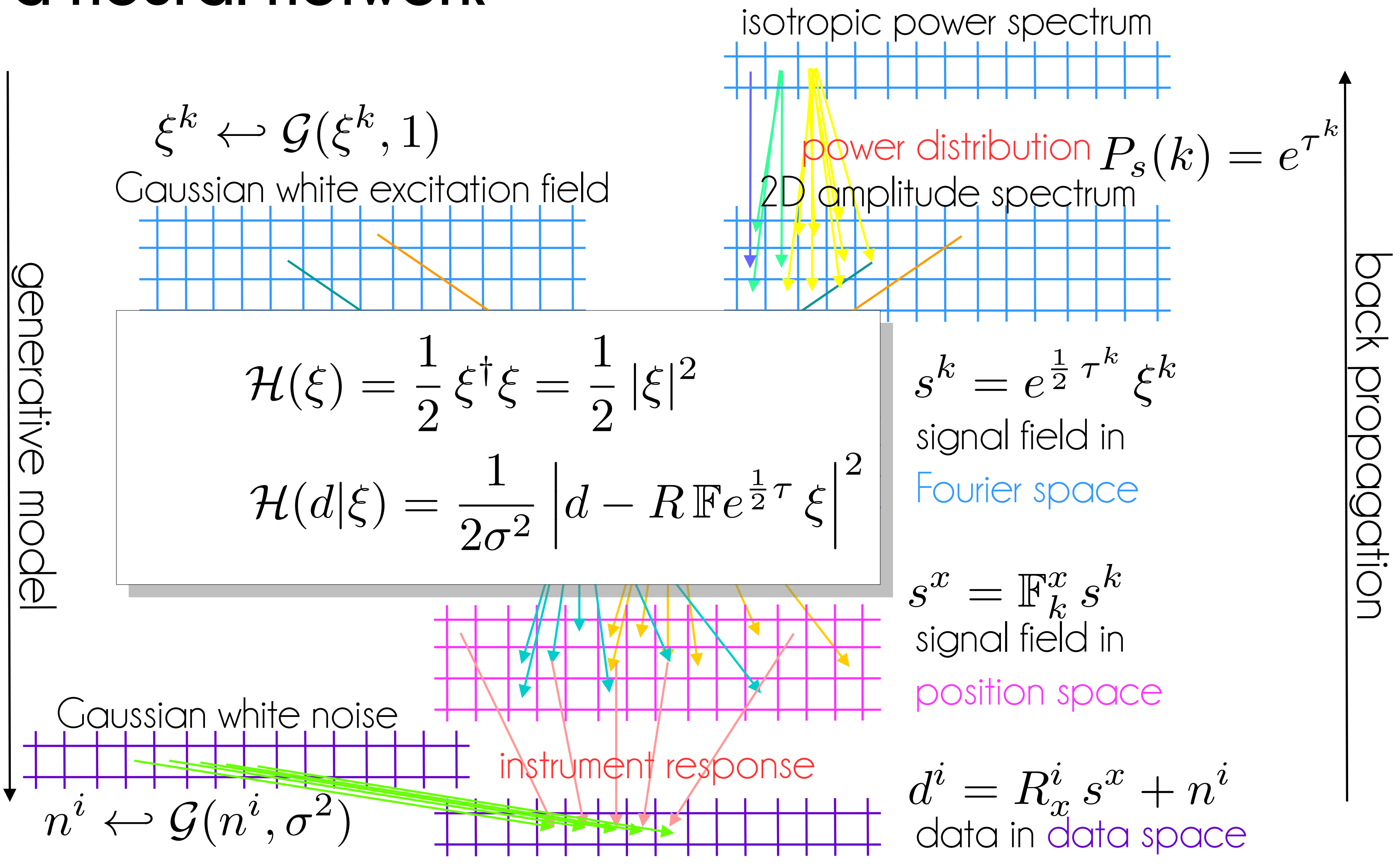
$$
\begin{aligned}
d &= R\,s + n && \text{data} \\
\mathcal{P}(d, s|R, S, N) &= \mathcal{G}(s, S)\,\mathcal{G}(d - R\,s, N) && \text{prior \& likelihood} \\
\mathcal{P}(s|d, R, S, N) &= \mathcal{G}(s - m, D) && \text{posterior} \\
m &= D\,j && \text{posterior mean} \\
j &= R^{\dagger} N^{-1} d && \text{information source} \\
D &= \left(S^{-1} + R^{\dagger} N^{-1} R\right)^{-1} && \text{information propagator}
\end{aligned}
$$



IFT as a neural network

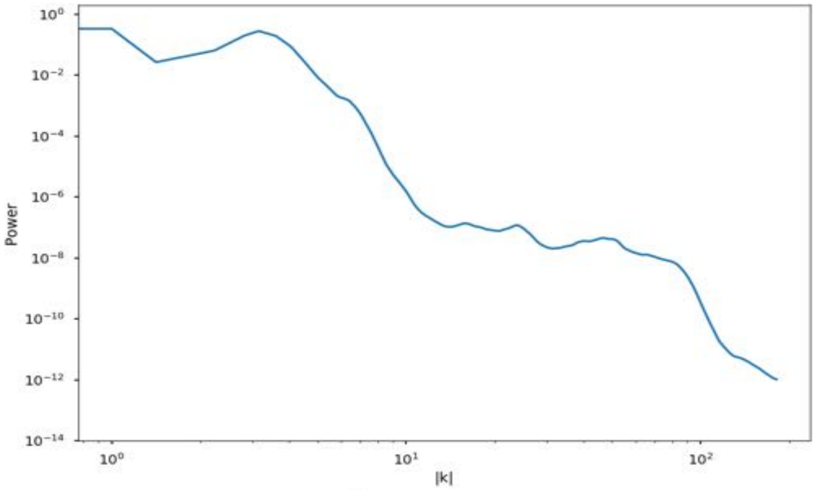


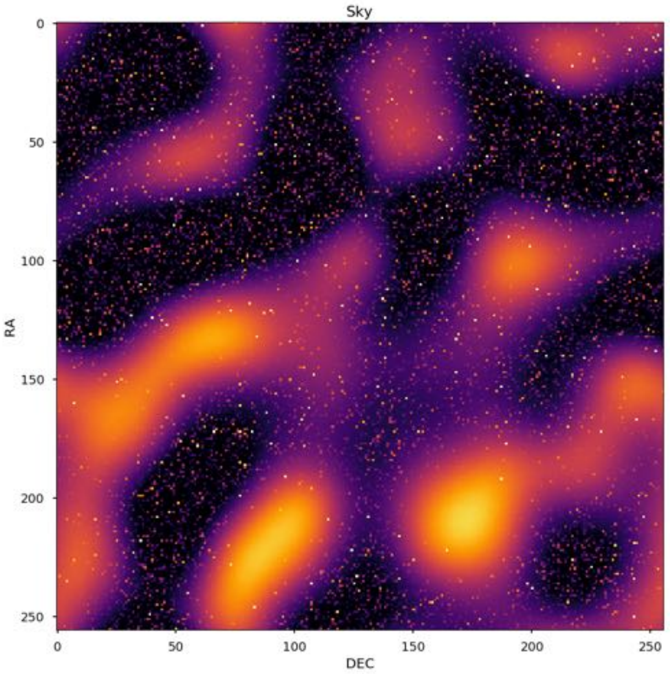
IFT as a neural network

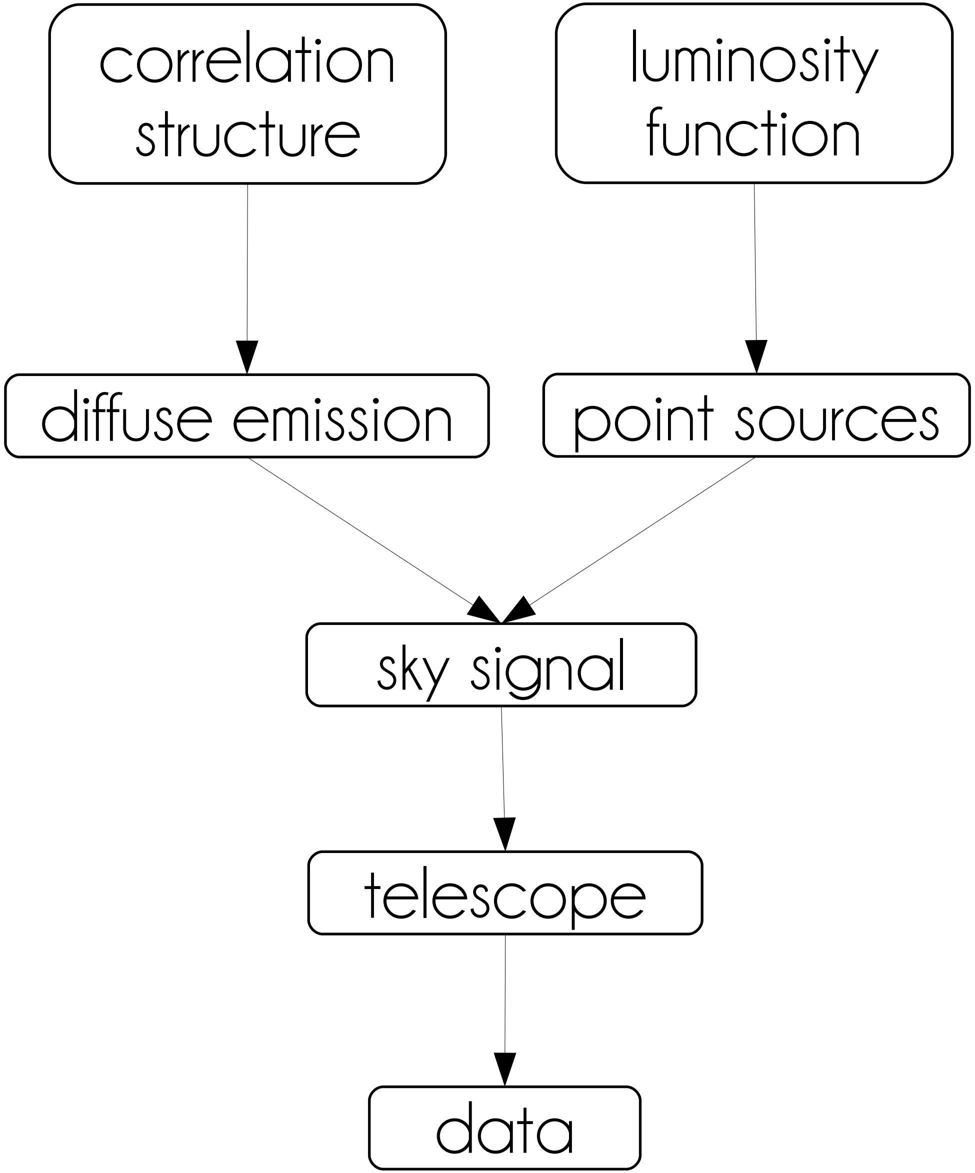


NIFTy tutorial part 1

linear reconstructions


Power
|k|


Sky
RA
DEC


correlation structure
luminosity function
diffuse emission
point sources
sky signal
telescope
data

$\mathcal{P}(d|s)$ Data model

known → $d = R\, e^{s} + n$

unknown → $\lambda = R\, e^{s}$

$\mathcal{P}(s) = \mathcal{G}(s,\, S)$ unknown

$$\mathcal{P}(d|\lambda) = \prod_i \frac{\lambda_i^{d_i}}{d_i!}\, e^{-\lambda_i}$$

Information

$$\mathcal{H}(\boldsymbol{d}, \boldsymbol{s}, \boldsymbol{\tau}) = -\log \mathcal{P}(\boldsymbol{d}, \boldsymbol{s}, \boldsymbol{\tau})$$

$$= 1^\dagger \left[\log(d!) + \boldsymbol{R}\left(\mathrm{e}^{\boldsymbol{s}} + \mathrm{e}^{\boldsymbol{u}}\right)\right] - \boldsymbol{d}^\dagger \log\left[\boldsymbol{R}\left(\mathrm{e}^{\boldsymbol{s}} + \mathrm{e}^{\boldsymbol{u}}\right)\right]$$

$$+ \frac{1}{2}\boldsymbol{s}^\dagger \boldsymbol{S}^{-1}\boldsymbol{s} + \frac{1}{2}\log\left(\det\left[\boldsymbol{S}\right]\right)$$

$$+ (\boldsymbol{\alpha} - \boldsymbol{1})^\dagger \boldsymbol{\tau} + \boldsymbol{q}^\dagger \mathrm{e}^{-\boldsymbol{\tau}} + \frac{1}{\ldots}\boldsymbol{\tau}^\dagger \boldsymbol{T}\boldsymbol{\tau}$$

$$+ (\boldsymbol{\beta} - \boldsymbol{1})^\dagger \ldots$$

$$\boldsymbol{S} = \sum_k \mathrm{e}^{\tau_k} \ldots$$

- Convert into **generative model**
- Compress information into Gaussian via **Metric Gaussian Variational Inference**

