Principles of Optical Interferometry

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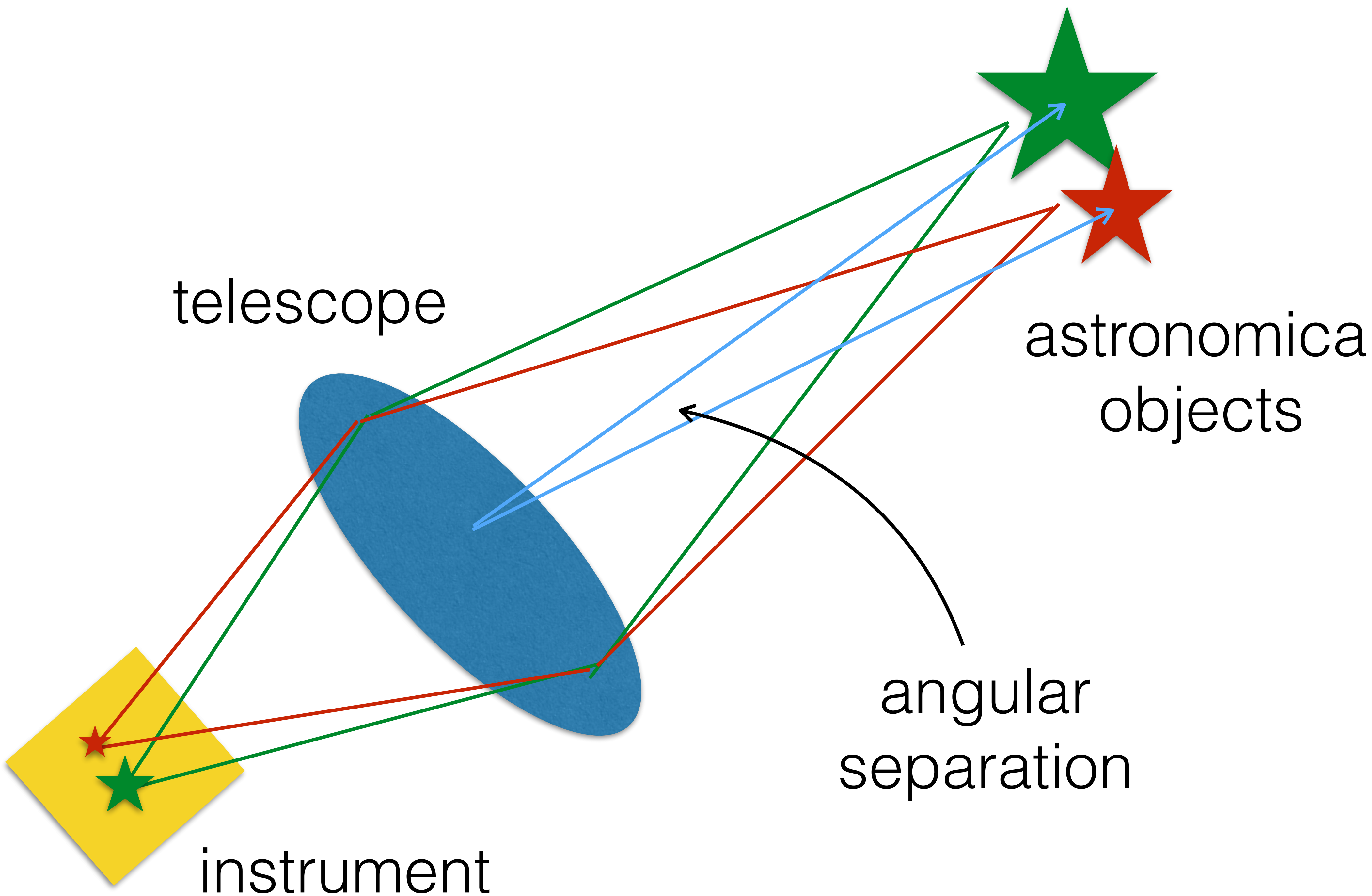
The Need for Angular Resolution

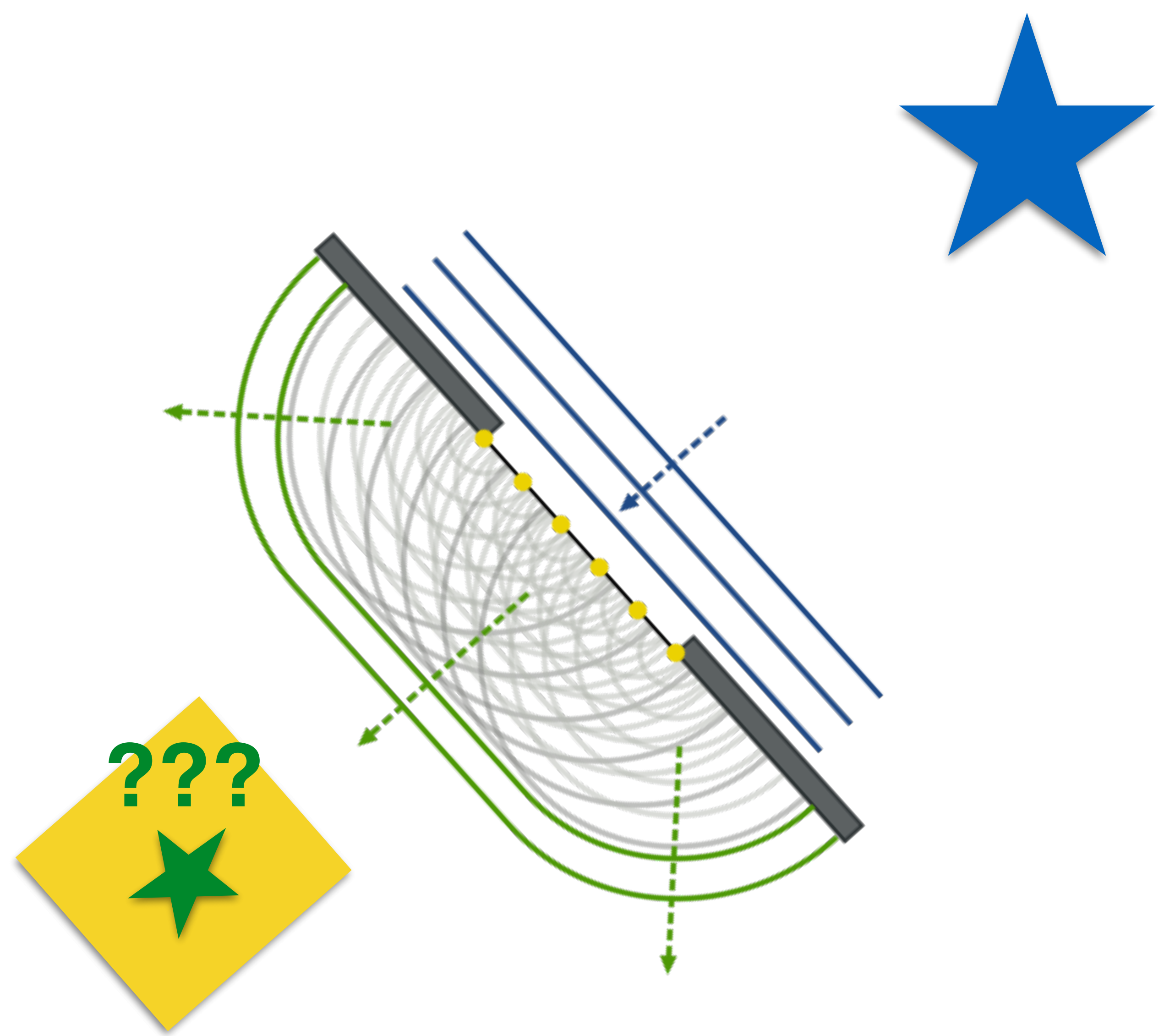
Angular resolution



Diffraction

- light as a wavefront

- each points of the aperture emits an hemispherical wave



illustrations from http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle

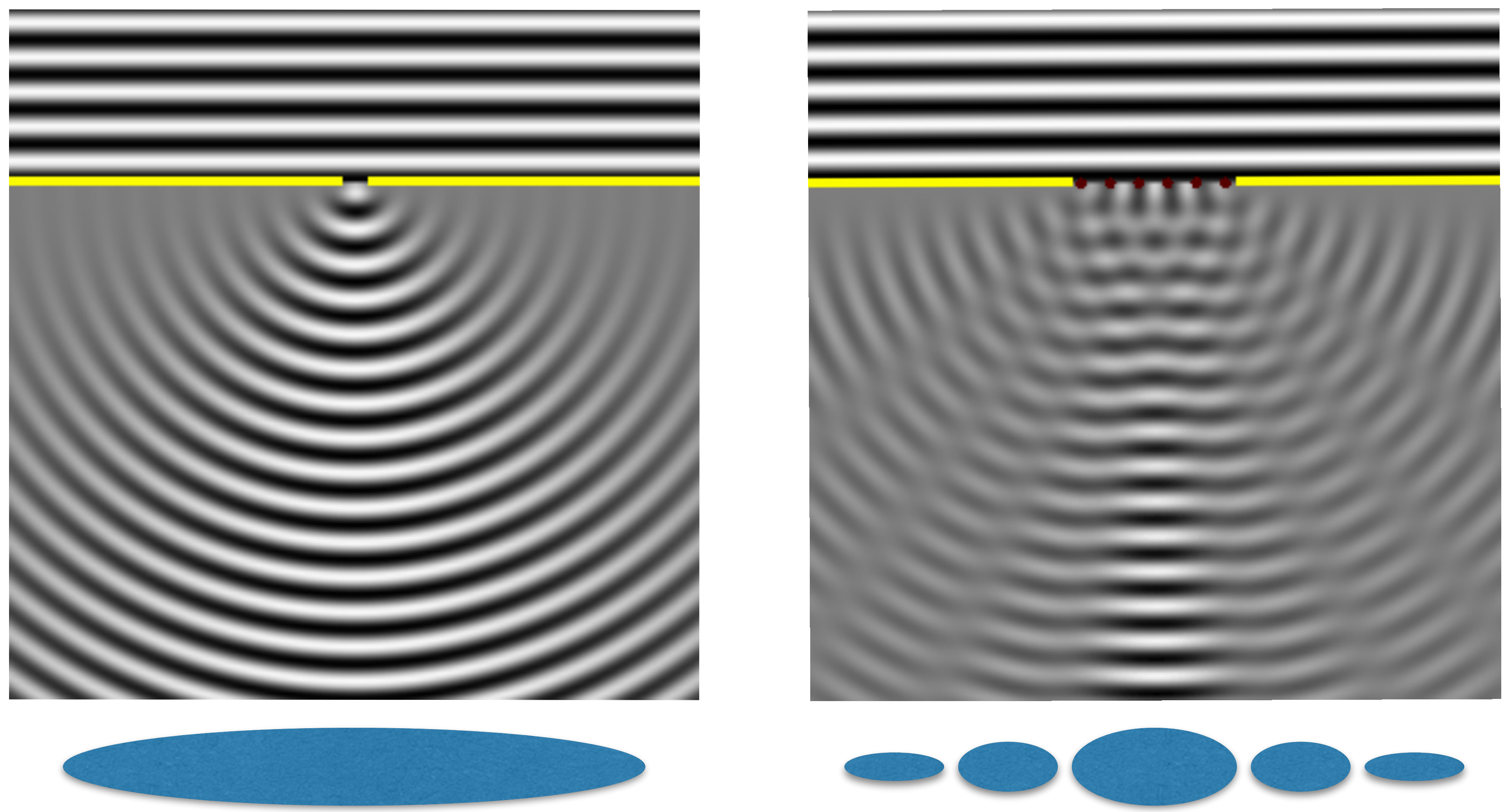
Larger apertures produce finer images

incoming wave

aperture

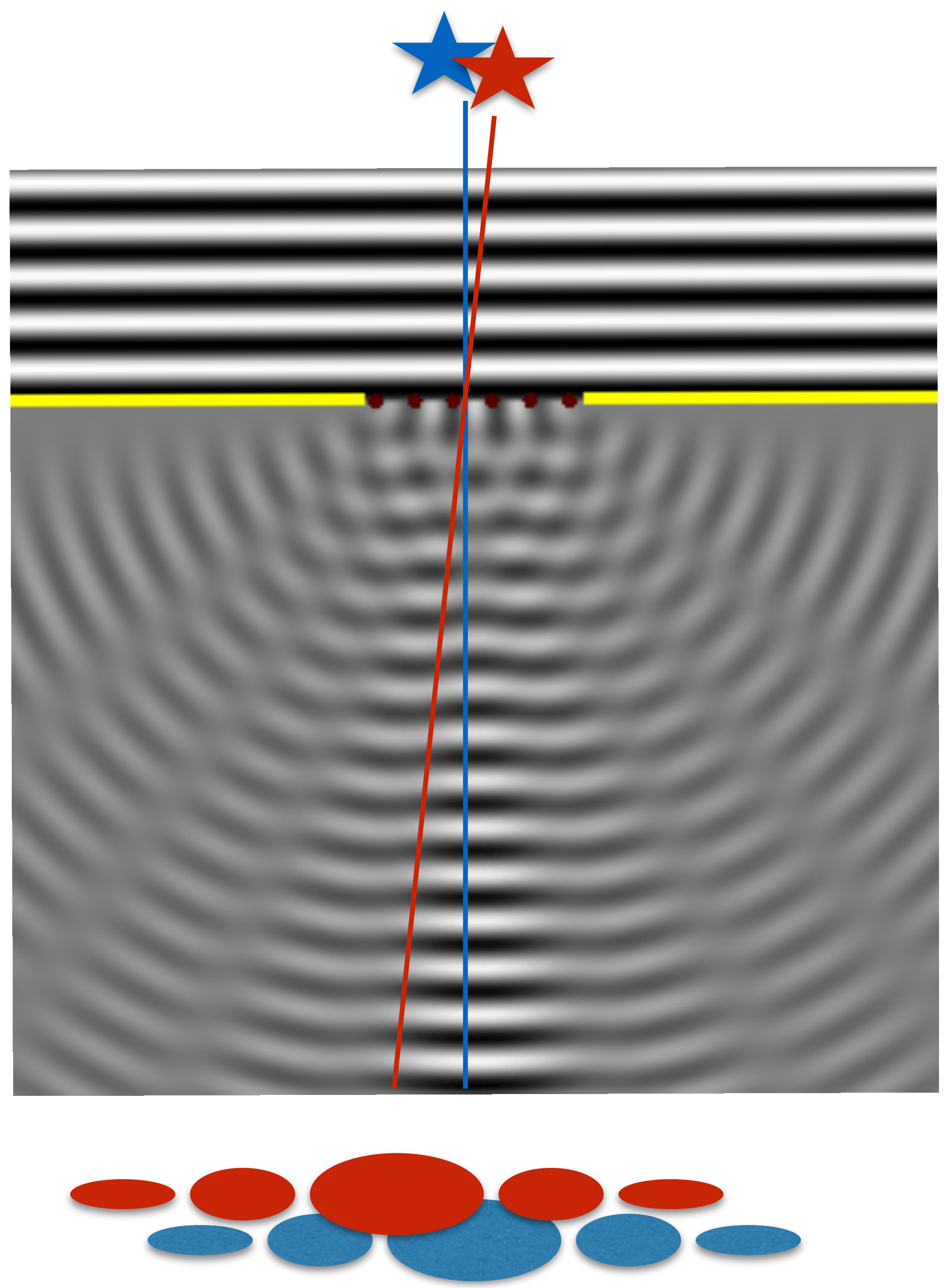
diffracted wave

image



illustrations from http://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle

Diffraction limits the angular resolution

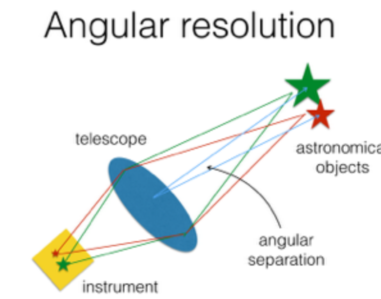


Confusion!