Variational Bayes

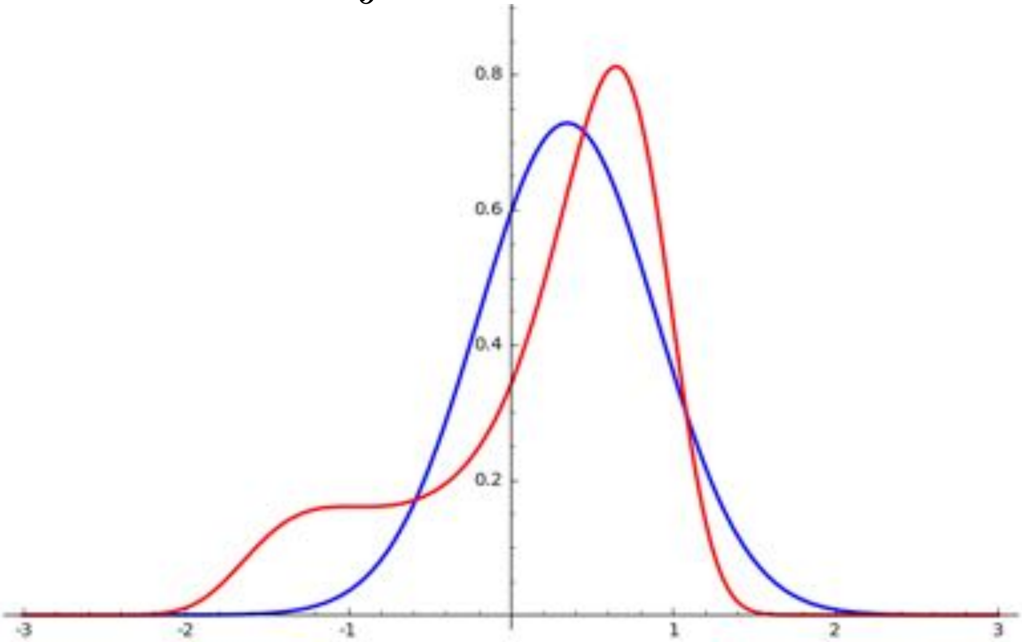
$\mathcal{P}(s|d)$

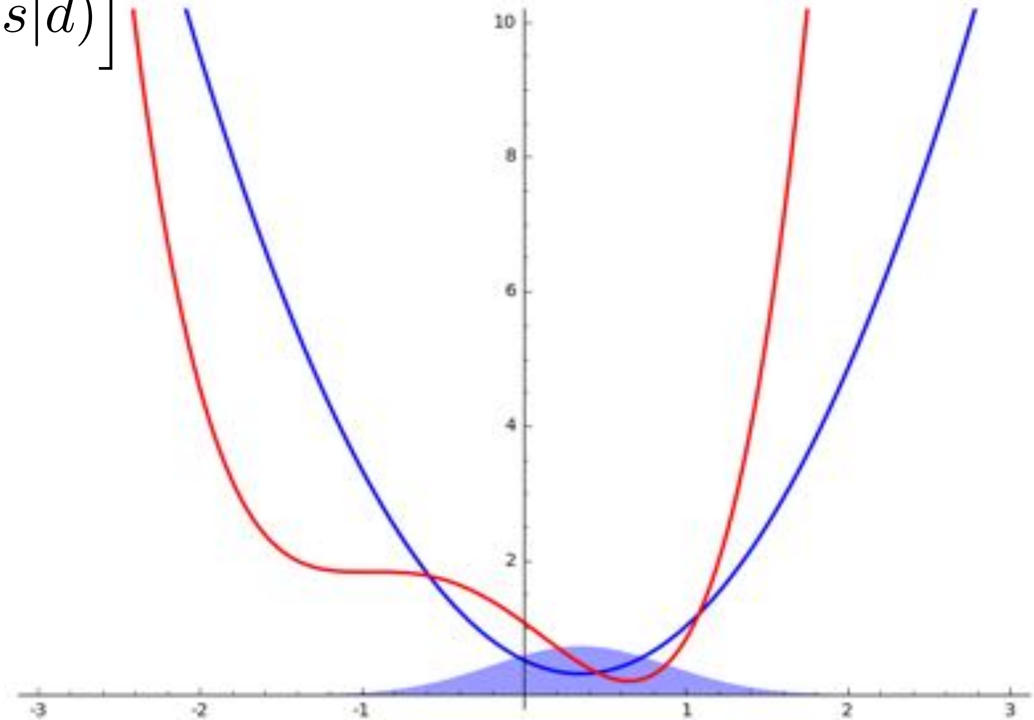
$\widetilde{\mathcal{P}}(s|d) = \mathcal{G}(s-m, D)$

$\mathcal{H}(s|d)$

$\widetilde{\mathcal{H}}(s|d) \widehat{=} \frac{1}{2}(s-m)^{\dagger} D^{-1} (s-m)$

$$\mathrm{KL}(\widetilde{\mathcal{P}}, \mathcal{P}) = \int \mathcal{D}s\, \widetilde{\mathcal{P}}(s|d) \left[\mathcal{H}(s|d) - \widetilde{\mathcal{H}}(s|d) \right]$$





Metric Gaussian Variational Bayes

$\mathcal{P}(s|d)$

$\widetilde{\mathcal{P}}(s|d) = \mathcal{G}(s - m, D)$

$\mathcal{H}(s|d)$

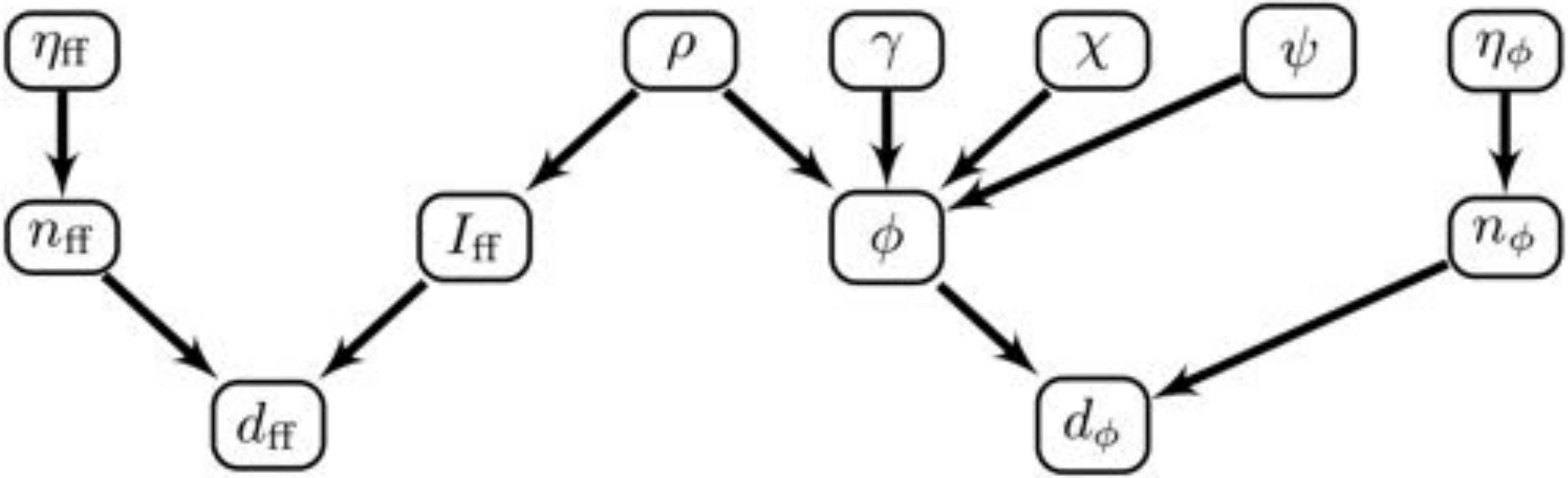
$\widetilde{\mathcal{H}}(s|d) \widehat{=} \frac{1}{2}(s-m)^\dagger D^{-1}(s-m)$

Knollmüller & Enßlin (2019)

$$\mathrm{KL}(\widetilde{\mathcal{P}}, \mathcal{P}) = \int \mathcal{D}s \, \widetilde{\mathcal{P}}(s|d) \left[\mathcal{H}(s|d) - \widetilde{\mathcal{H}}(s|d)\right]$$

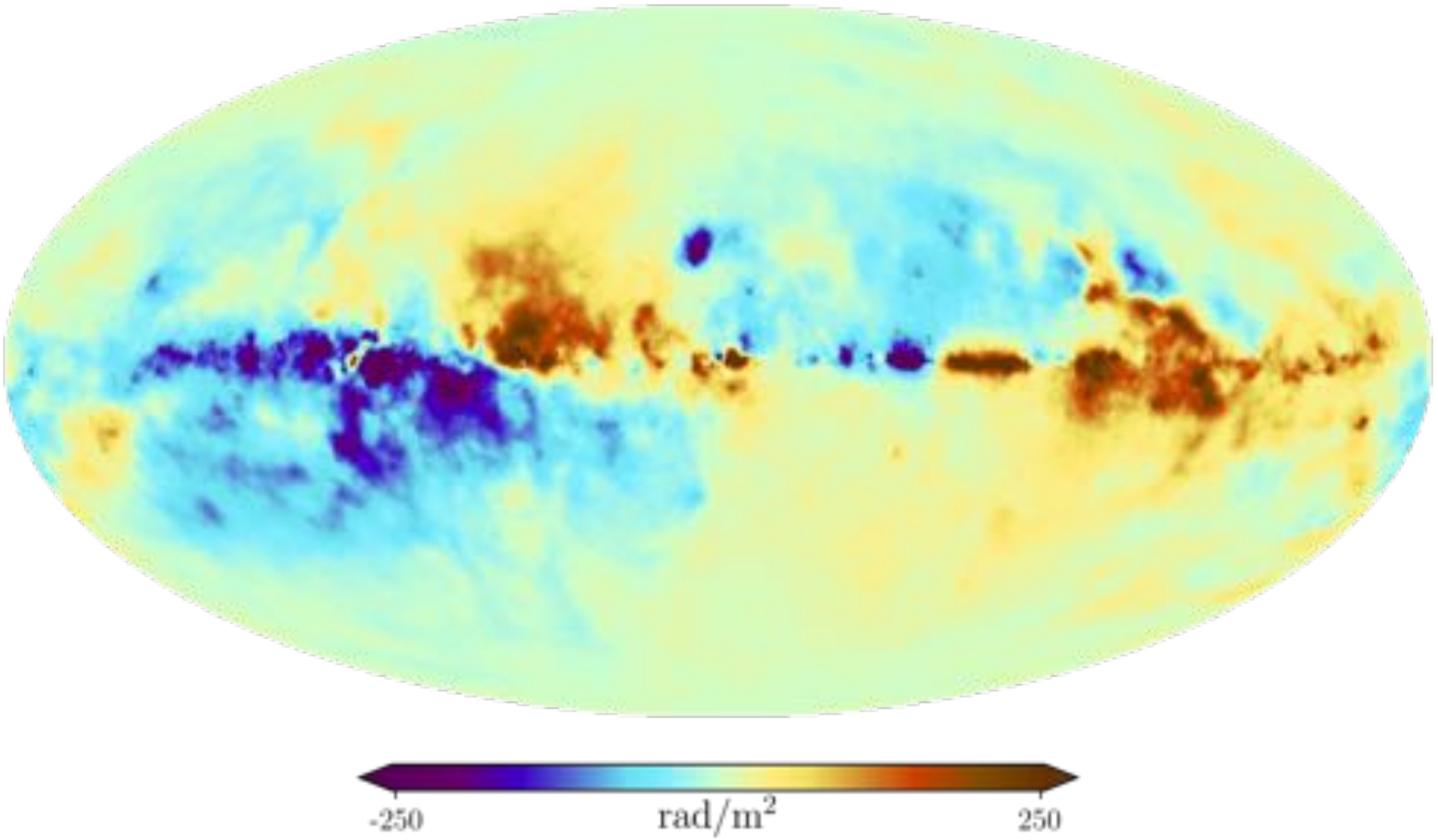
$$D \approx \left\langle \frac{\partial \mathcal{H}(d,s)}{\partial s} \frac{\partial \mathcal{H}(d,s)}{\partial s}^\dagger \right\rangle_{(d|s=m)}^{-1}$$

Hierarchical Bayesian Model

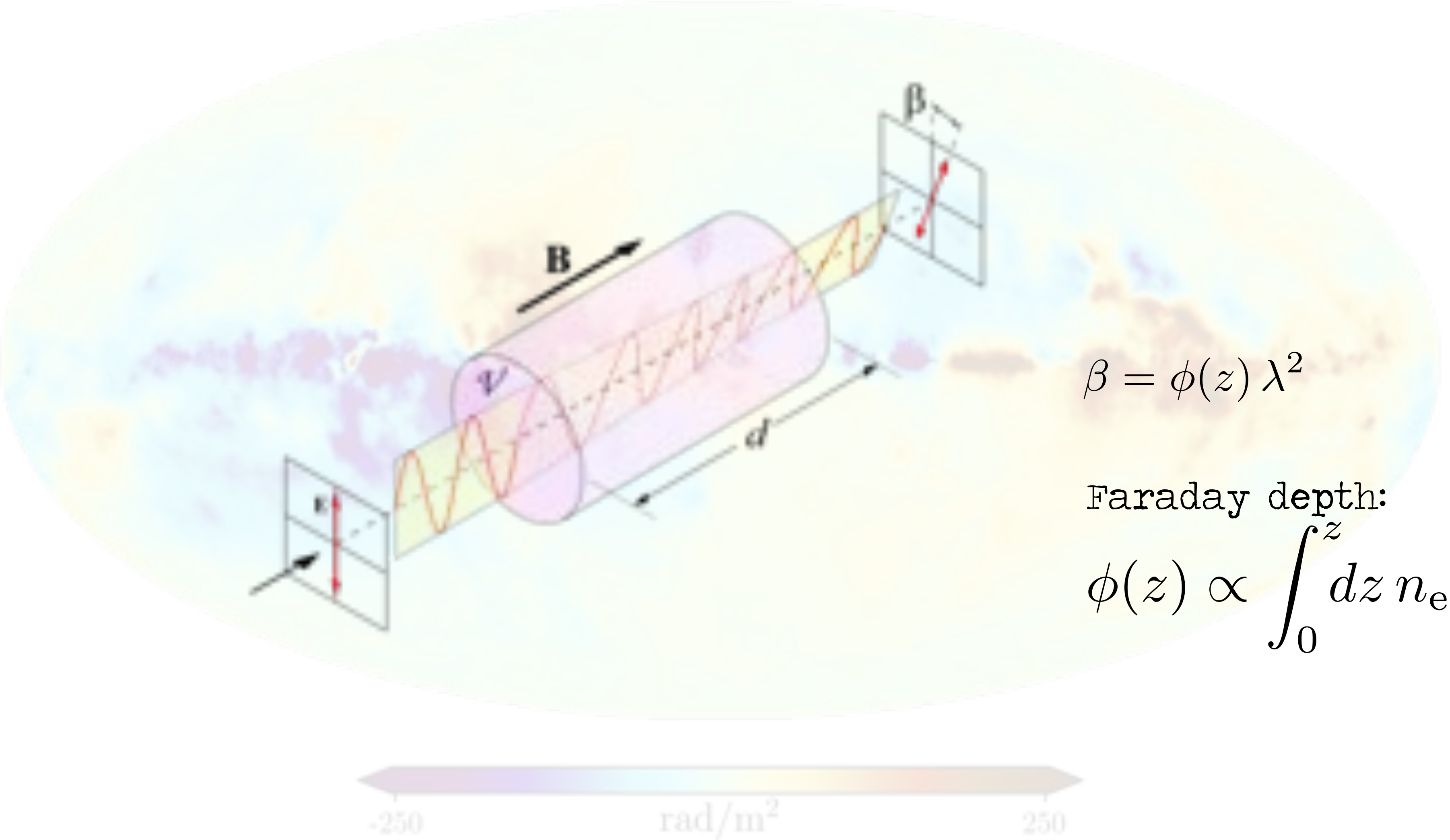


Galactic Faraday Sky

Hutschenreuter & Enßlin (2019)



Faraday Effect



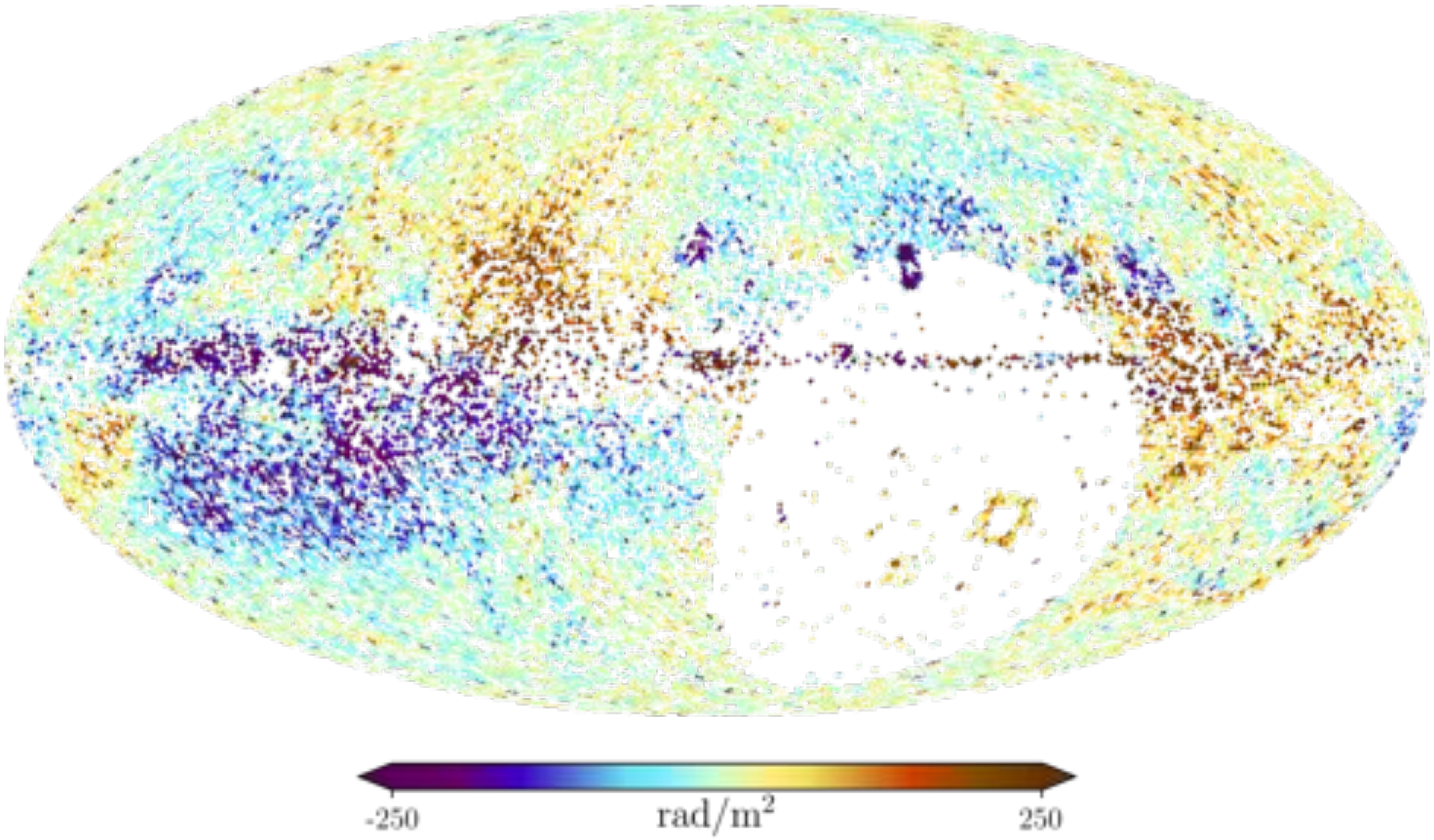
$$\beta = \phi(z)\,\lambda^2$$

Faraday depth:

$$\phi(z) \propto \int_0^z dz\, n_\mathrm{e}\, B_z$$

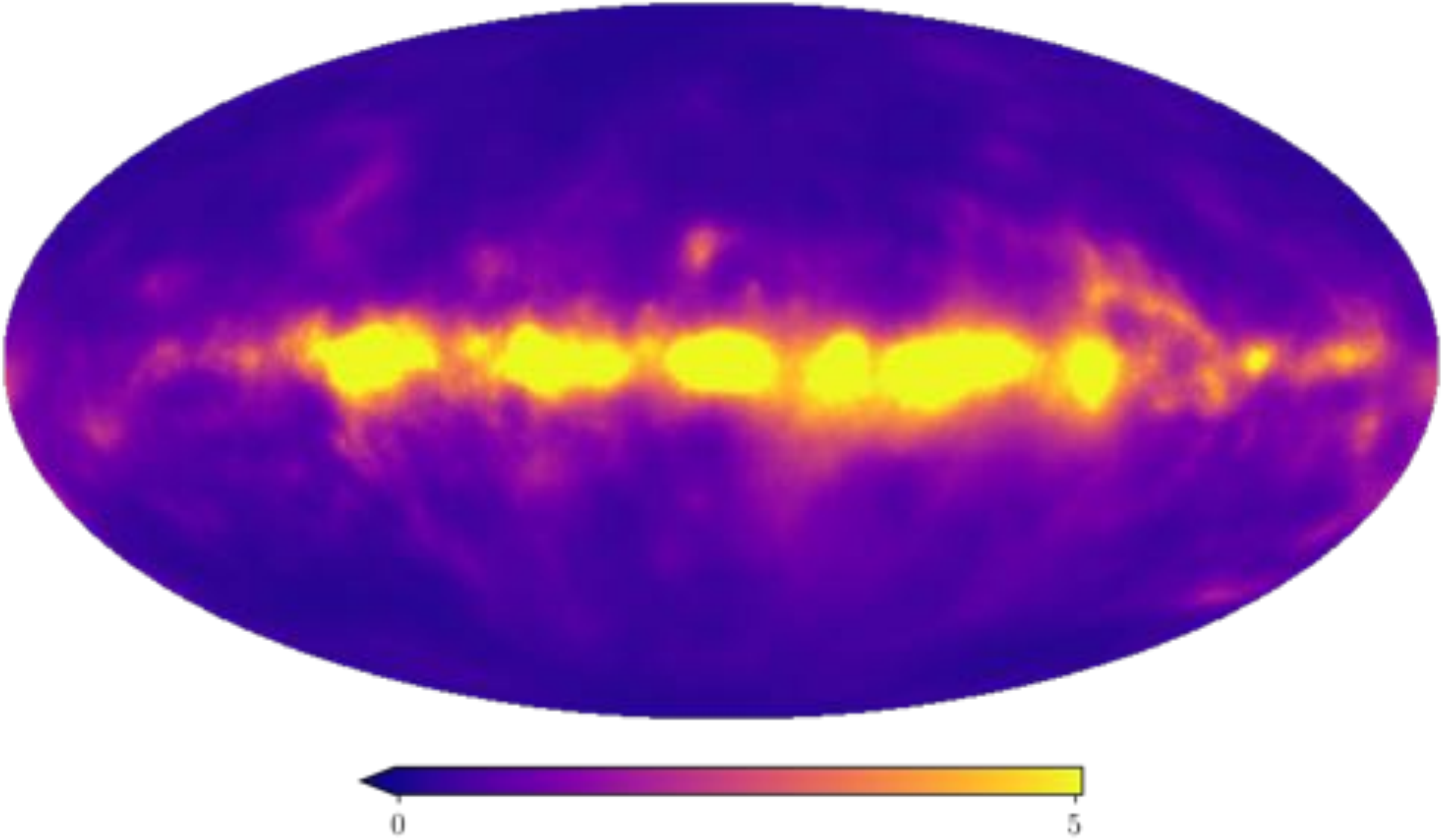
Faraday Data

Oppermann et al. (2012)