Image formation

- For a incoming **plane wavefront** (point like object)
- Each point of the **entrance pupil** generate a small wave
- $I \sim \iint pupil(x,y)e^{i(xa+by)}dxdy$
- The image of a point like object is the Fourier Transform of the pupil: **Point Spread Function**
- The observed image is the convolution of the true image by the PSF

Angular resolution

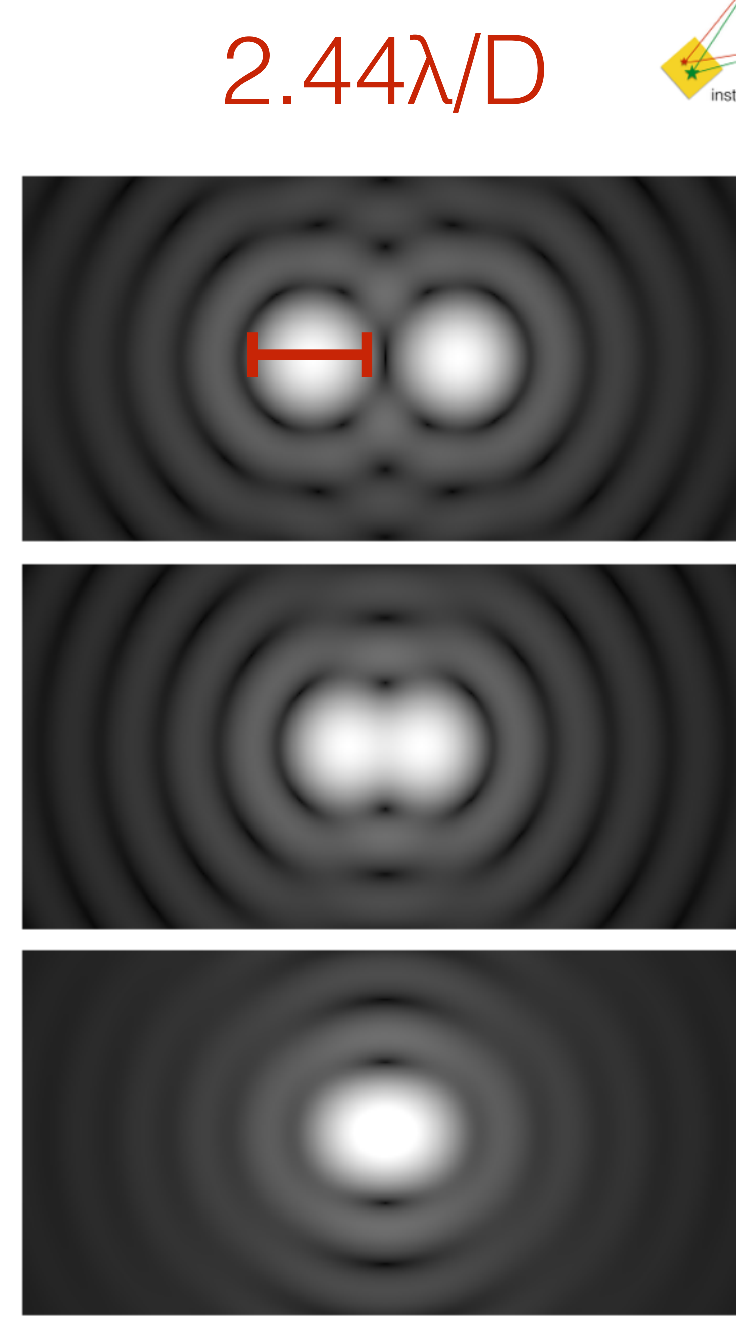


- The PSF of a **circular aperture** in an **Airy pattern**

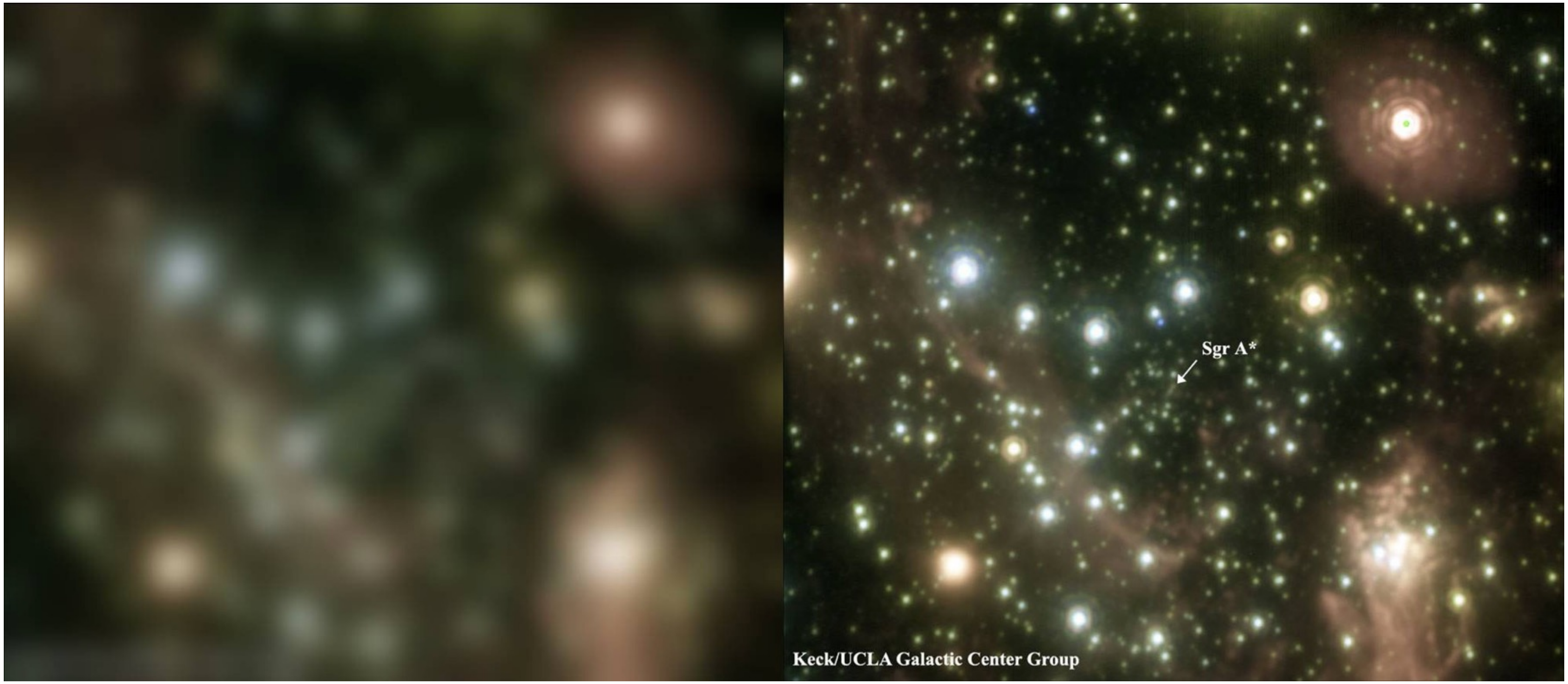
- For diameter D and wavelength λ:

  - **First null occurs at angle 1.22λ/D** (radians)

  - The input aperture act as a **low pass filter with cutoff frequency λ/B**



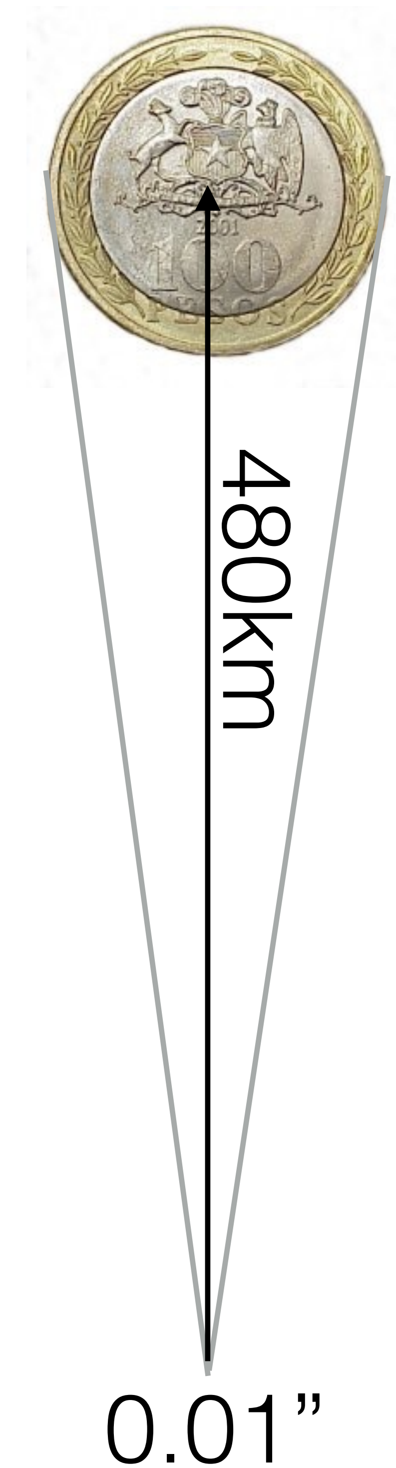
The need for angular resolution



Galactic Centre seen without and with Adaptive Optics

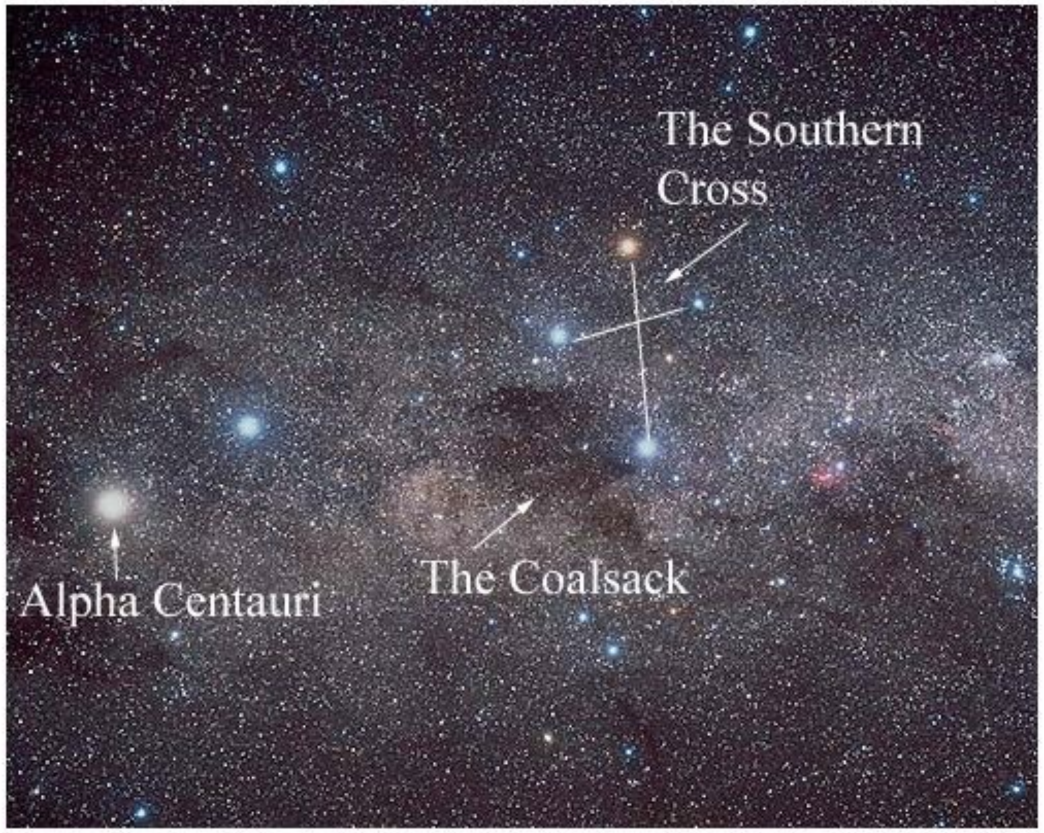
Limitations to angular resolution

- **Diffraction: ~λ/D**
  - D=10m, λ=500nm → 3e-6° ~ 0.01”
- **Atmospheric turbulence**
  - typically limits to ~1” (in the visible)
  - best sites down to ~0.5” or less
  - adaptive optics bring diff limit for λ~1μm



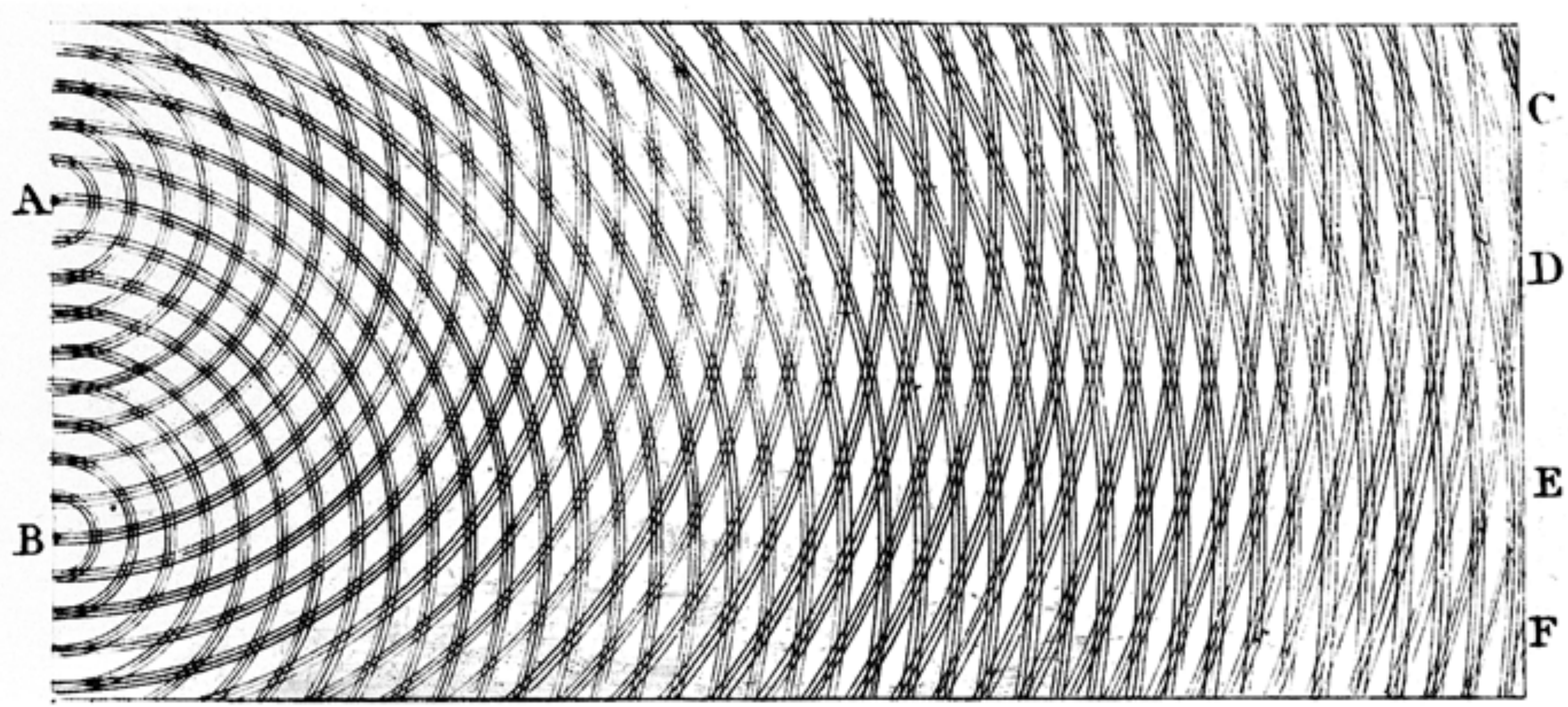
What is the angular size of stars?