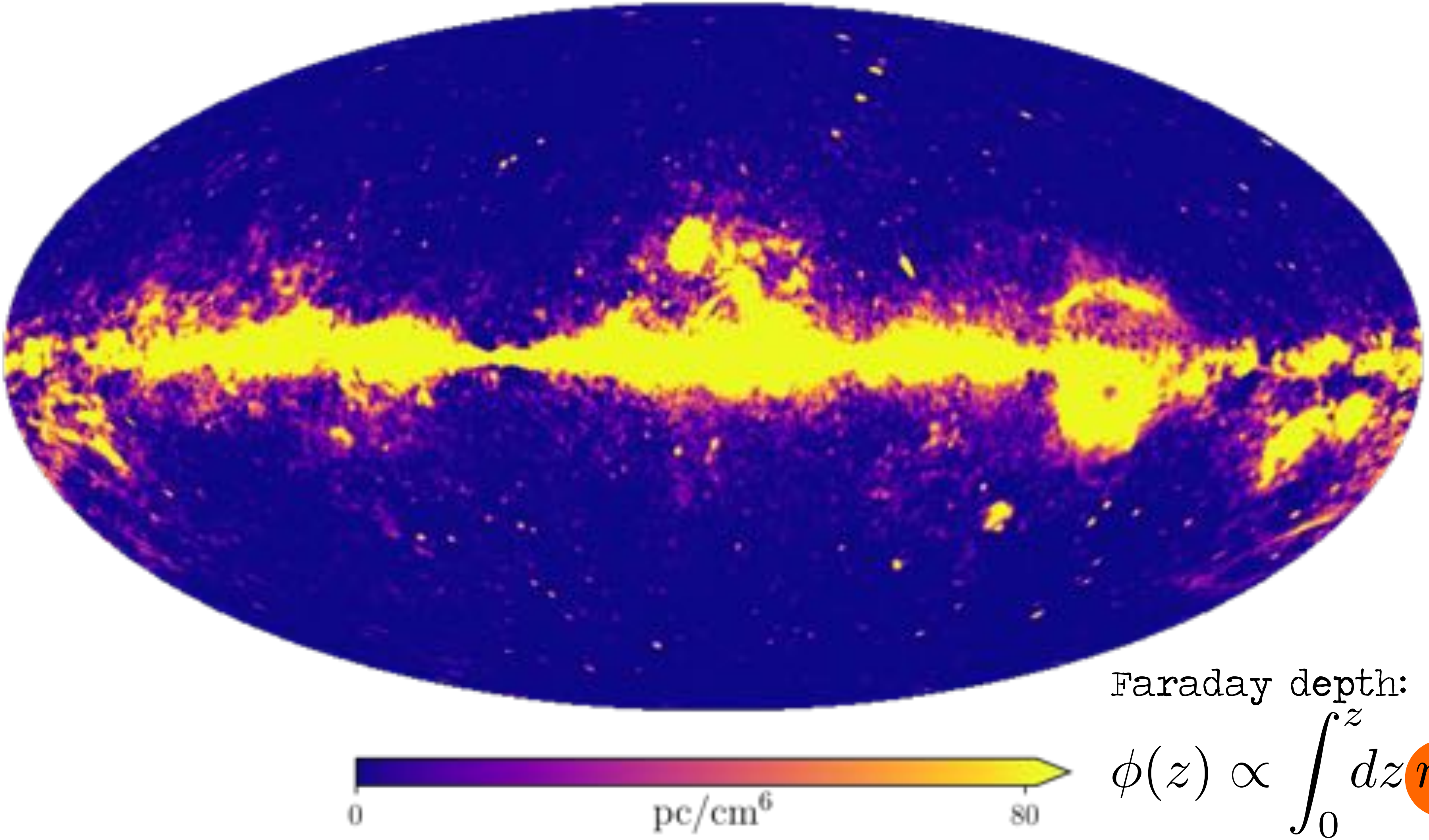
Faraday Amplitude Field

Hutschenreuter & Enßlin (2019)



Planck Free-Free Emission

Hutschenreuter & Enßlin (2019)



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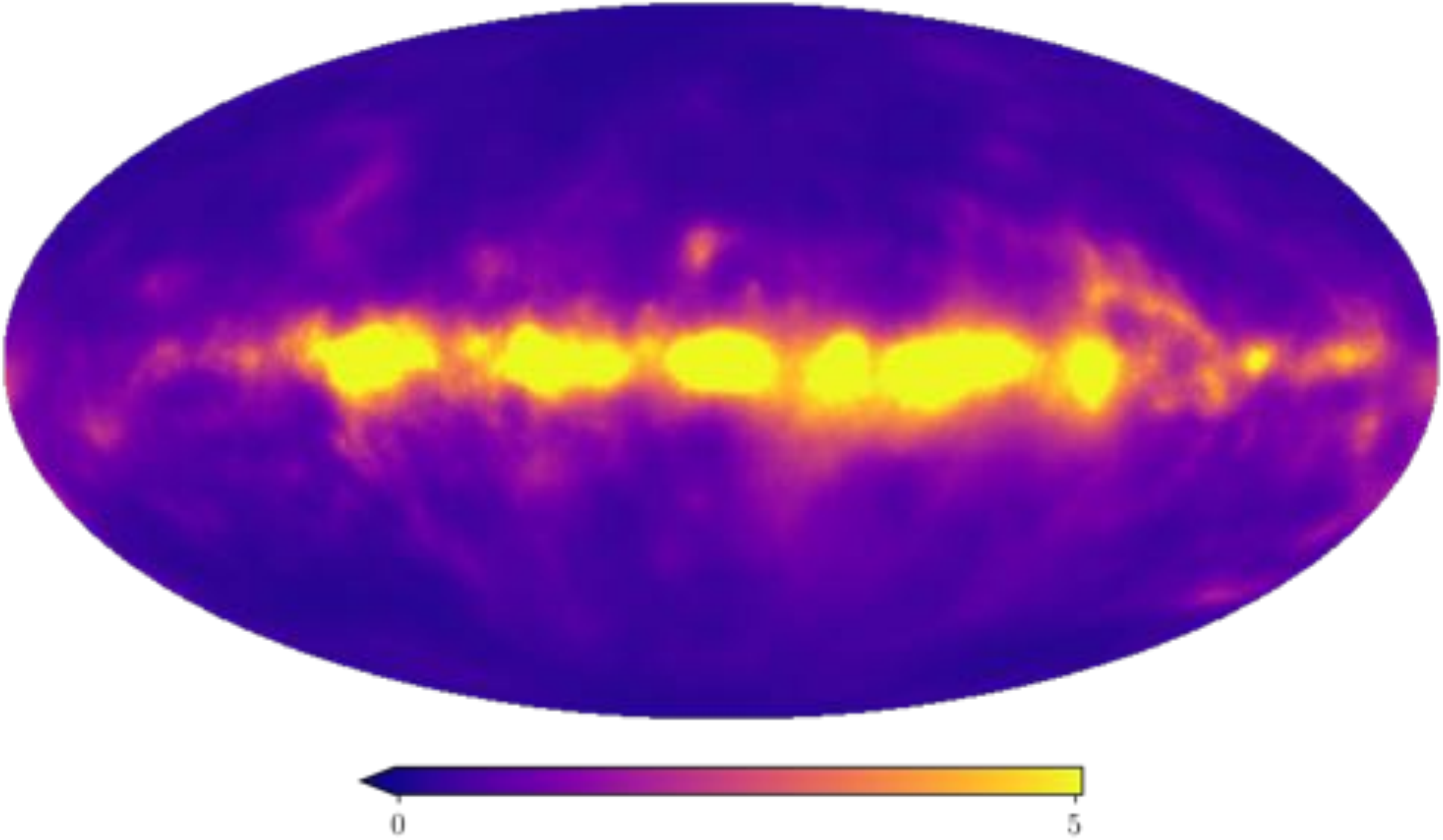
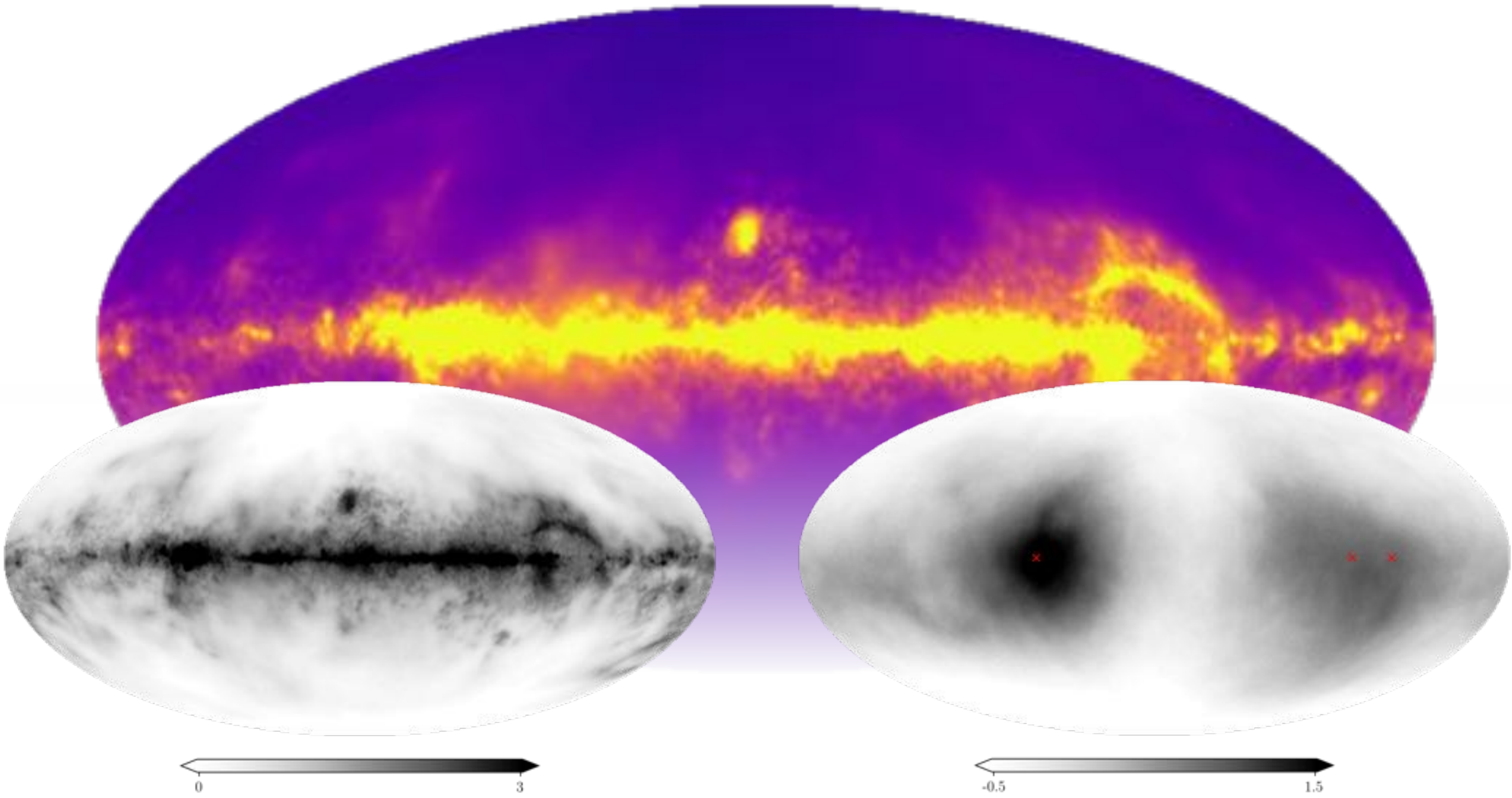
Faraday Amplitude Field

Hutschenreuter & Enßlin (2019)



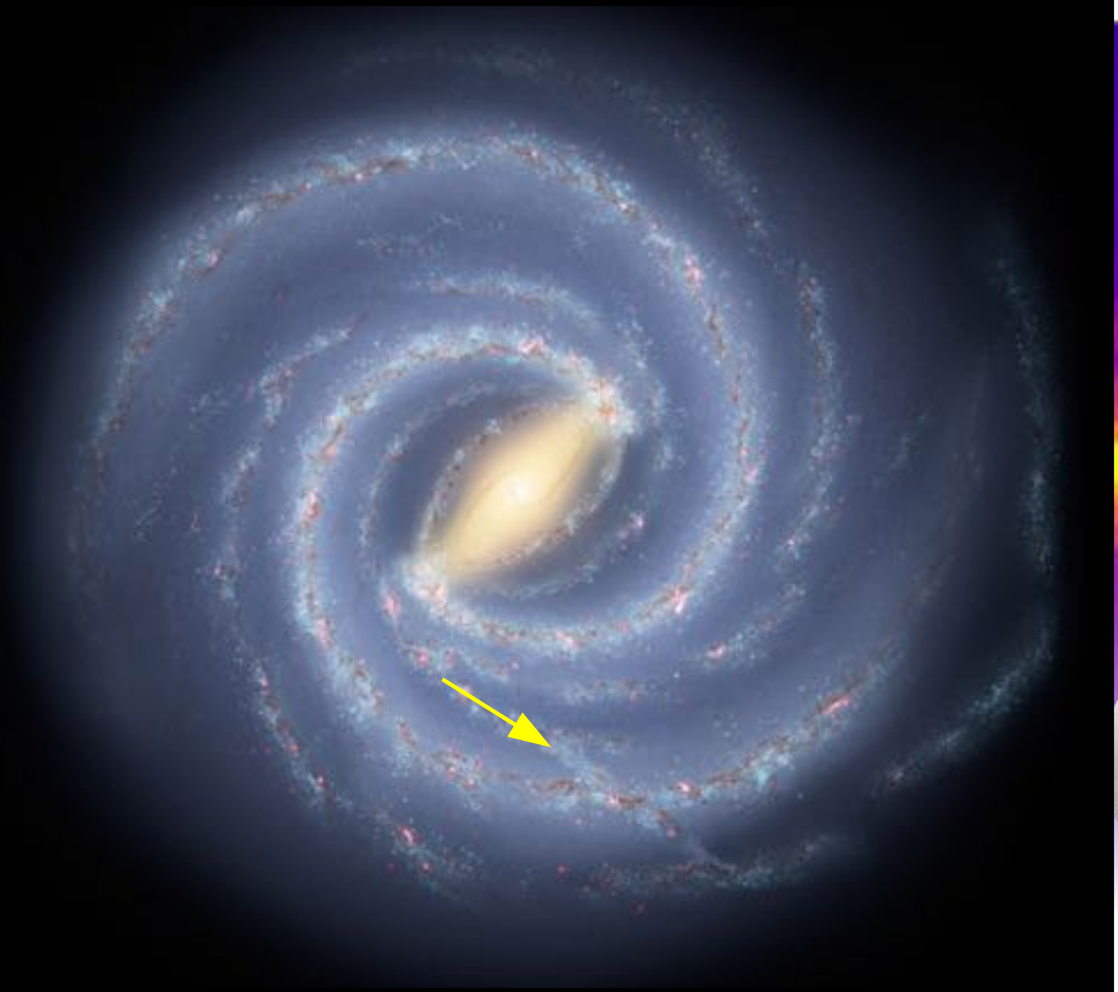
Faraday Amplitude Field

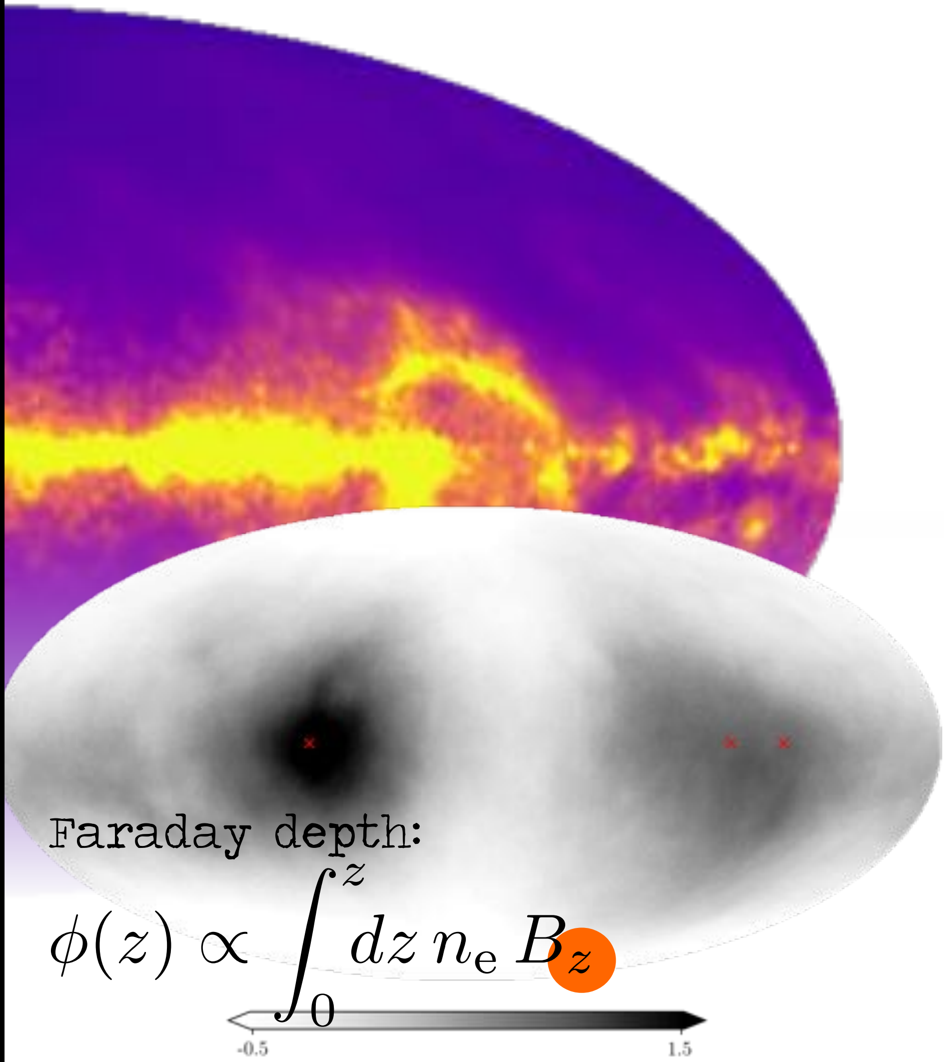
Hutschenreuter & Enßlin (2019)



…tude Field

Hutschenreuter & Enßlin (2019)





Faraday depth:

$$\phi(z) \propto \int_0^z dz\, n_e\, B_z$$

Data Fusion

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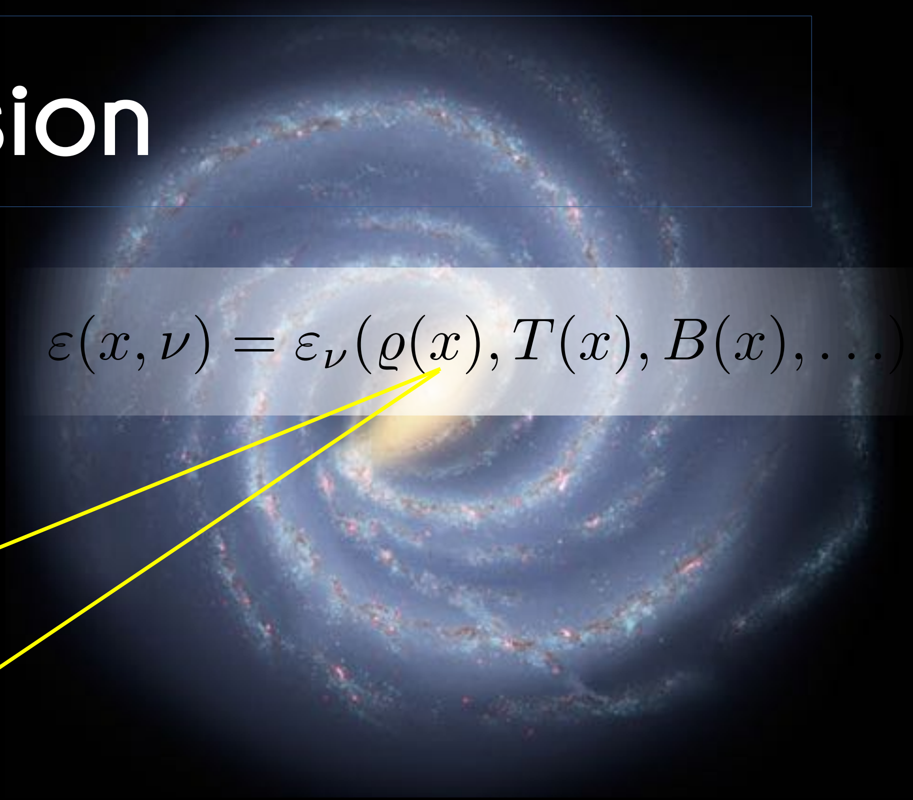
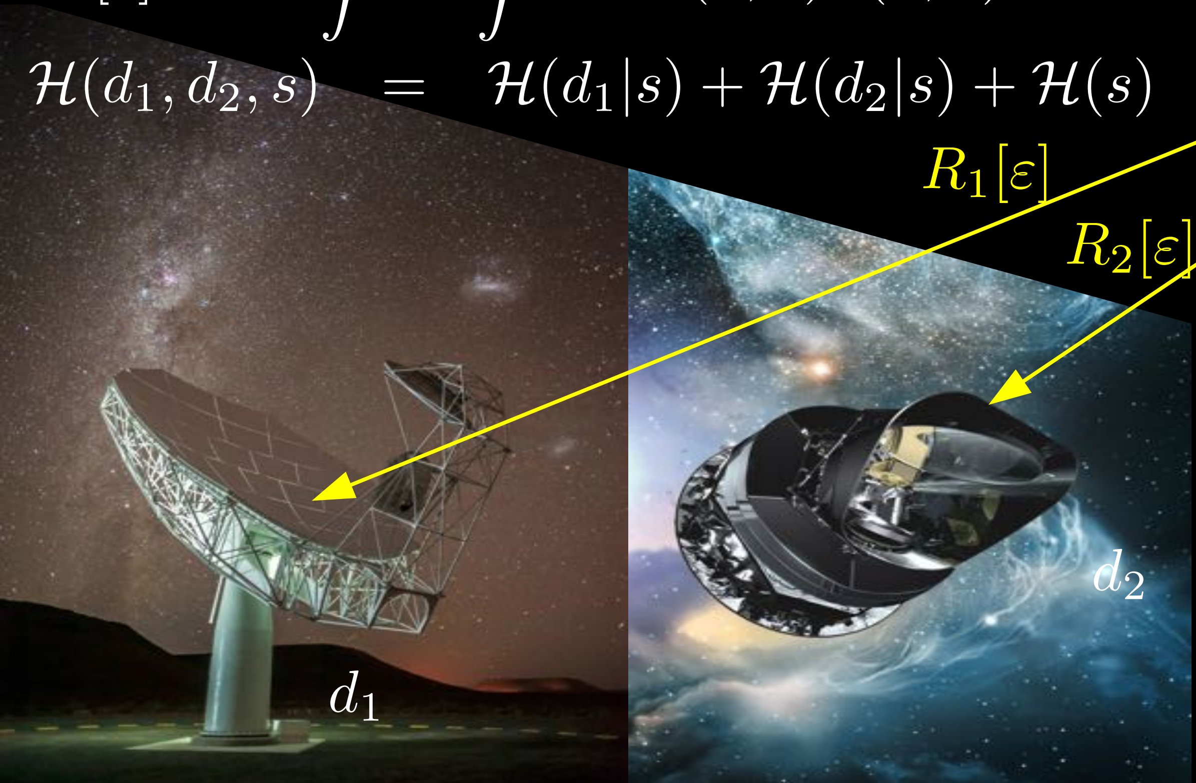
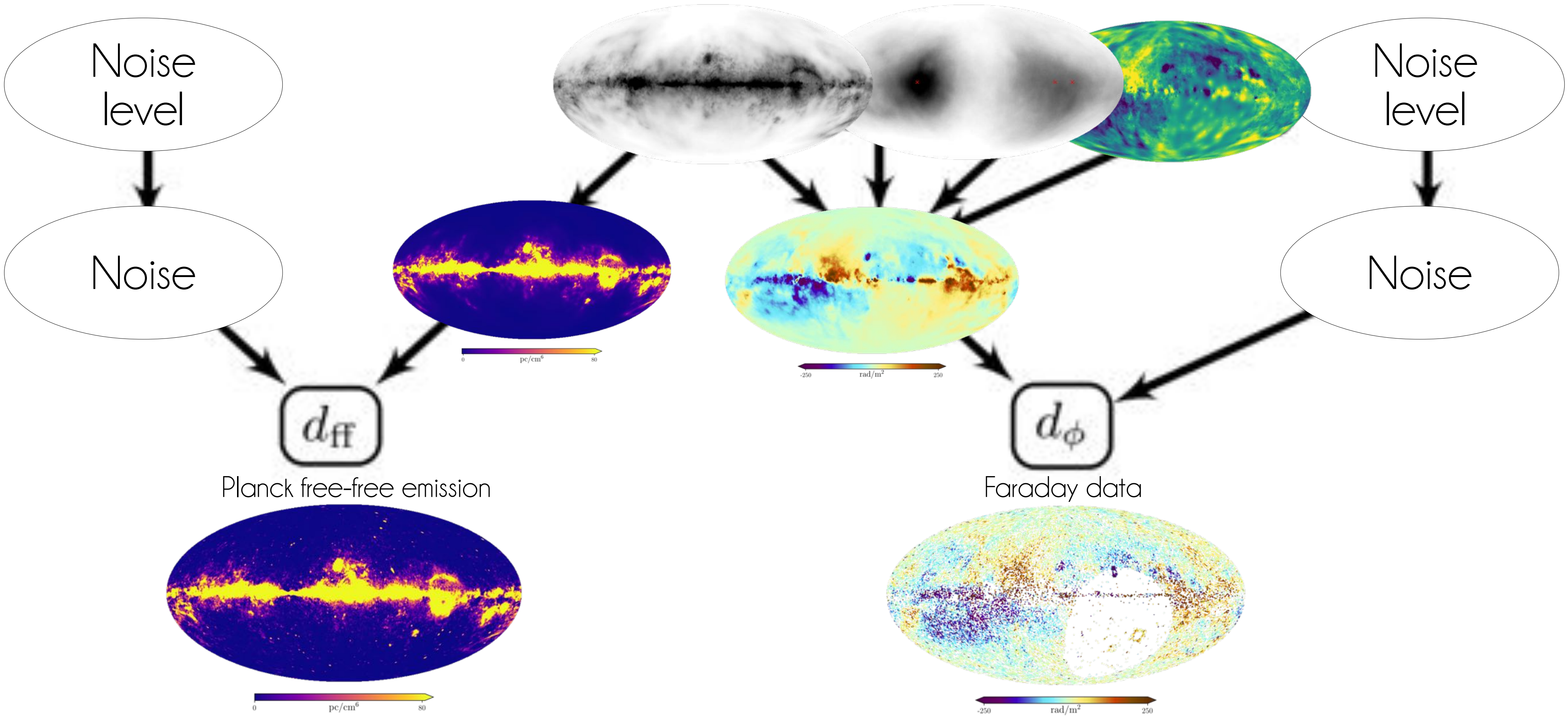