- The Sun is 30’ at 1au (~1/100 radian)
- Alpha Cen is ~300000 times further away
- The Sun appears 0.006” in diameter from Alpha Cen



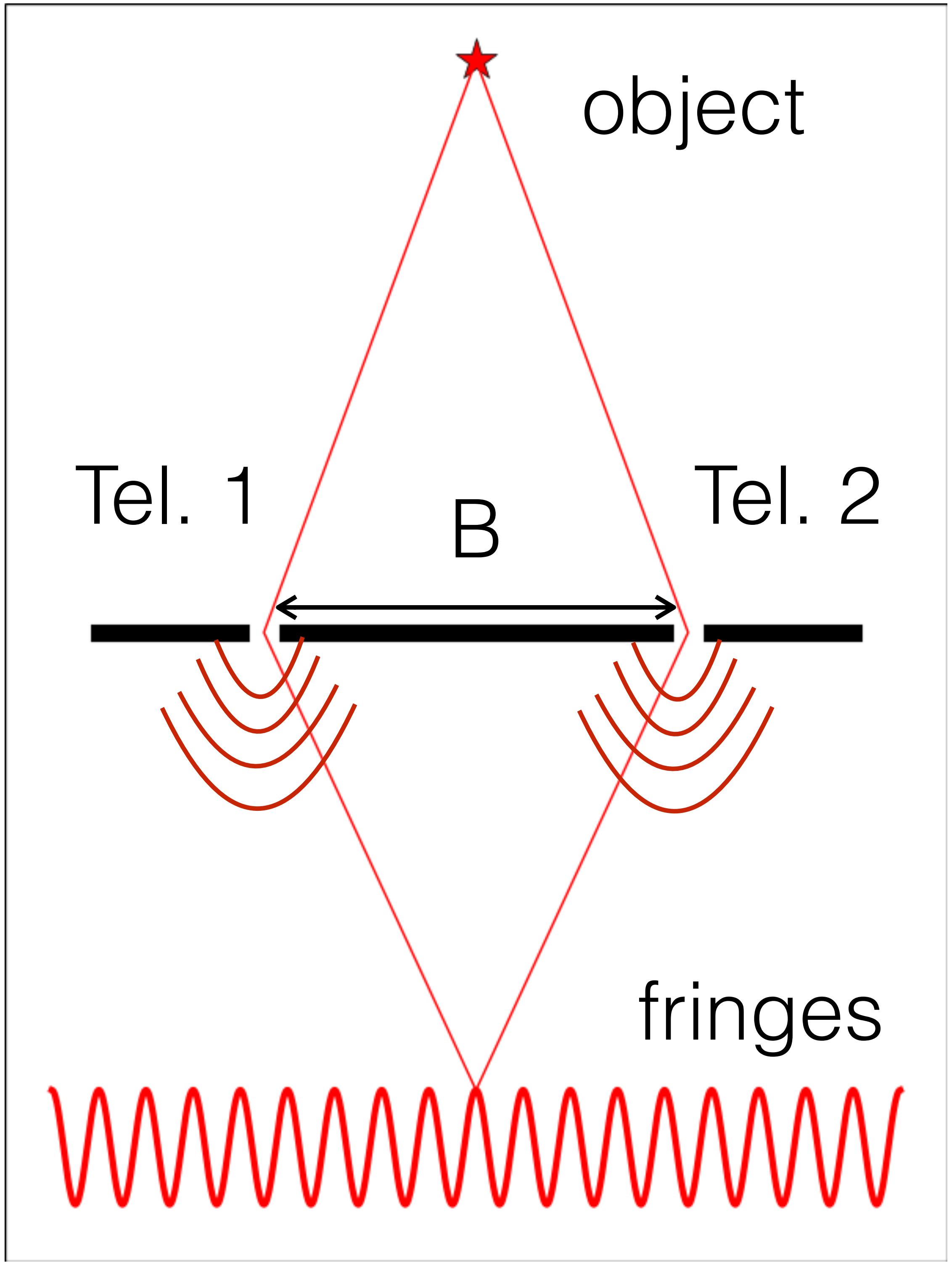
Basic Principles of Interferometry

Young's experiment

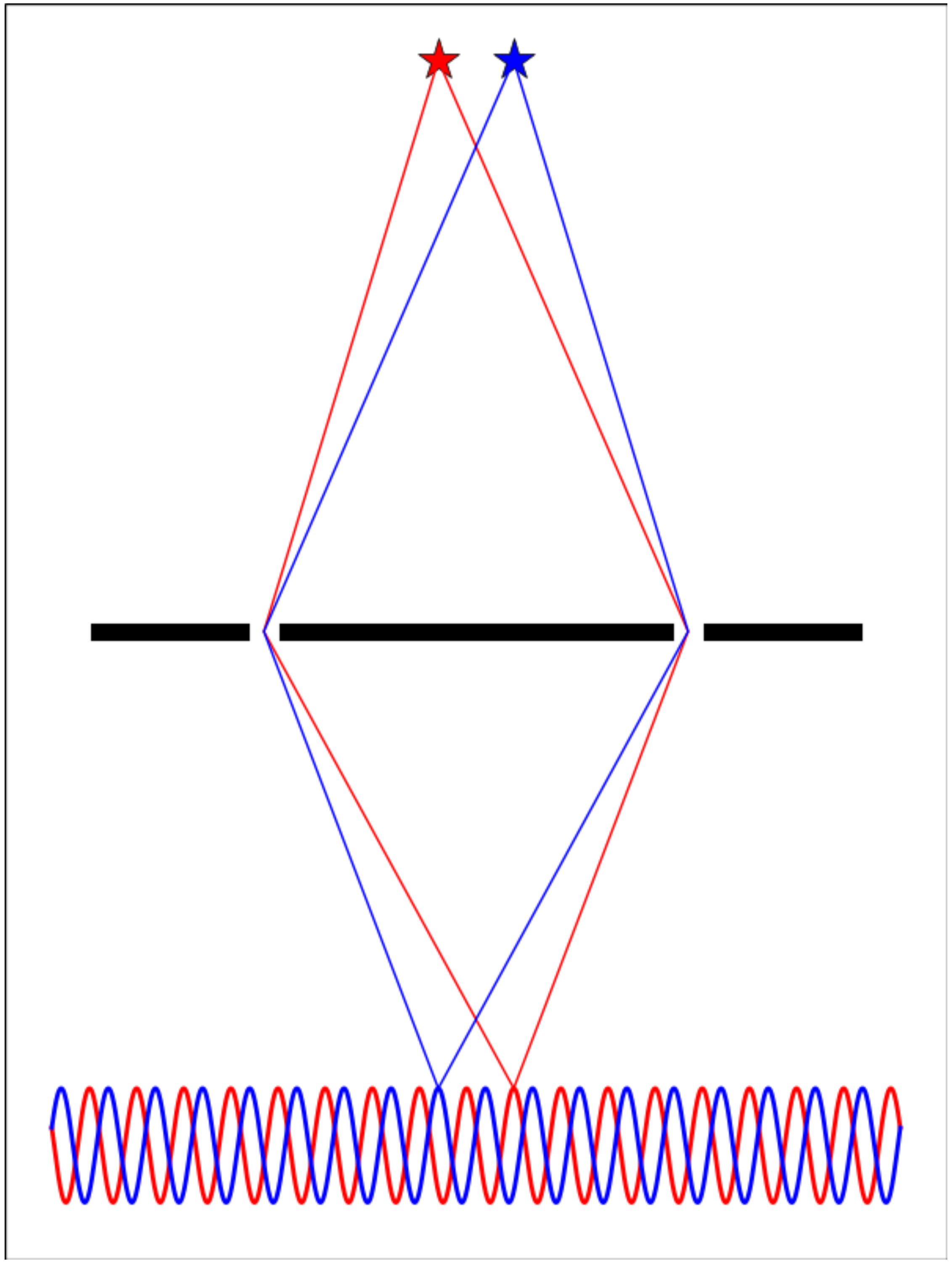


"On the Theory of Light and Colours"
Thomas Young, 1801

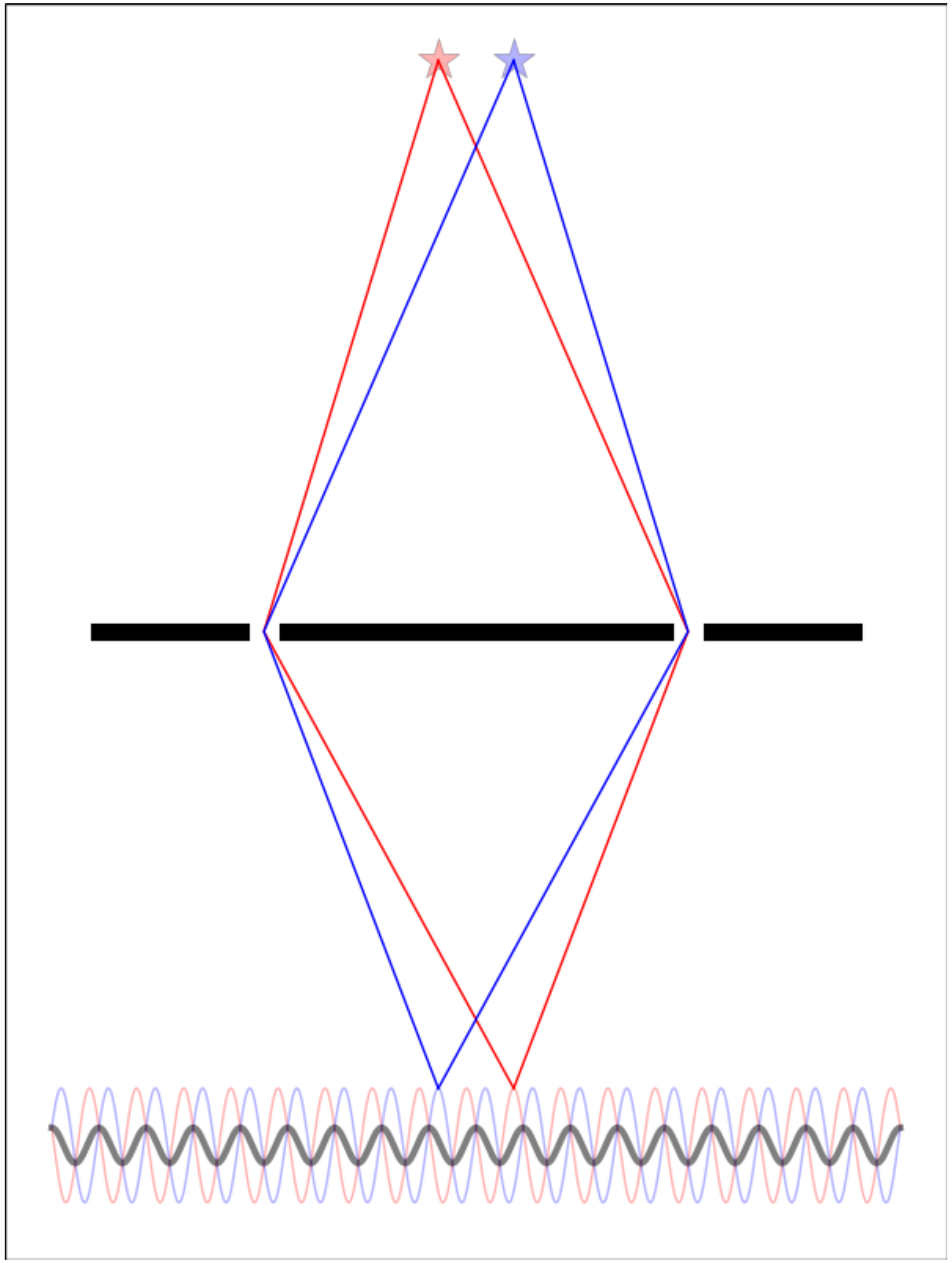
A Simple interferometer



Angular Resolution



Angular Resolution



Basic principles

- Fringes replace images
- 2 objects separated by angle **s** will produce fringes offset by **B.s**
- Fringes will disappear for **B.s = λ/2**
- If fringes amplitude is measured with accuracy, separation power < **λ/2B**

Accurate Formulation

- The complex visibility is the normalized Fourier Transform of the brightness distribution:

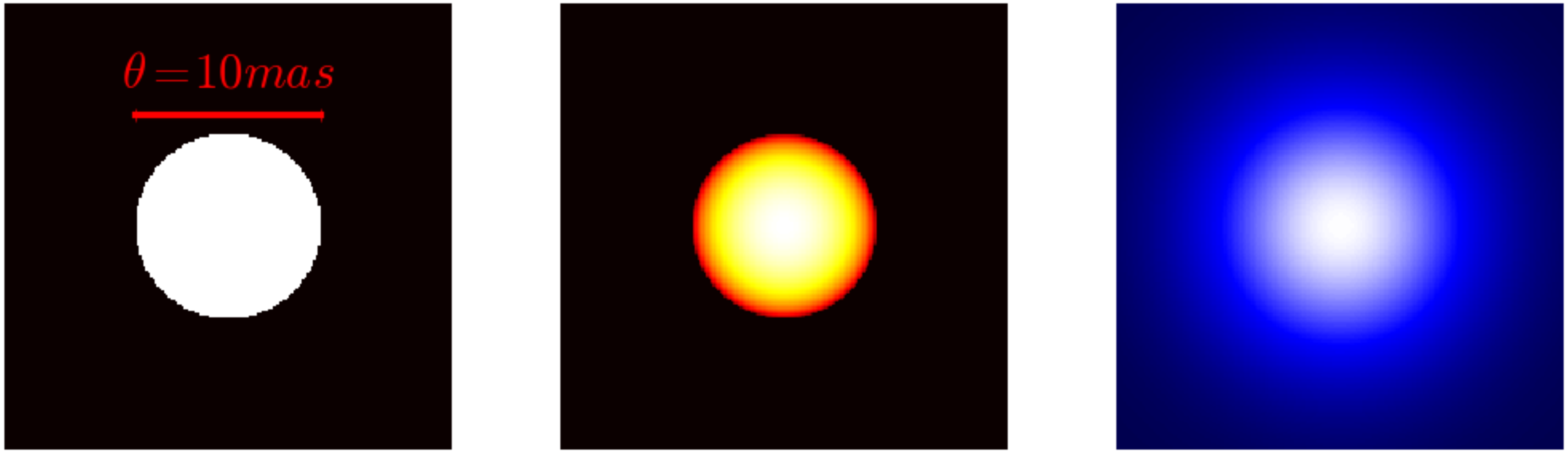
$$V(u,v,\lambda) = \frac{\int I(x,y,\lambda)e^{-i(xu+yv)/\lambda}dxdy}{\int I(x,y,\lambda)dxdy}$$

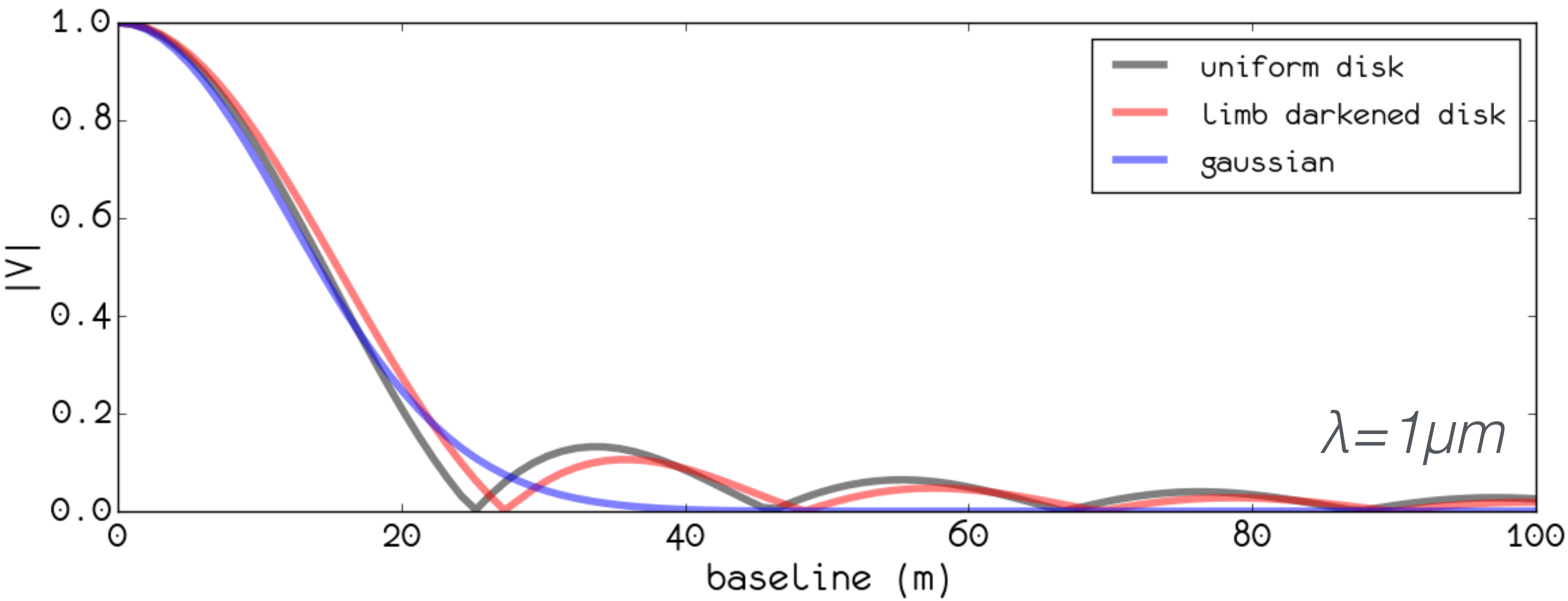
- This is the van Cittert-Zernike Theorem

$$V_{\mathrm{UD}}(B,\lambda) = \left|2\frac{J_1(x=\pi B\theta/\lambda)}{x}\right|$$

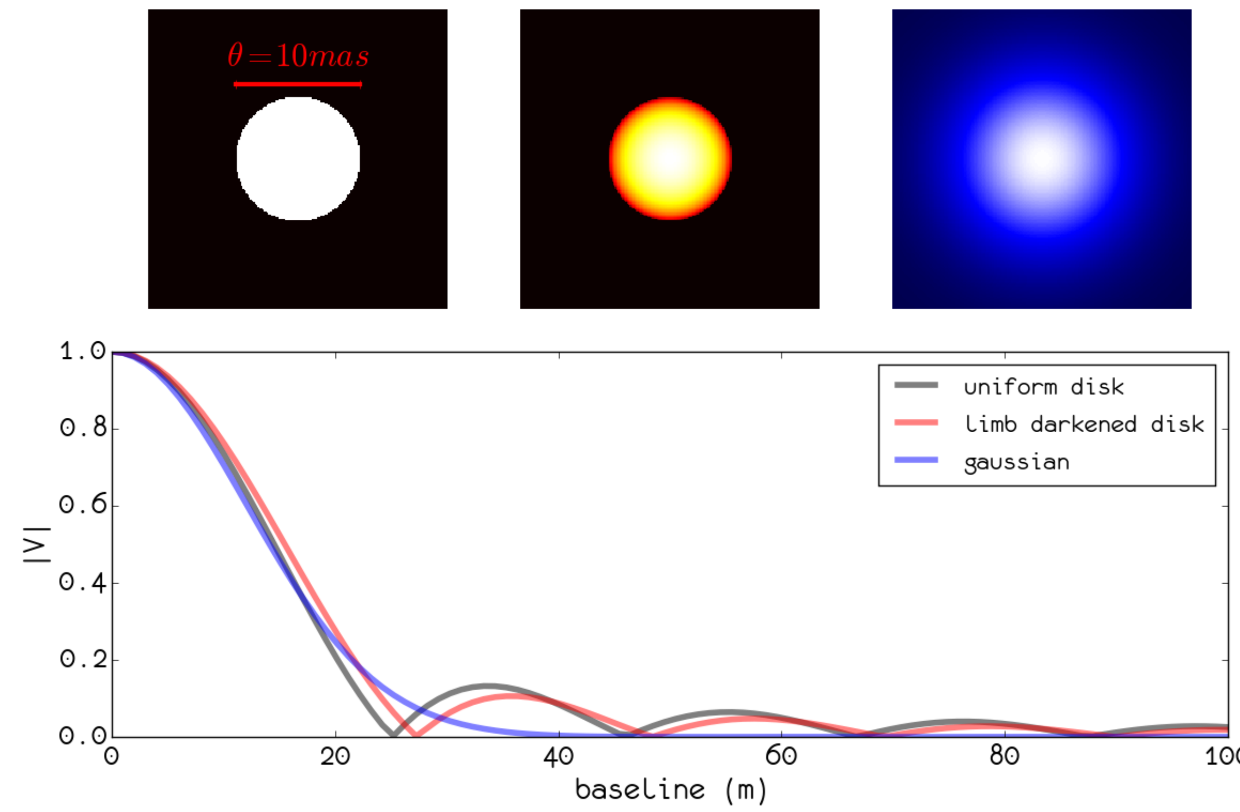
centro-symetric:
Hankel transform

$$V(B,\lambda) = \frac{\int I(r,\lambda)J_0(rB/\lambda)rdr}{\int I(r,\lambda)dr}$$



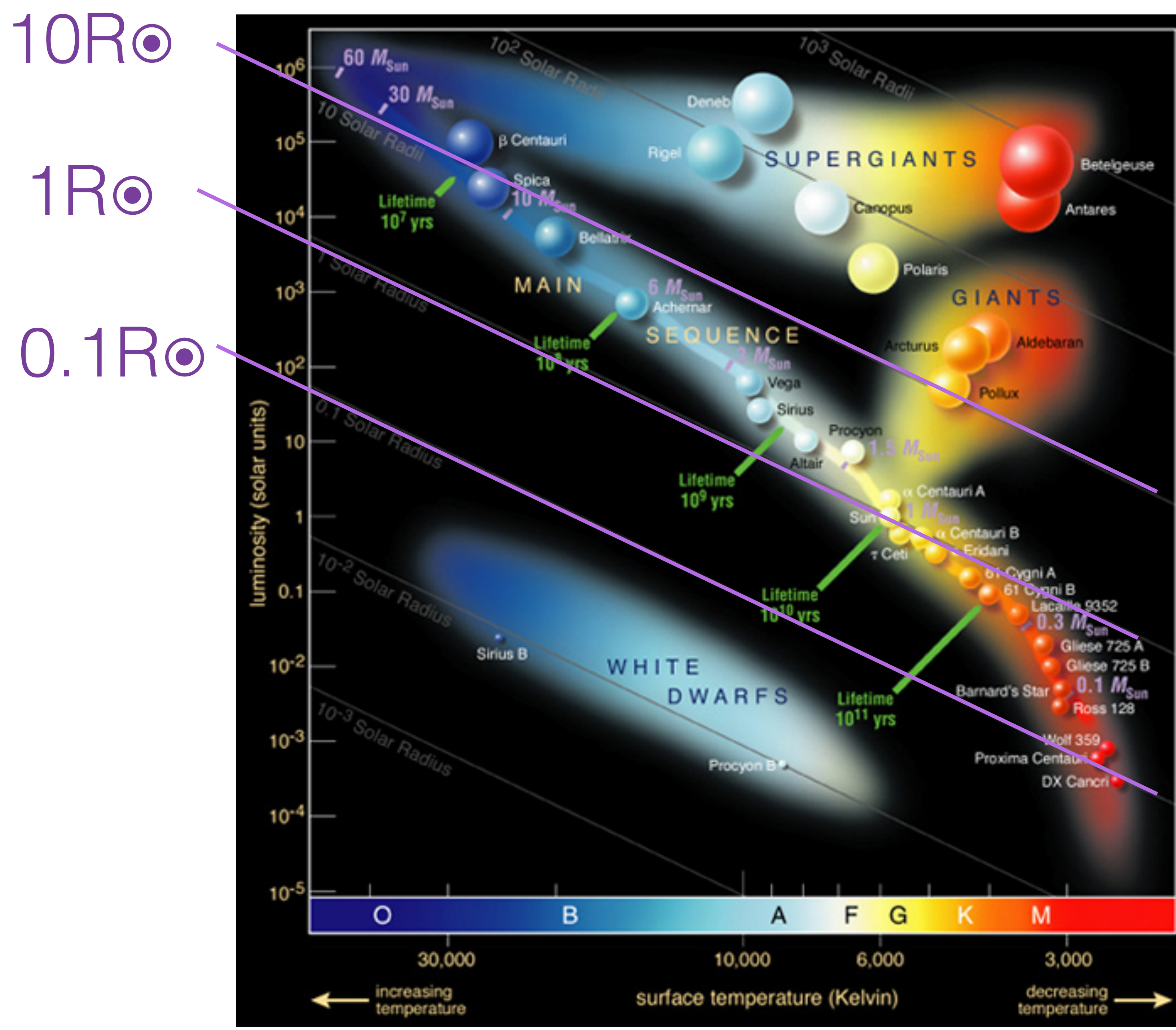


- Measuring diameters == inverting V(B,Θ,λ)
- True stars are NOT uniform disks
- limb darkening
    - lowers the visibility lobes
    - bias the diameter measurements



diameters of stars

- Relation between fundamental parameters of stars: **Temperature**, **Luminosity**, **Radius**
- Absolute L ~ R^2T^4
- Apparent L ~ Θ^2T^4



What have we seen so far?

Telescope of **diameter D**:

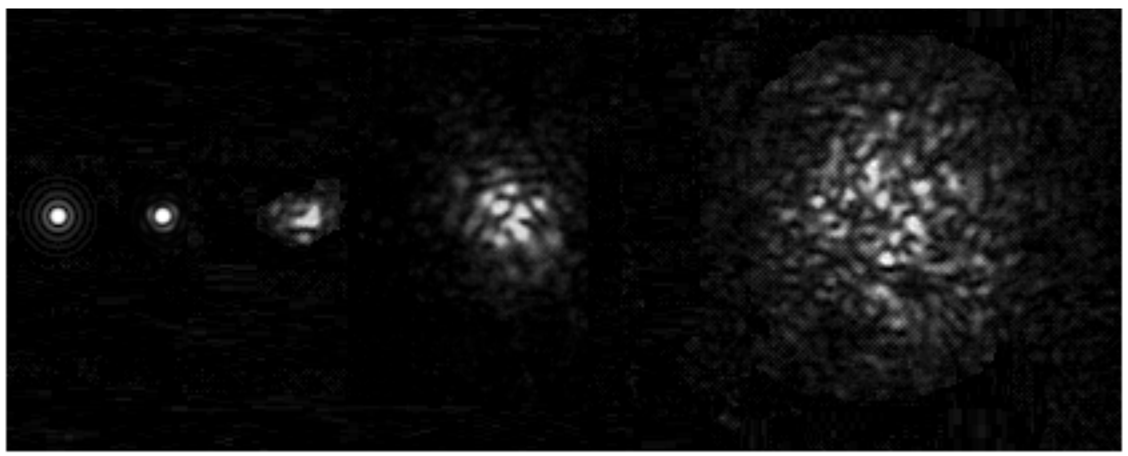
- **low pass** spatial frequency filter
- angular resolution ~**λ/D**
- strongly limited by atmospheric turbulence

Interferometer of **baseline B**:

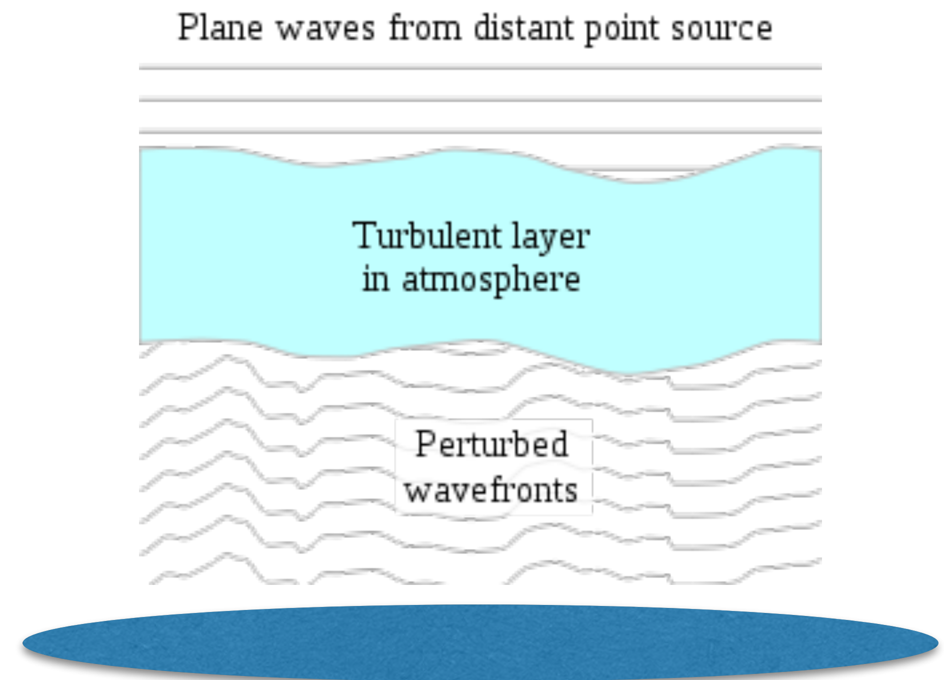
- **band pass** spatial frequency filter
- angular resolution ~**λ/B**
- effect of atmospheric turbulence???