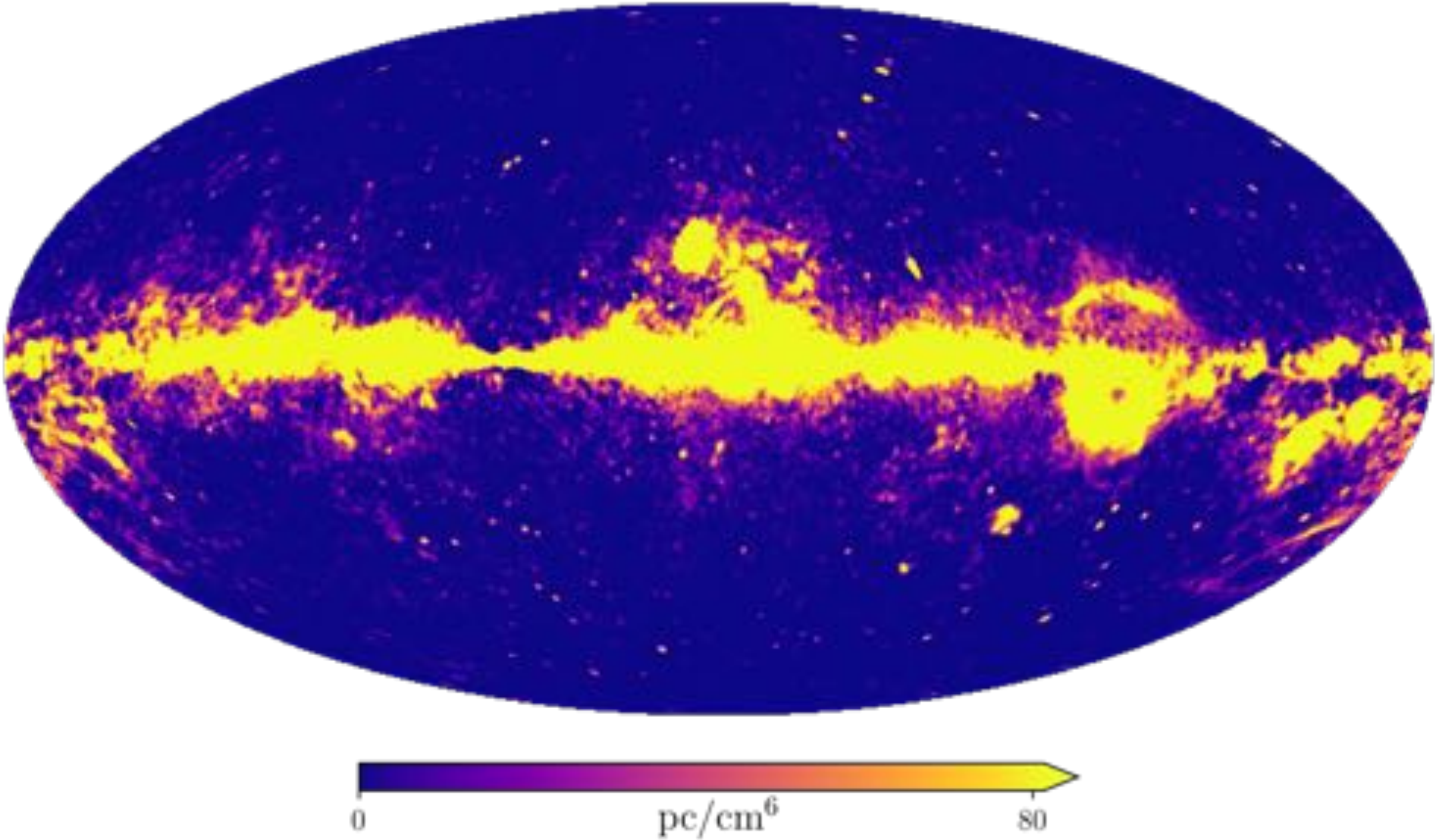


Hierarchical Bayesian Model

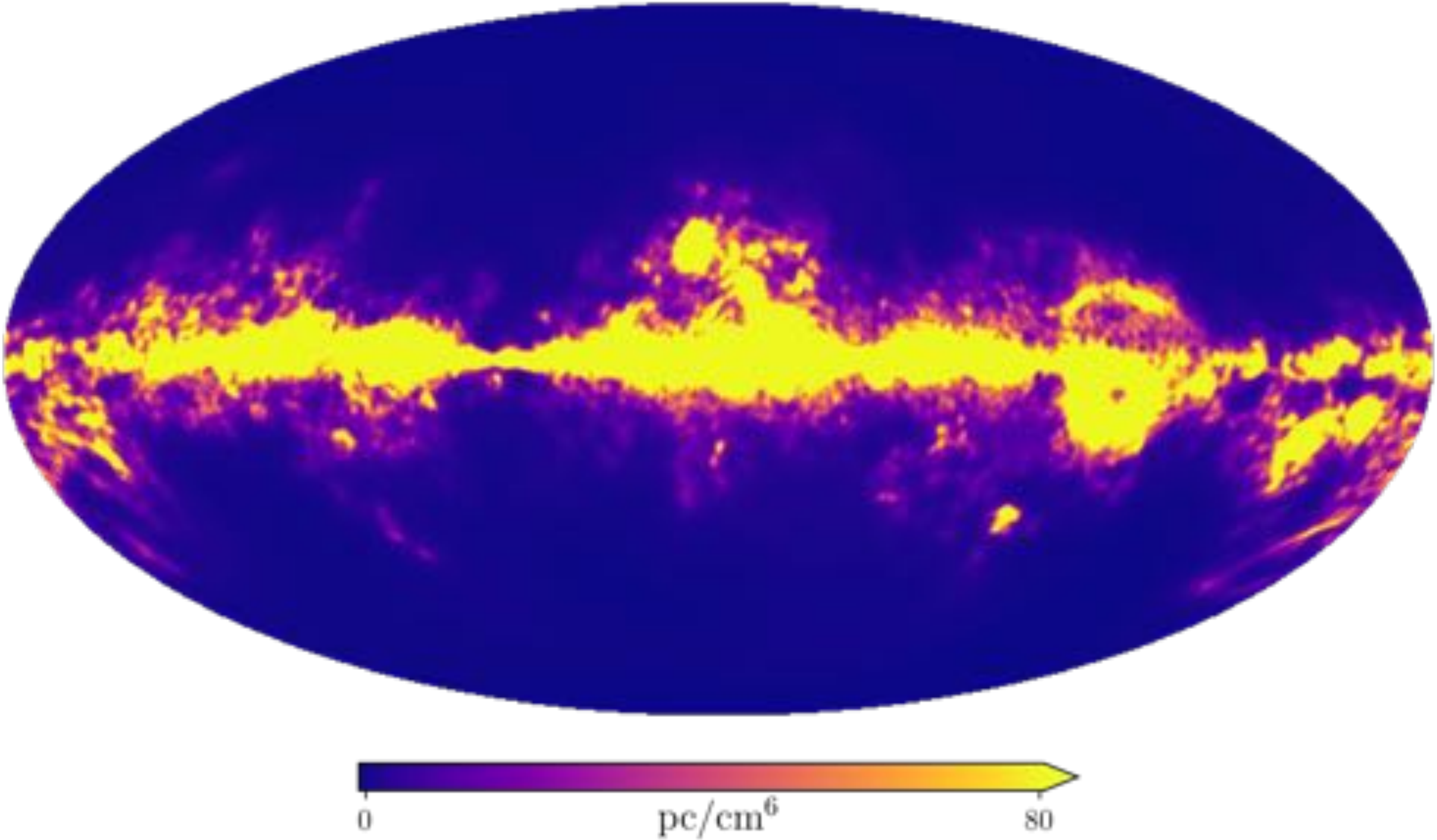


Planck free free map

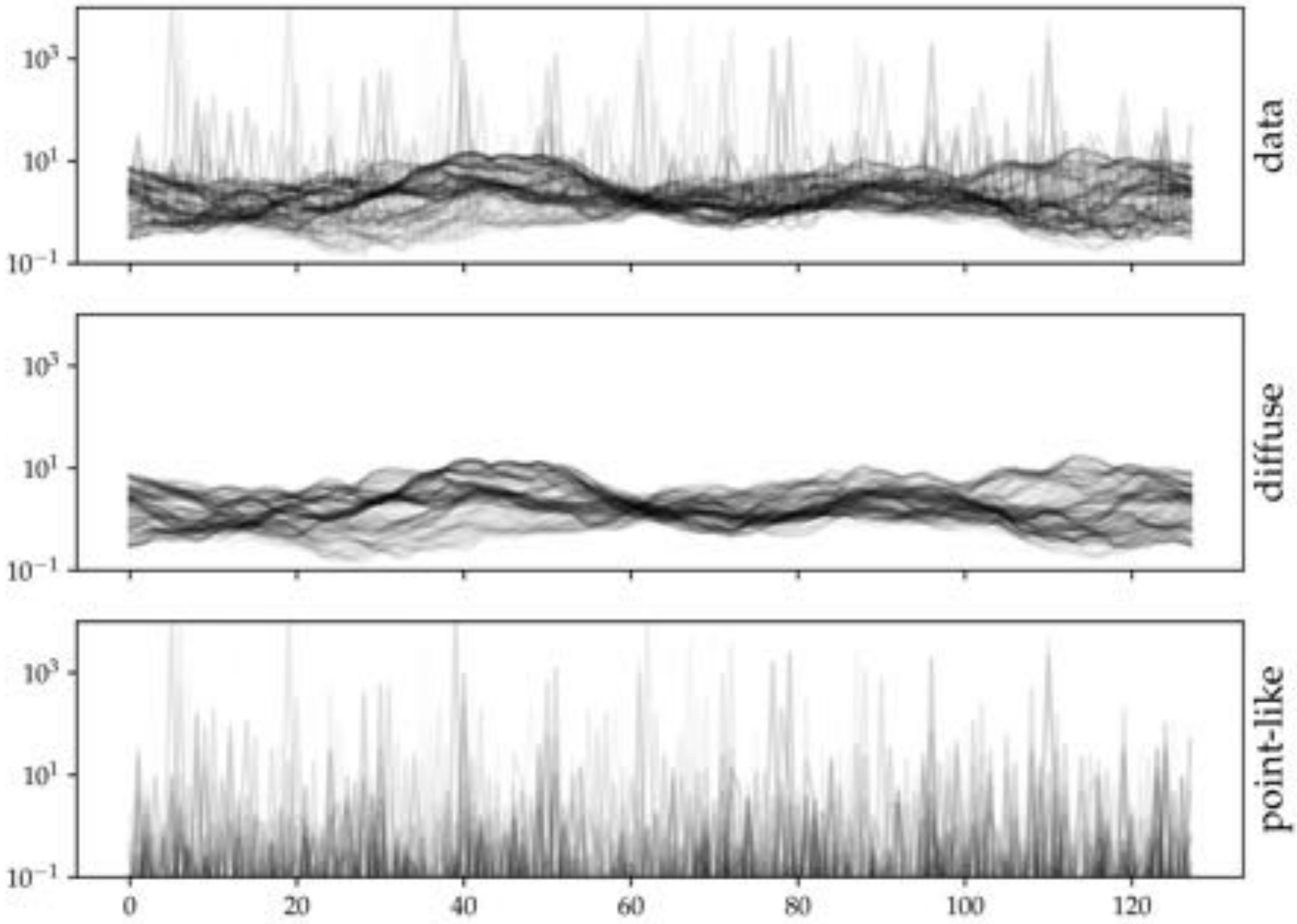
Hutschenreuter & Enßlin (2019)