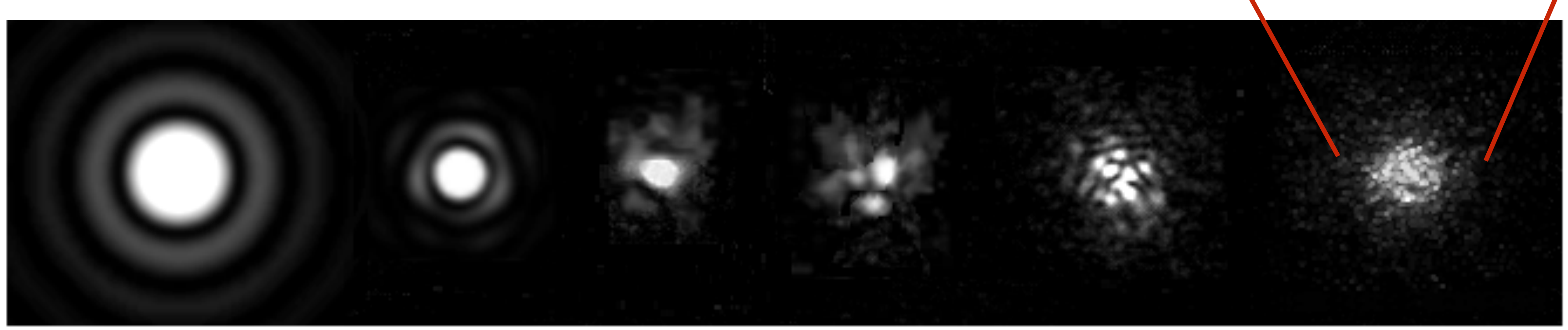
Practical Use of Optical Interferometry

Turbulence: single aperture



Single Aperture images
as turbulence degrades

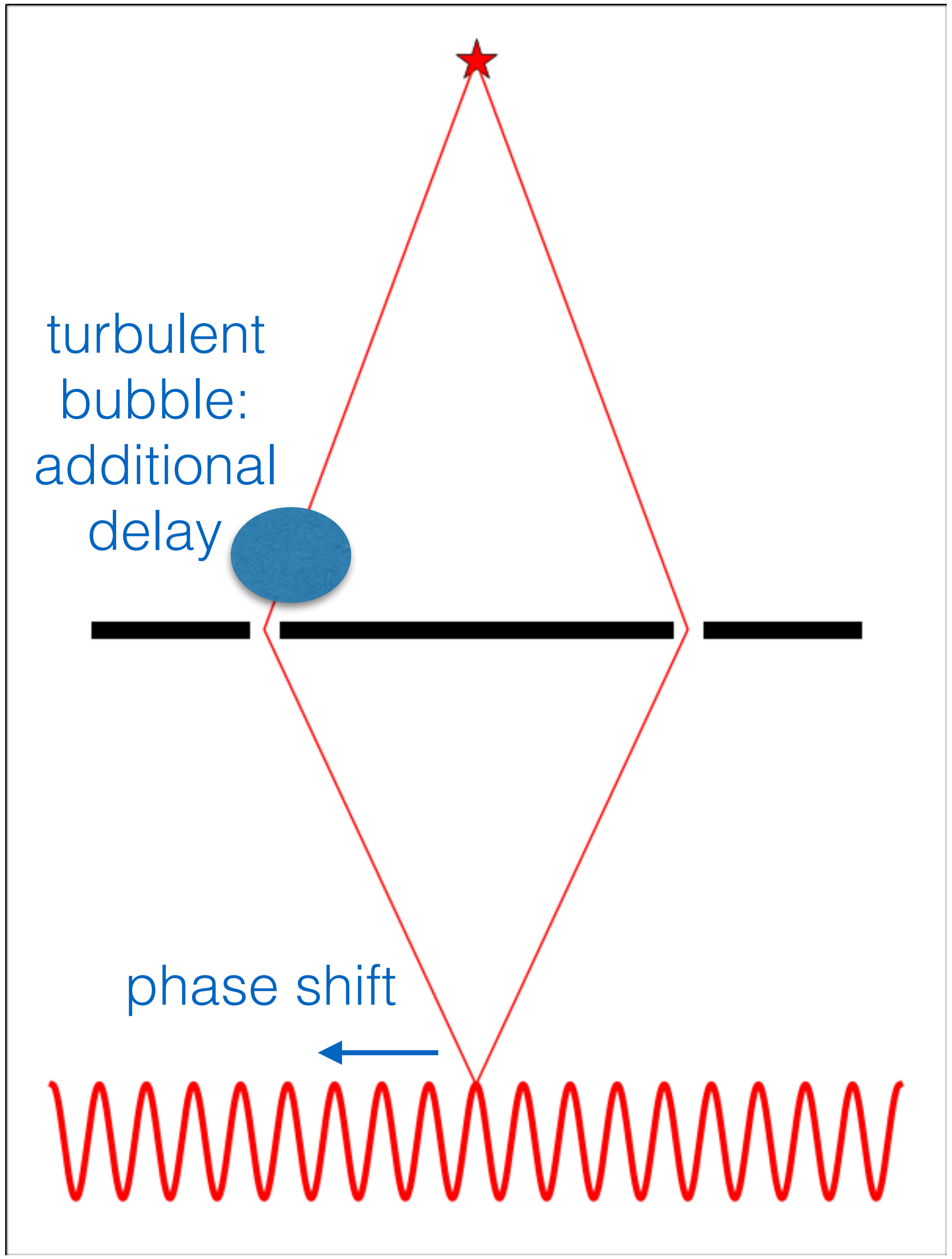




Single Aperture images as telescope aperture increases

1rst order: Piston

- First order: difference of refraction index produced **differential delay**
- Additional delay produce a **phase shift** in fringes.

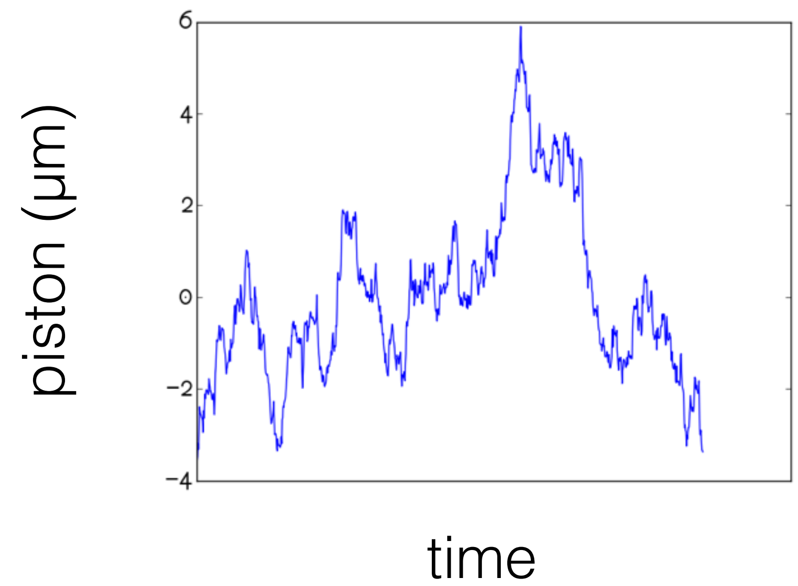


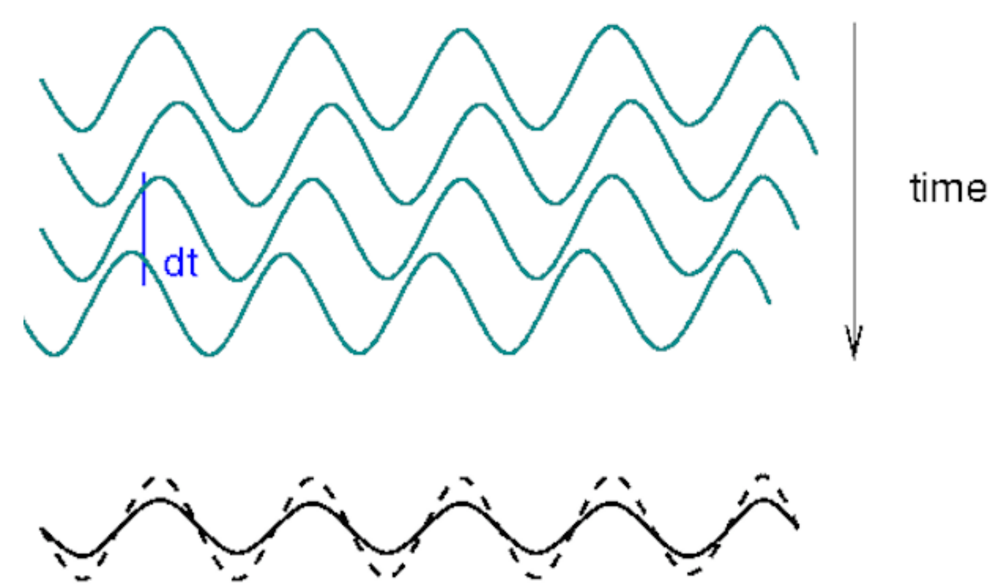
Piston effects

- Turbulence has a **low frequency excess** spectrum

- averaging in time: **contrast is lowered** and **phase information is lost**

- The longer the exposure time, the worse the contrast loss (!)





time averaged fringes

Piston effect

Why do radio interferometer measure phases?

- in cm regime, piston << λ
- in mm regime, piston dominated by water vapor fluctuations, which can be measured by radiometer (ALMA)
- in optical and near-IR amplitude of piston is >> λ

Fundamental effect of turbulence

Turbulence has a negative power law power spectrum. There is more effect at low frequencies than high frequencies (*Excess Low Freq. noise*)

- Short exposures freeze the turbulence
- Long exposures result in loss of fringe contrast

The need for calibration

- Measuring diameters rely on inverting V(B,Θ,λ)
- Measure of V(B,Θ,λ) is biased by atmosphere turbulence
- Necessity to measure objects with *known* V(B,Θ,λ): **calibrators**
- **Stellar calibrators** are preferably unresolved (V~1) to avoid relying on diameter estimation
- Calibrators should be observe under **same conditions** (instrument, turbulence) as the unknown target

What we have seen so far

- Fringe **visibility** is the main observable
- Visibility is the **Fourier Transform of the brightness distribution**
- Interferometer is a **spatial frequency band pass filter**
- 2T: **stellar angular diameters** can be inferred from measurements and known V(B,Θ,λ)
- Atmospheric piston leads to **loss of absolute phase** and the need for **short exposures** and **stellar calibrators**

The need for phase

- Binary star

star 1 *star 2* *modulation*

$$V(u,v,\lambda,x,y,f) = \frac{V_1(u,v,\lambda) + fV_2(u,v,\lambda)e^{2i\pi(xu+yv)/\lambda}}{1+f}$$

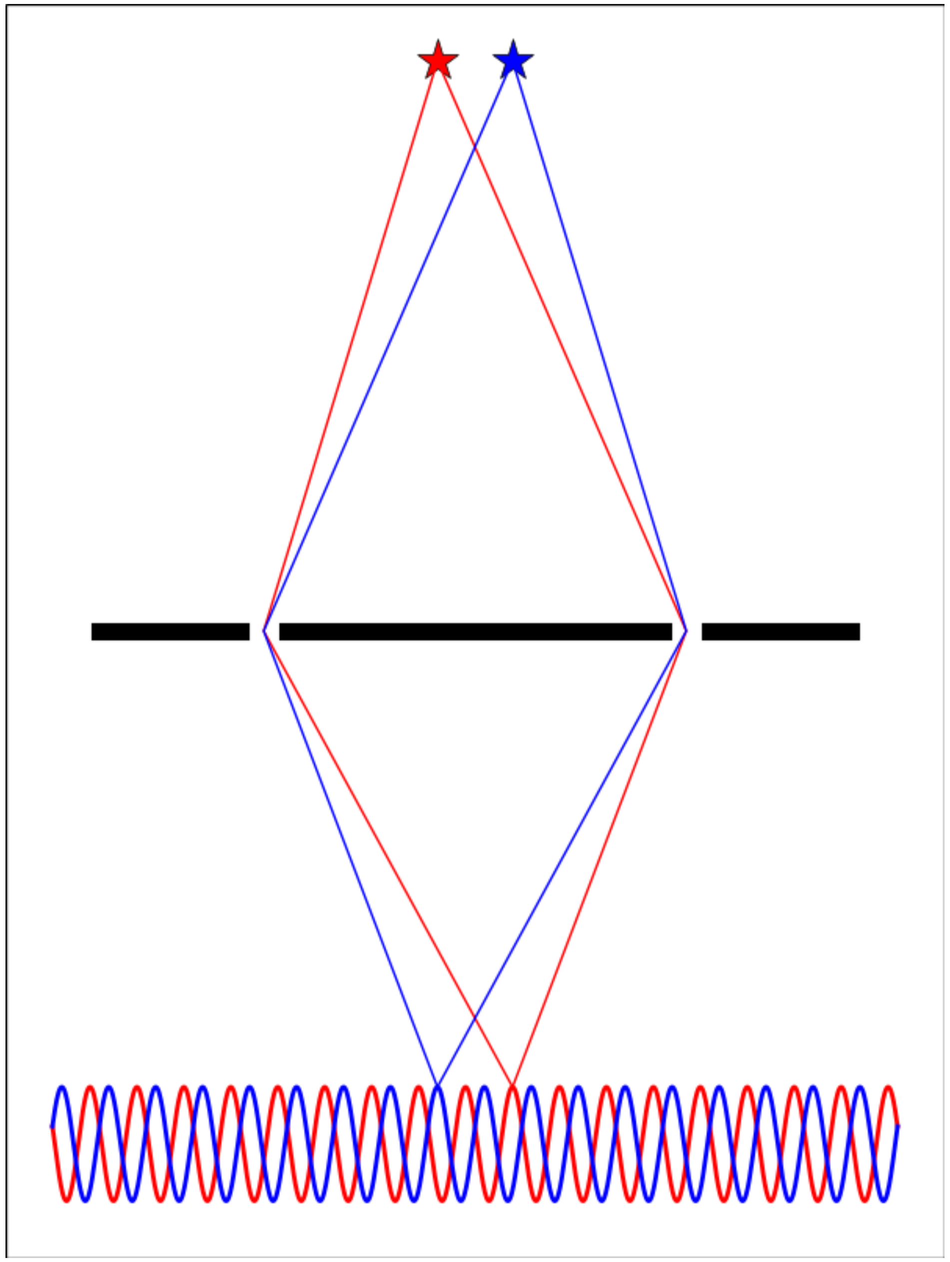
baseline wavelength separation flux ratio

- |V(x,y)| = |V(-x,-y)|

- If one measures only |V|, the asymmetry of an object cannot be assessed!

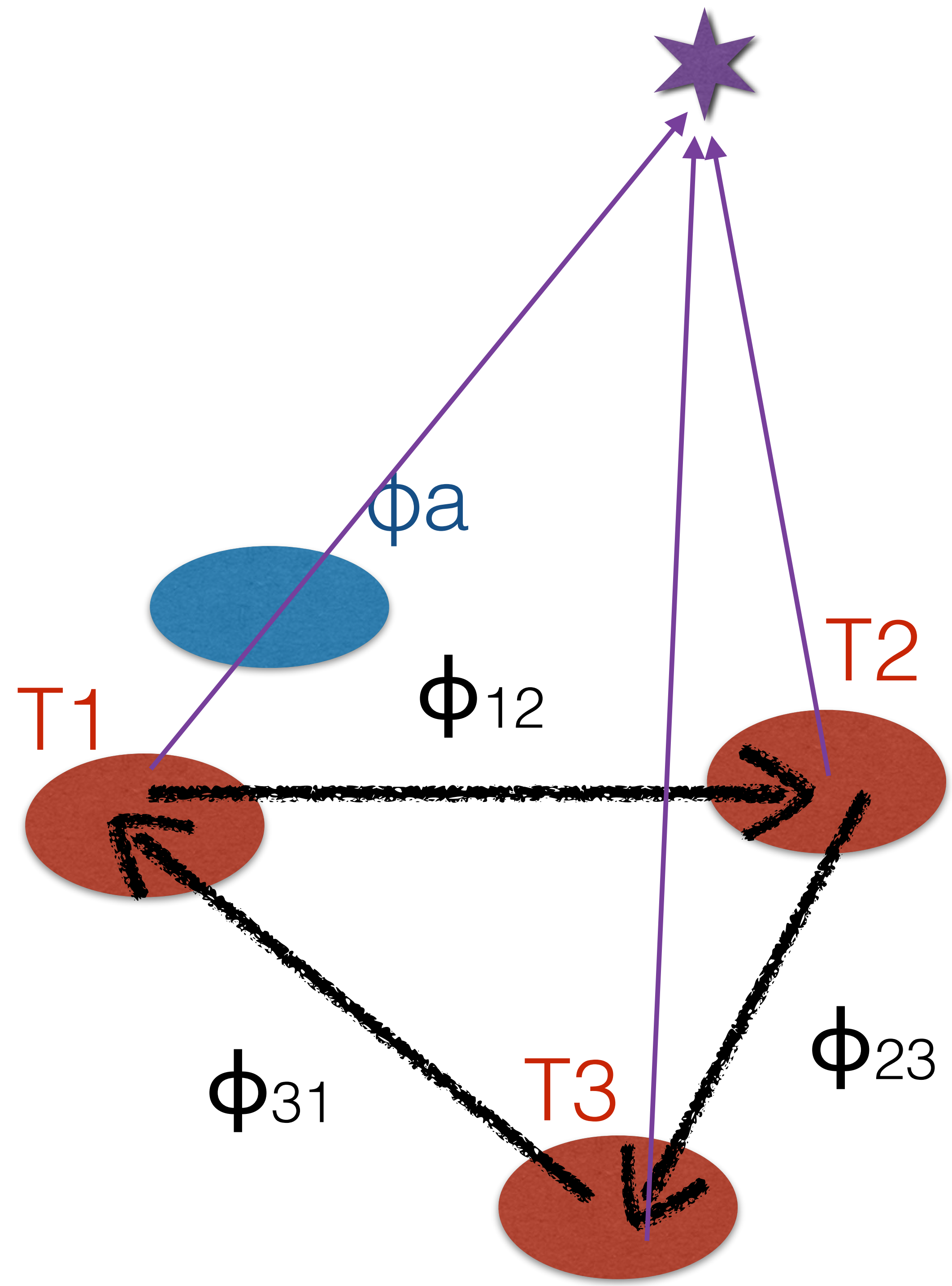
Phase = Astrometry

- The phase of the fringes is the astrometric position of the object
- Binary **fringe's contrast reduction results from a phase difference between the 2 objects**



Closure Phase

- For 2T, absolute phase is lost due to atmosphere
- For 3T there is a trick
- Measure phase sum in a **close triangle**
- CP = $(\phi_{12} + \phi_a) + (\phi_{23}) + (\phi_{31} - \phi_a) = \phi_{12}+\phi_{23}+\phi_{31}$
- <u>CP is insensitive to atmospheric turbulence</u>



Advanced implementation

Modern Instruments

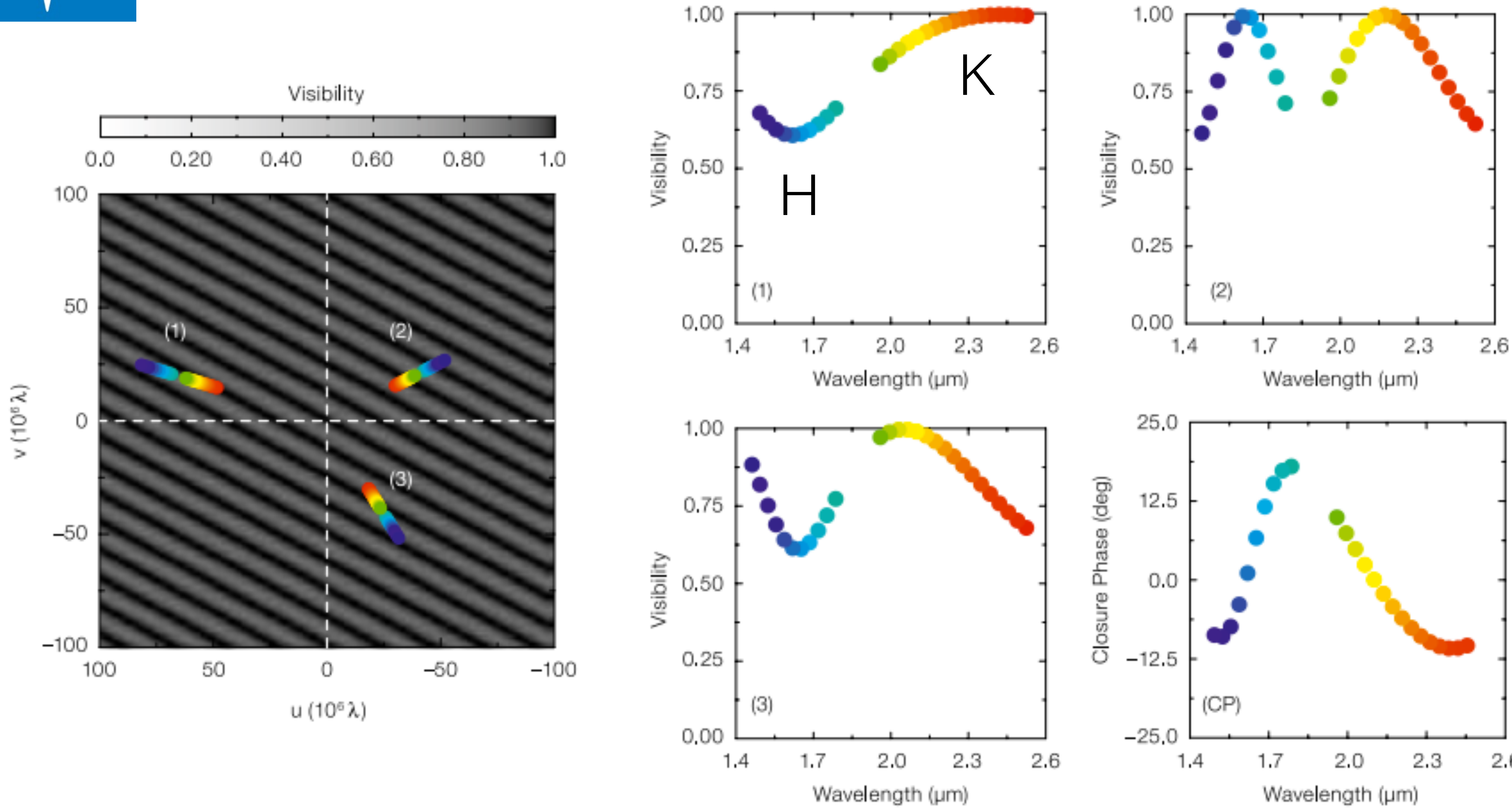
Most modern instruments contain some of the following:

- N Telescopes > 2
- Spectrally dispersed measurements
- Astrometry and/or phase referencing

Example: AMBER@VLTI

- AMBER combines 3 telescopes
- Near IR (J,H,K, λ=1→2.5μm)
- multiple spectral channels
- spectral resolution from R~30 to R~15000

Binary seen by AMBER



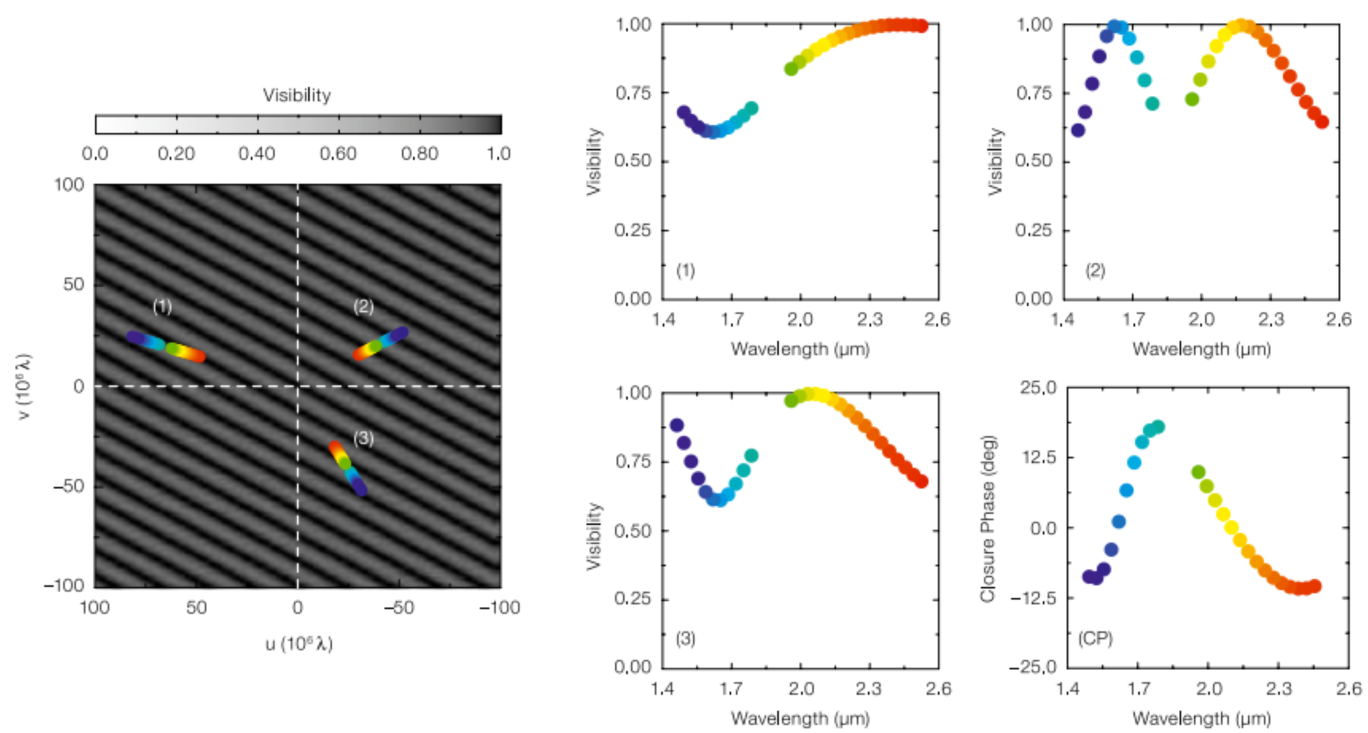
$$V(u,v,\lambda,x,y,f) = \frac{V_1(u,v,\lambda) + f V_2(u,v,\lambda) e^{2i\pi(xu+yv)/\lambda}}{1+f}$$

Kraus+ 2009 (The Messenger 136)

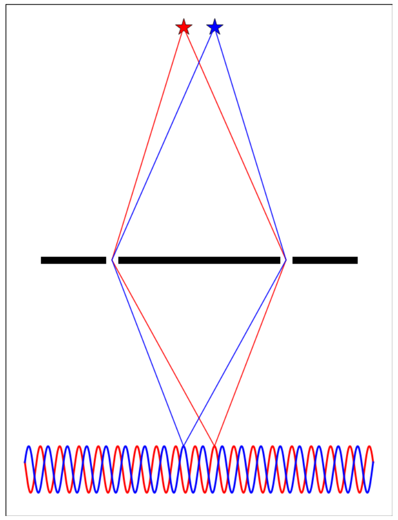
Achromatic objects

- Various spatial frequency B/λ simultaneously (**u,v coverage**)