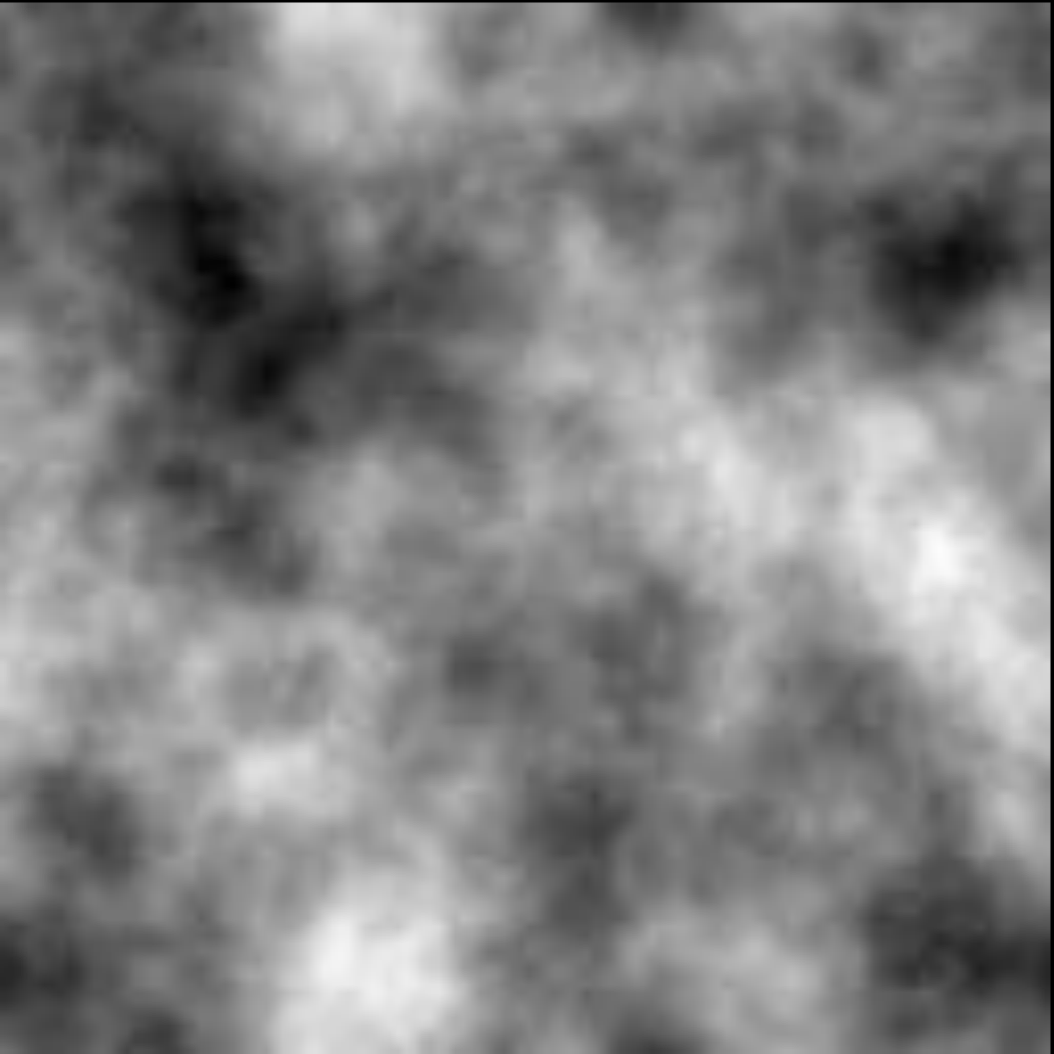
Inferred free free map Hutschenreuter & Enßlin (2019)