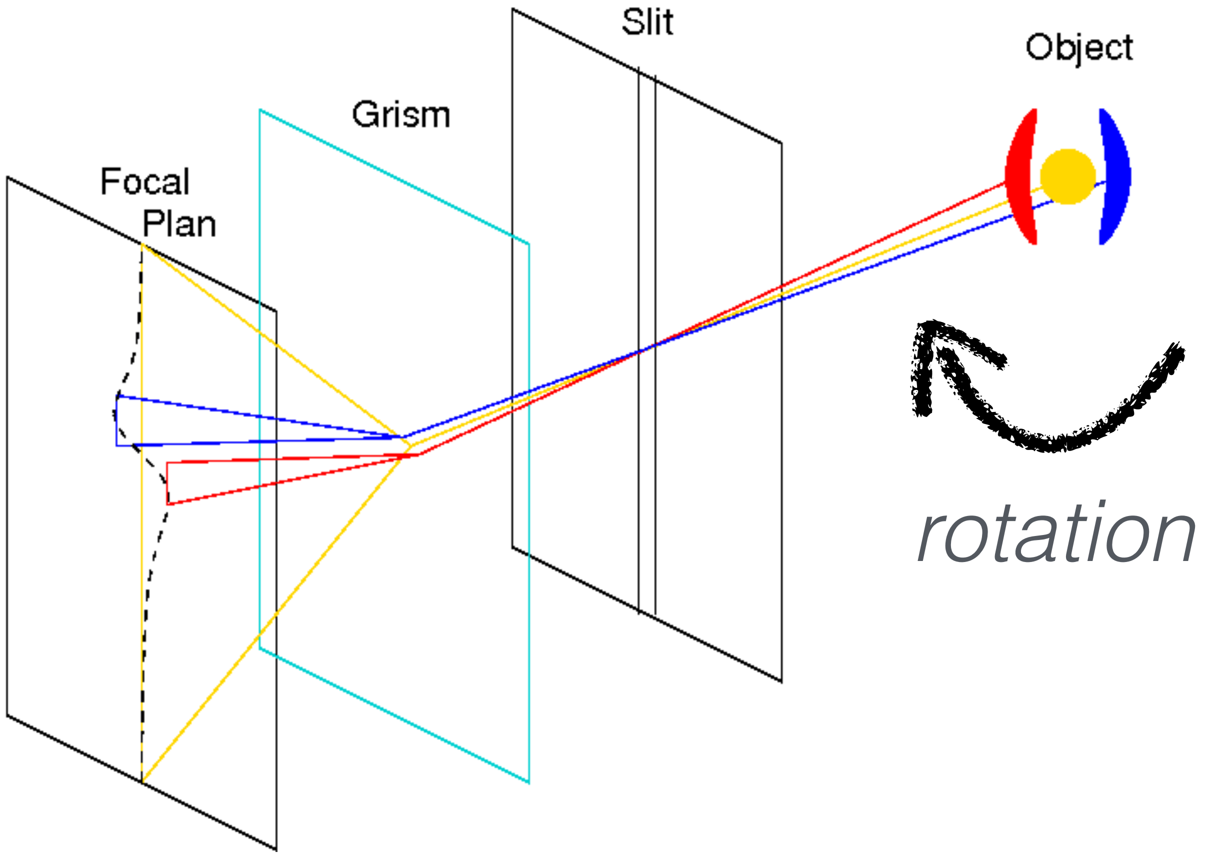
- **Differential measurement** (less sensitive to calibration uncertainty)



Spectro-astrometry



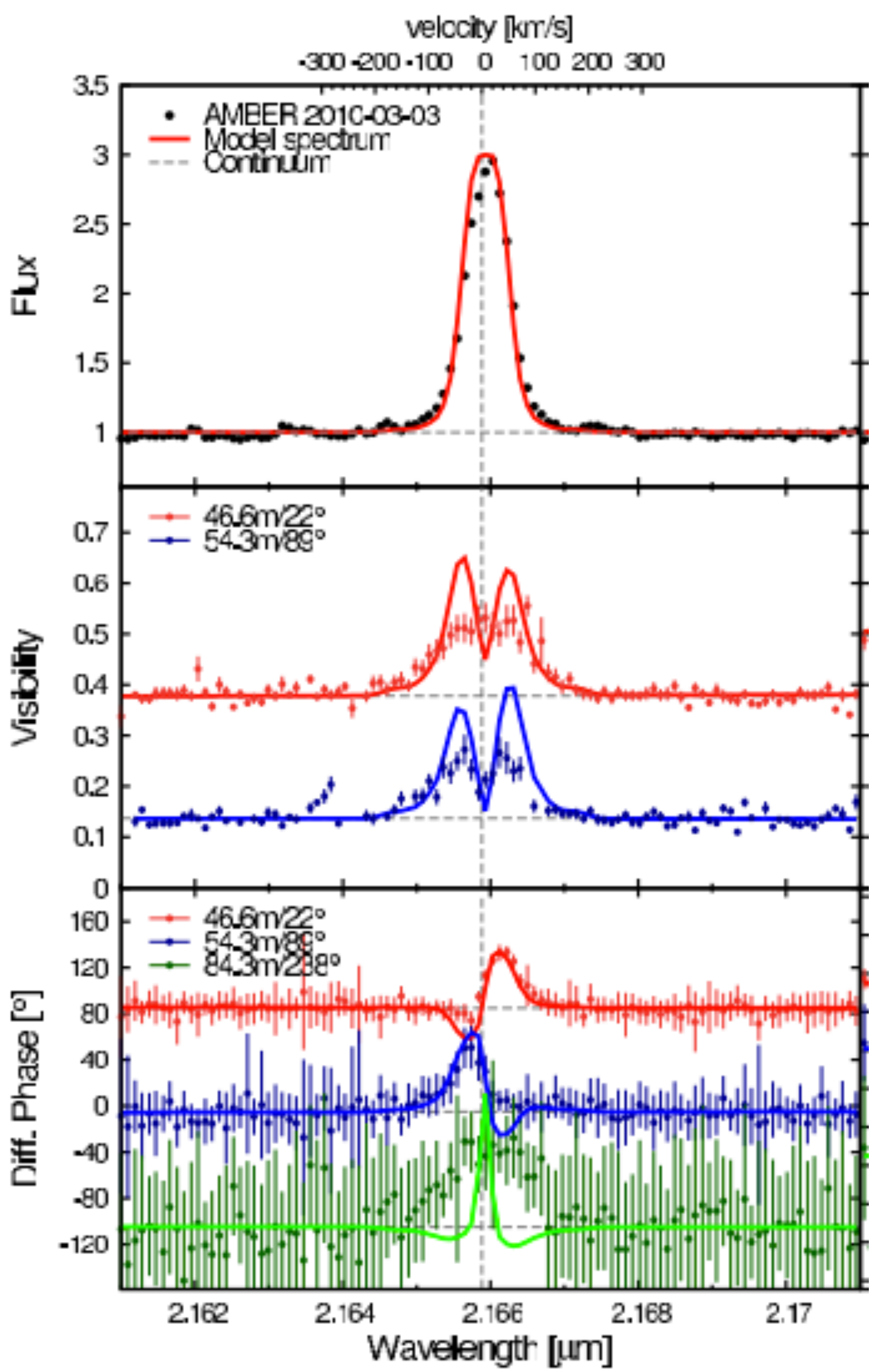
- Object in rotation (e.g. disk around a star)
- Spectral line with **Doppler shift**
- Phase of the fringes will vary throughout the line
- **Velocity field** and **geometric information** can be derived



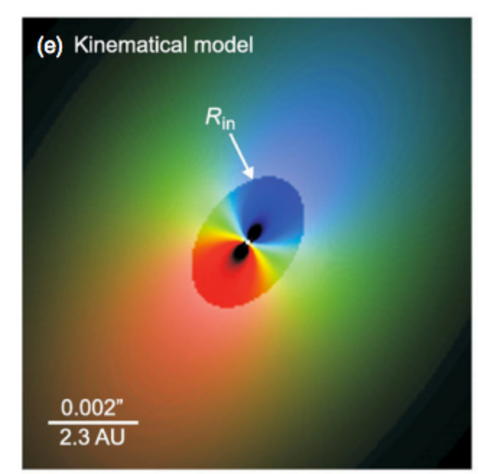
*fringes **differential** phase*

AMBER@VLTI

Herbig B[e] star (Kraus+ 2012)

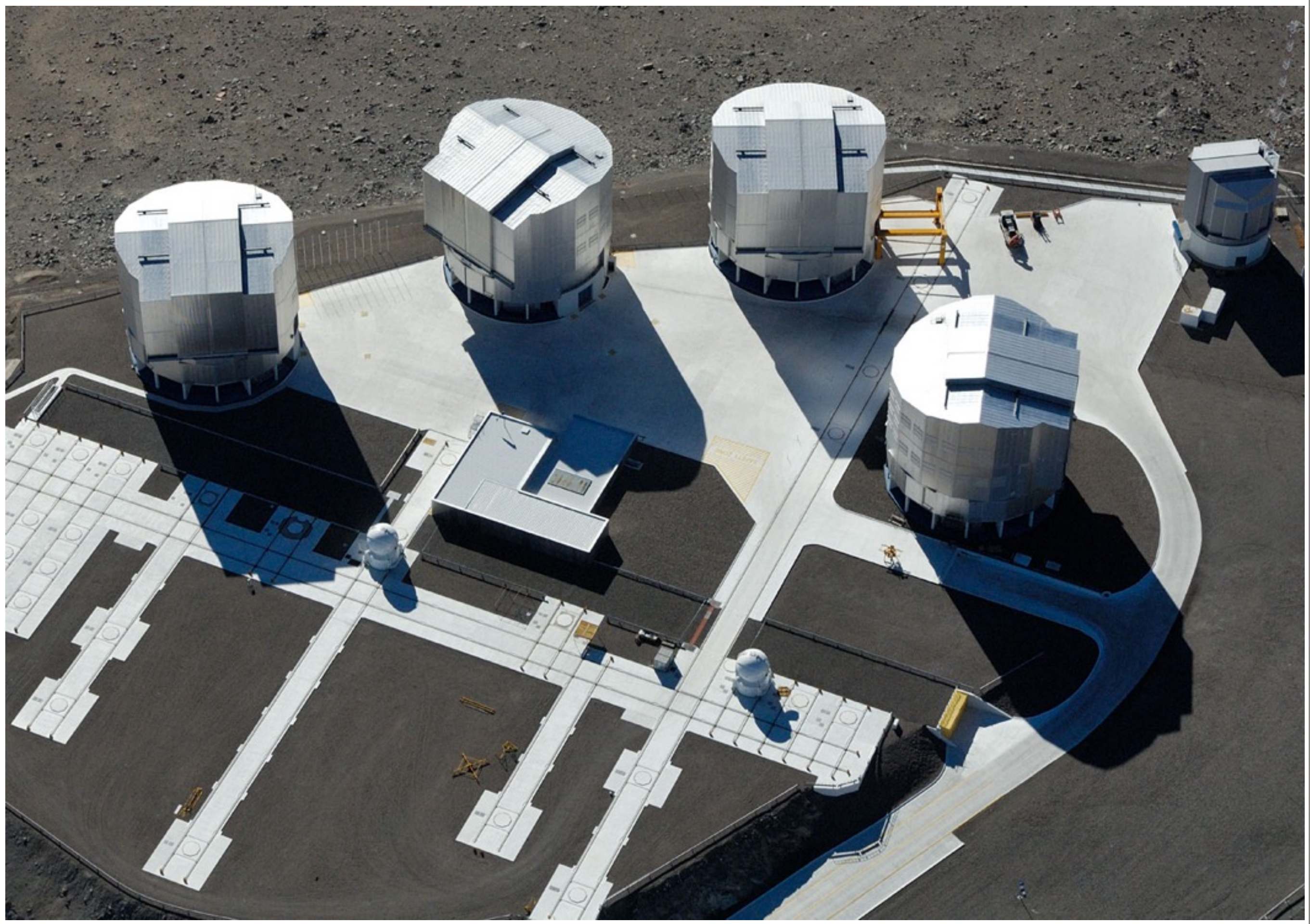


- Get differential phase in 3 baselines
- Allows to reconstruct the disk geometry
- Velocity field is Keplerian