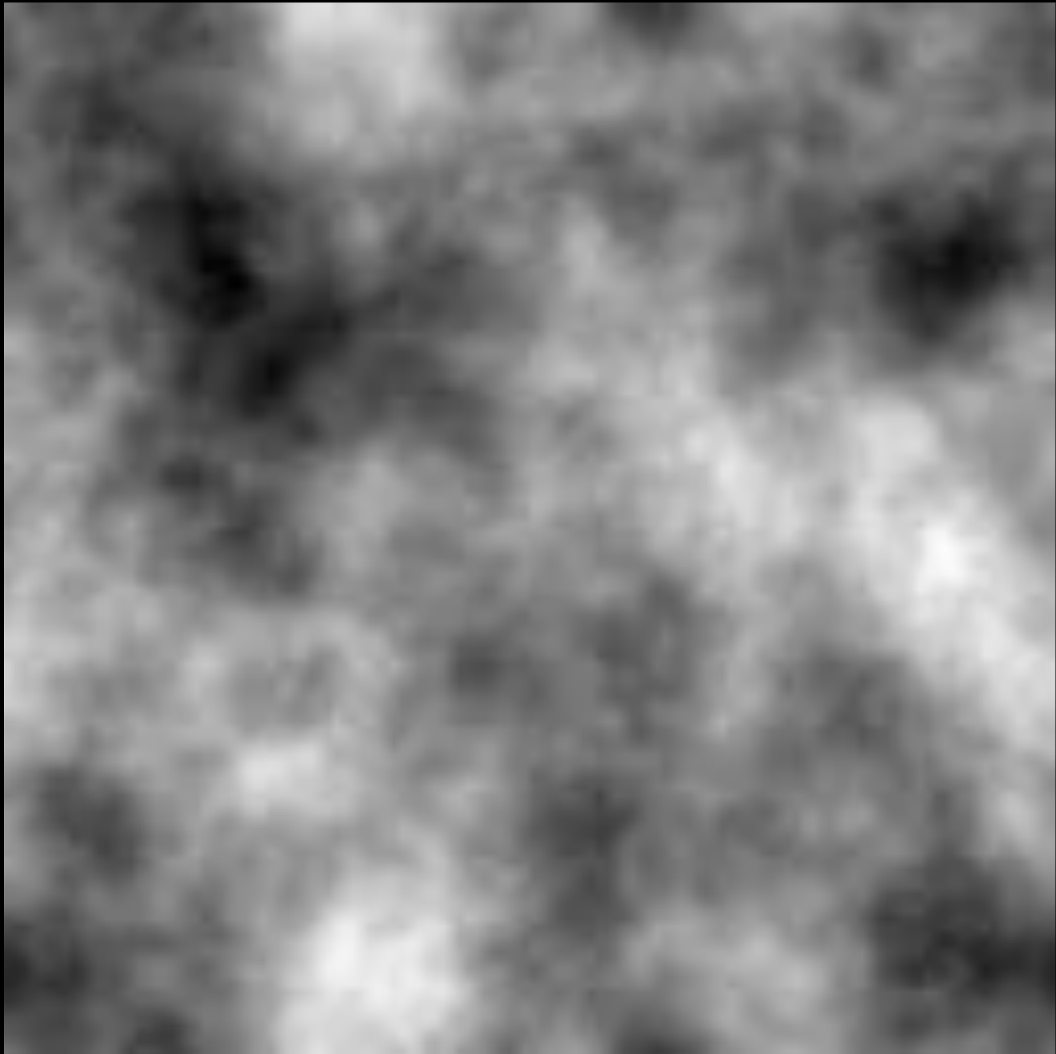
data and true components



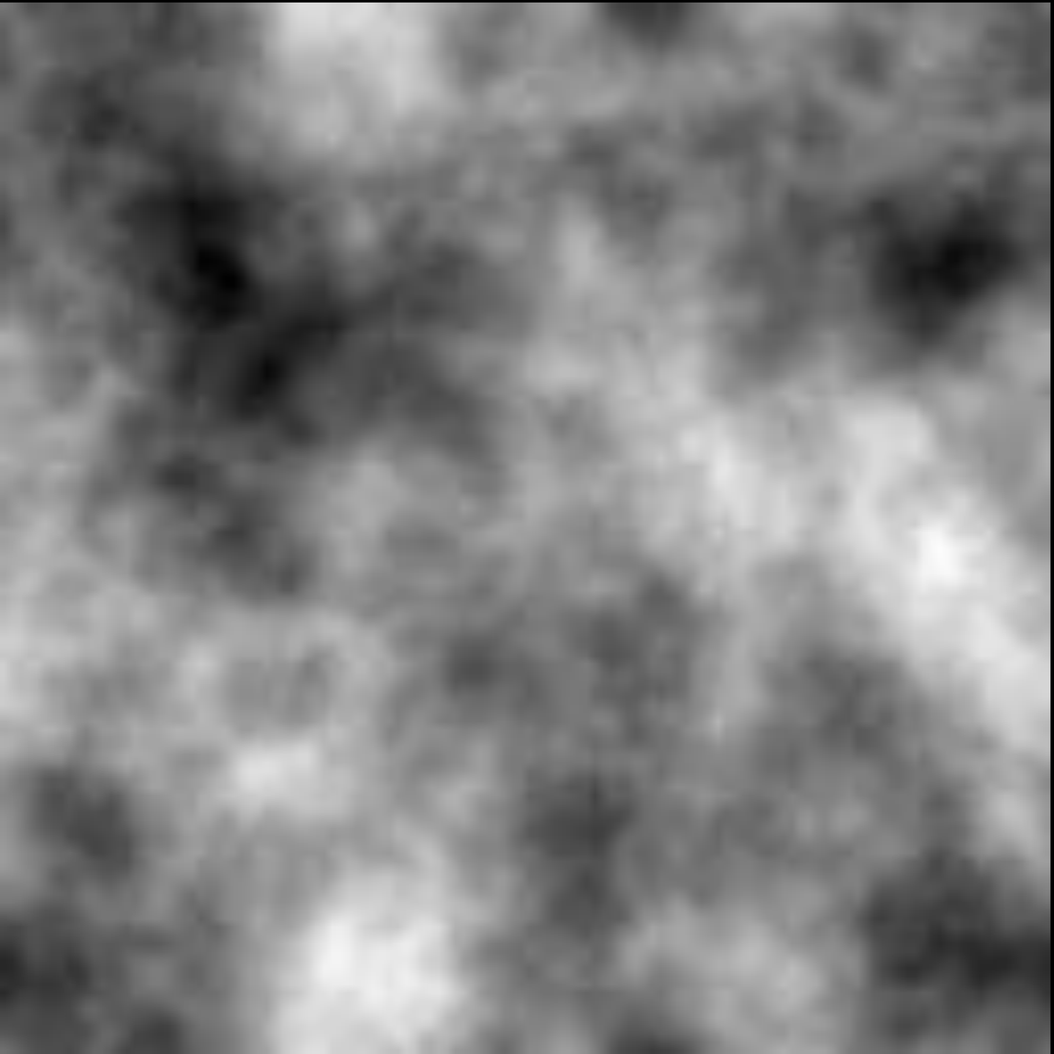
ground truth / starblade



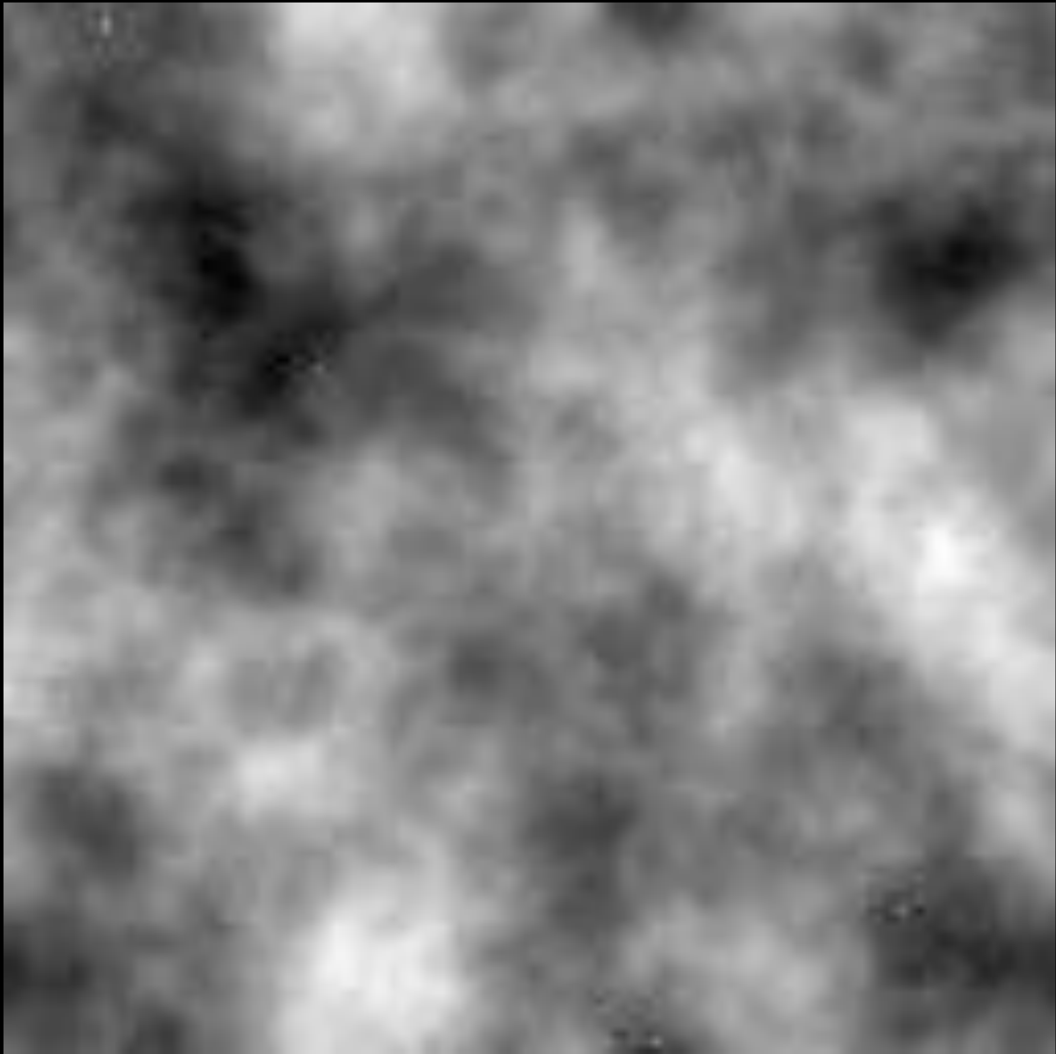
ground truth / autoencoder

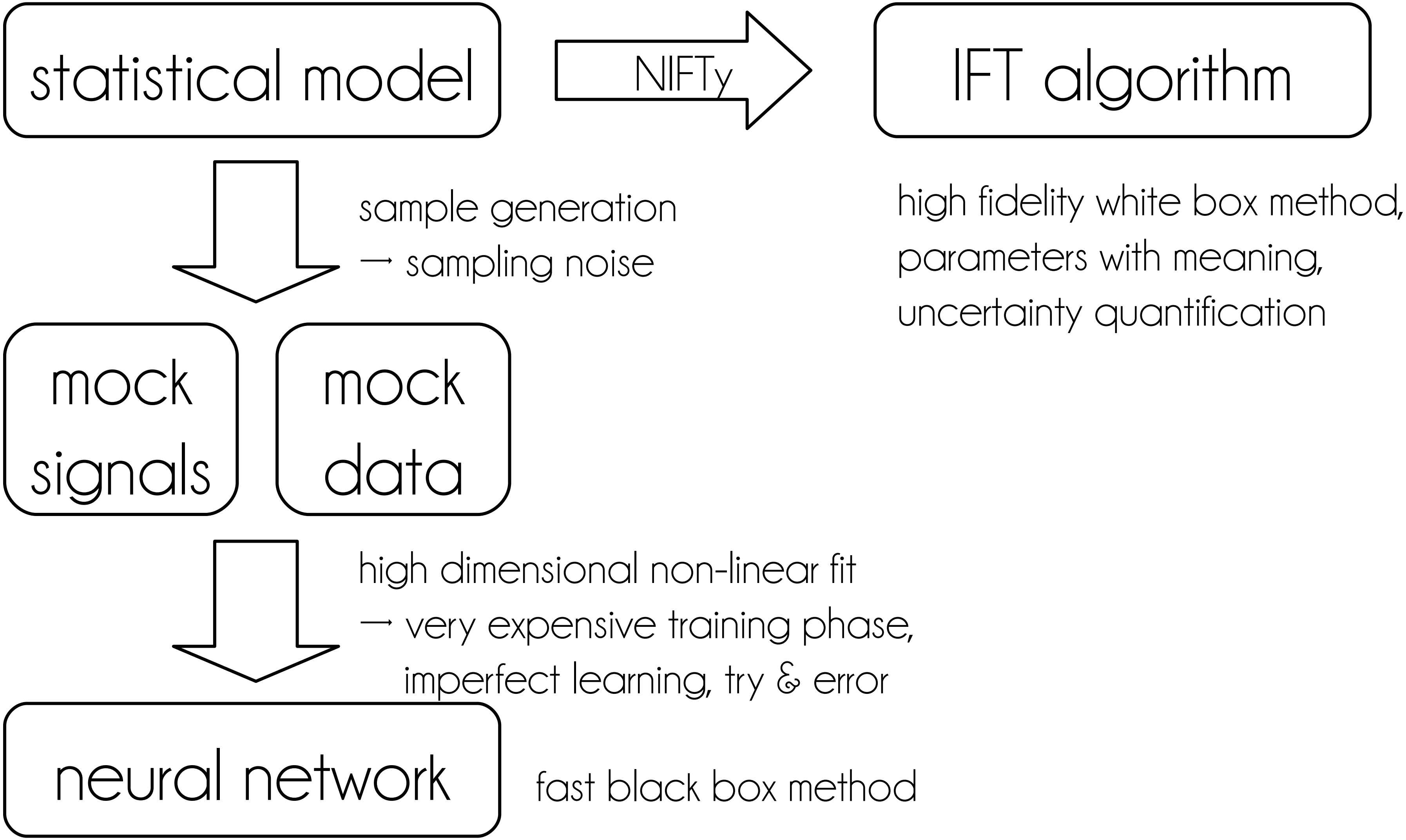


ground truth / starblade



ground truth / autoencoder

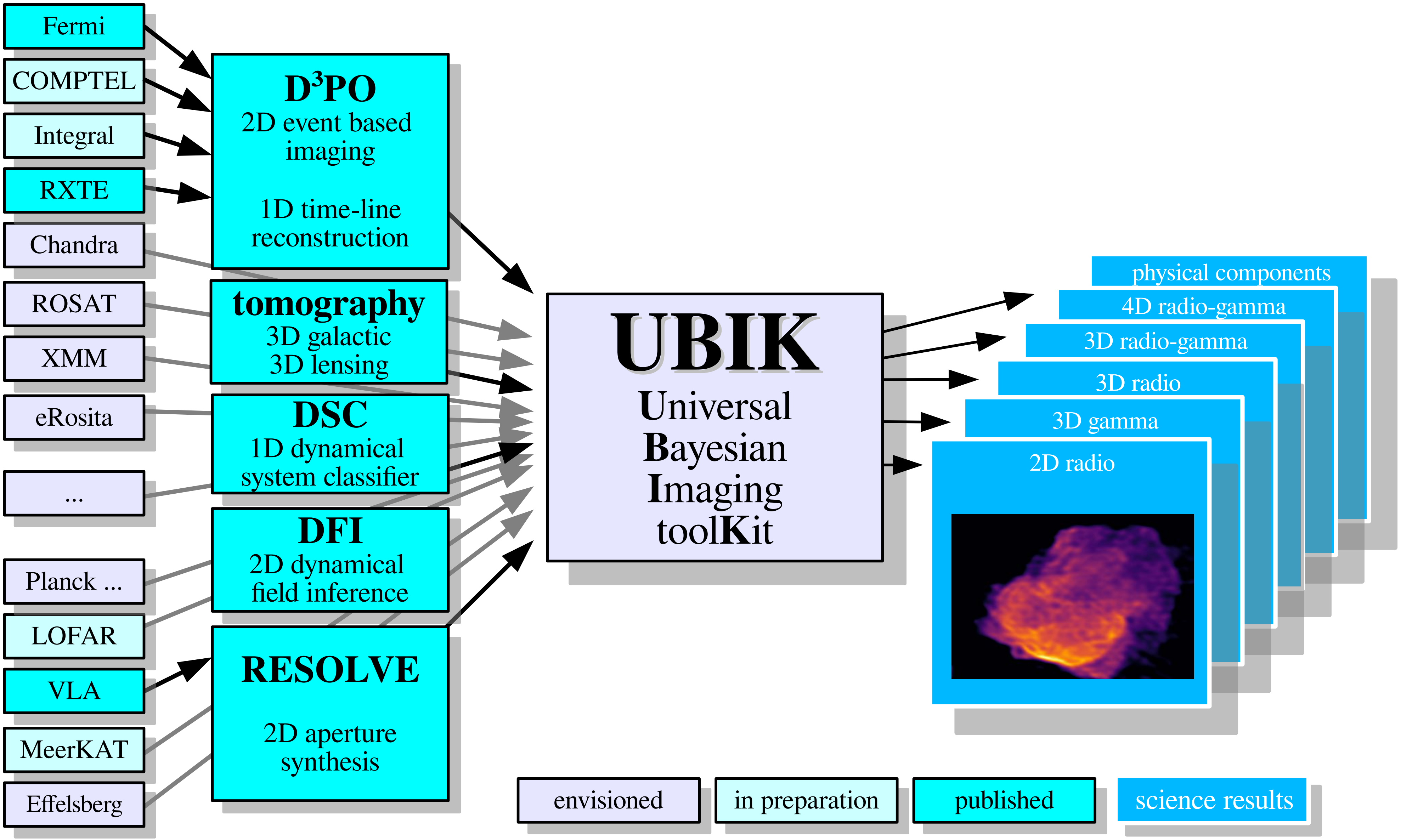




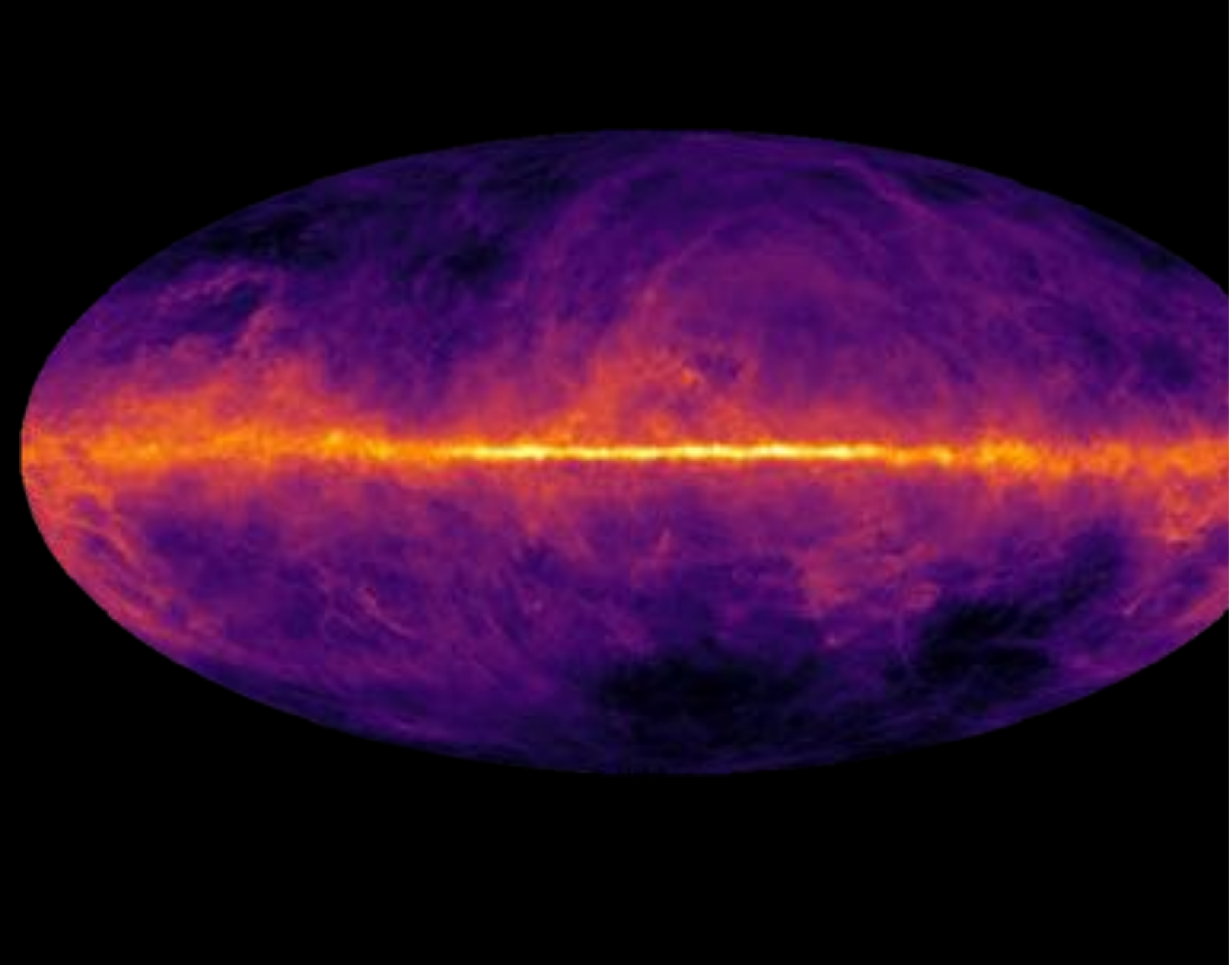
statistical model
NIFTy
IFT algorithm
high fidelity white box method,
parameters with meaning,
uncertainty quantification
sample generation
→ sampling noise
mock signals
mock data
high dimensional non-linear fit
→ very expensive training phase,
imperfect learning, try & error
neural network
fast black box method

NIFTy tutorial part 2

nonlinear reconstructions



Fermi
COMPTEL
Integral
RXTE
Chandra
ROSAT
XMM
eRosita
...
Planck ...
LOFAR
VLA
MeerKAT
Effelsberg
D³PO
2D event based imaging
1D time-line reconstruction
tomography
3D galactic
3D lensing
DSC
1D dynamical system classifier
DFI
2D dynamical field inference
RESOLVE
2D aperture synthesis
UBIK
Universal Bayesian Imaging toolKit
physical components
4D radio-gamma
3D radio-gamma
3D radio
3D gamma
2D radio
envisioned
in preparation
published
science results



Thank you!

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