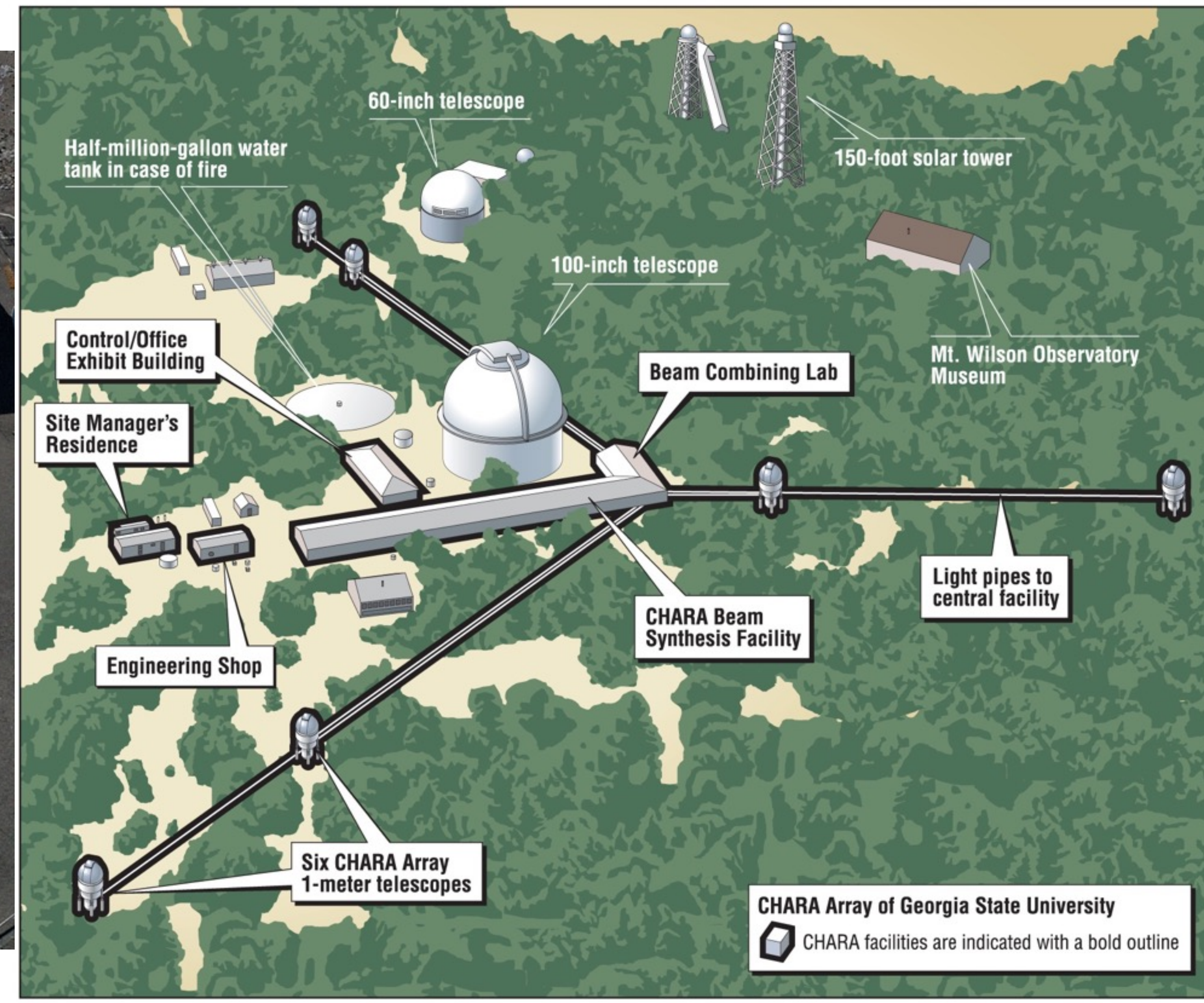


Real Interferometers

Real Interferometers

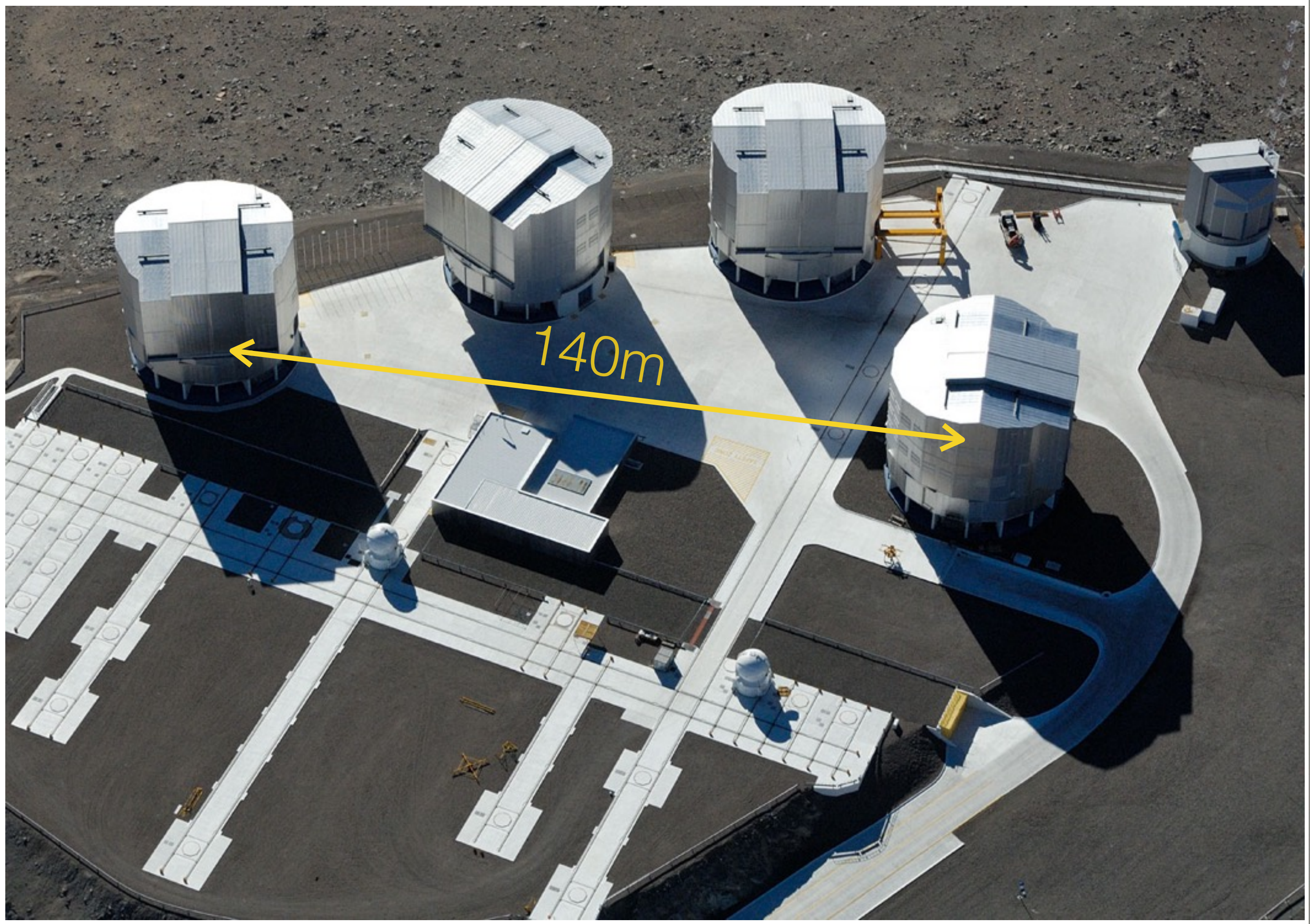


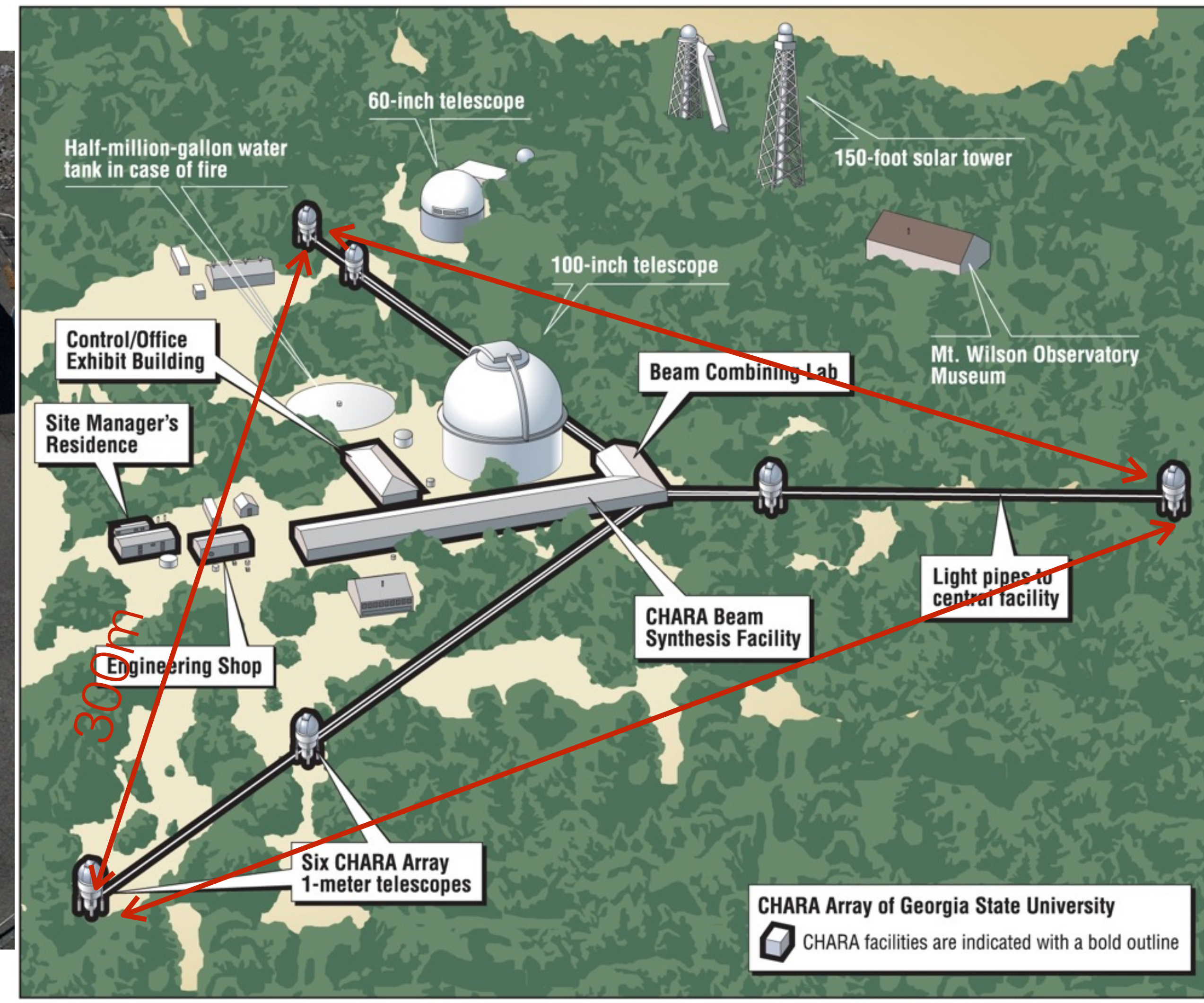
ESO VLTI



CHARA Array

Real Interferometers



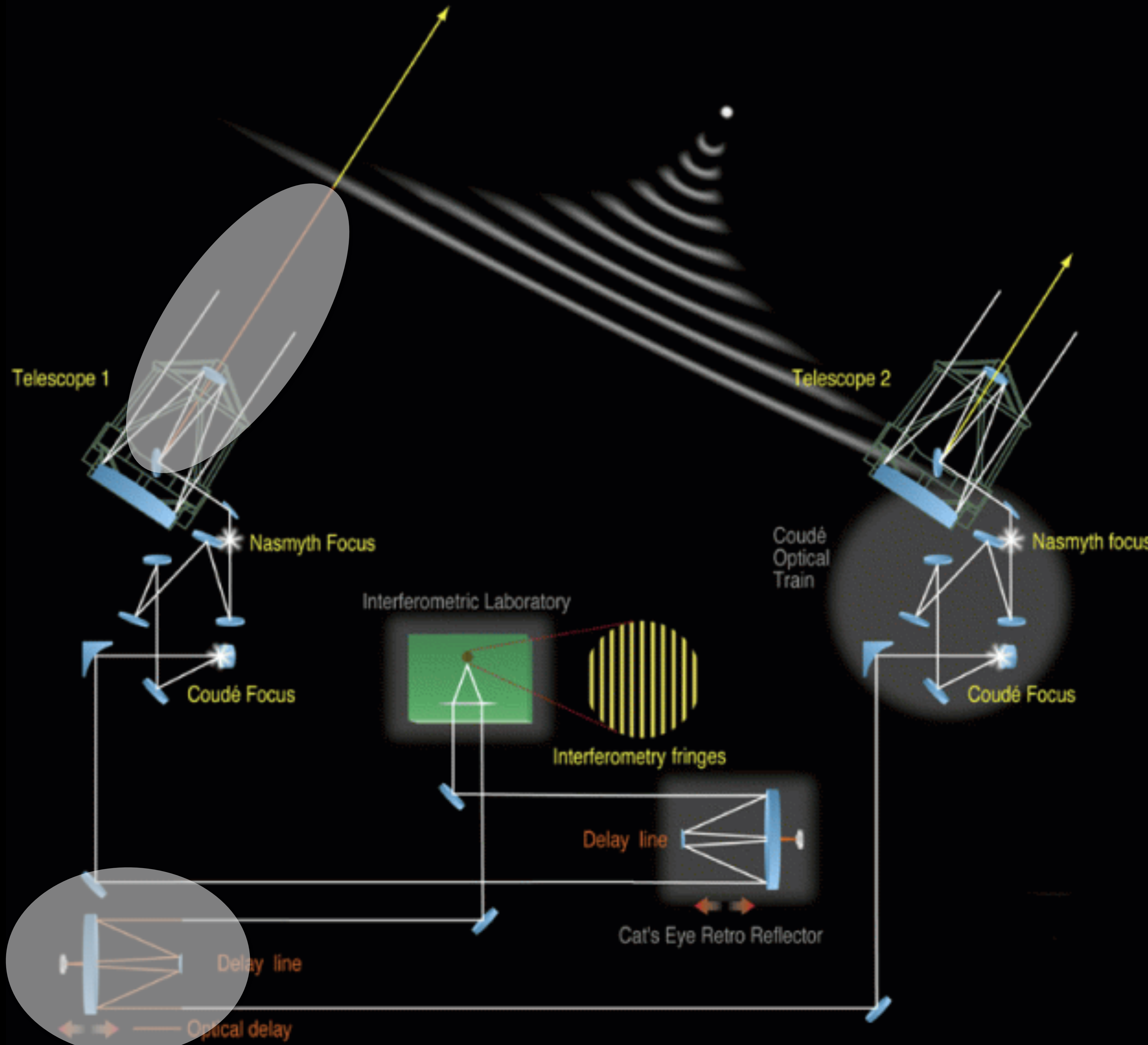


ESO VLTI

CHARA Array

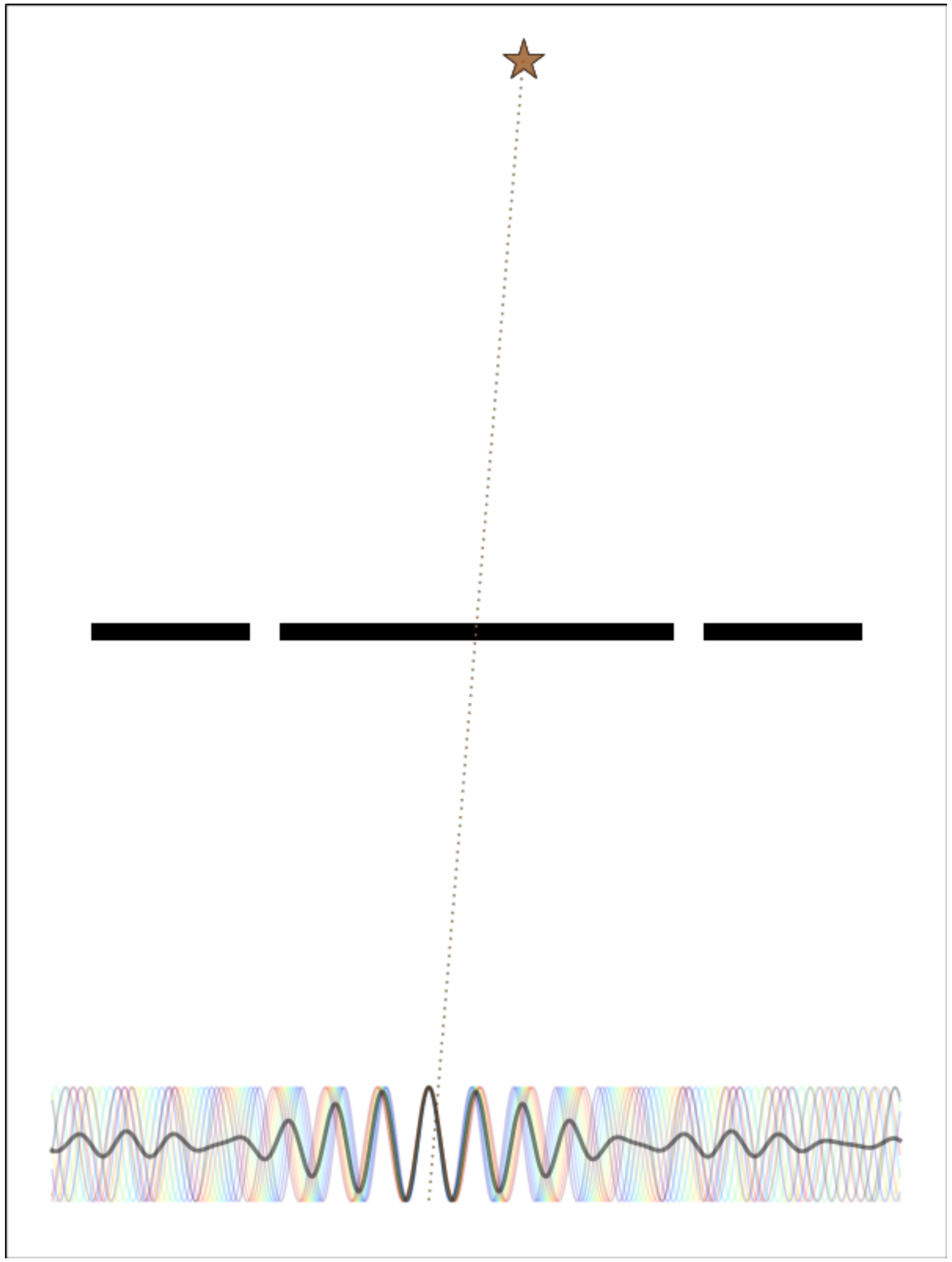
$\lambda/B \sim 1.5\mu m/140m \sim 2mas$

$\lambda/B \sim 0.8\mu m/330m \sim 0.5mas$

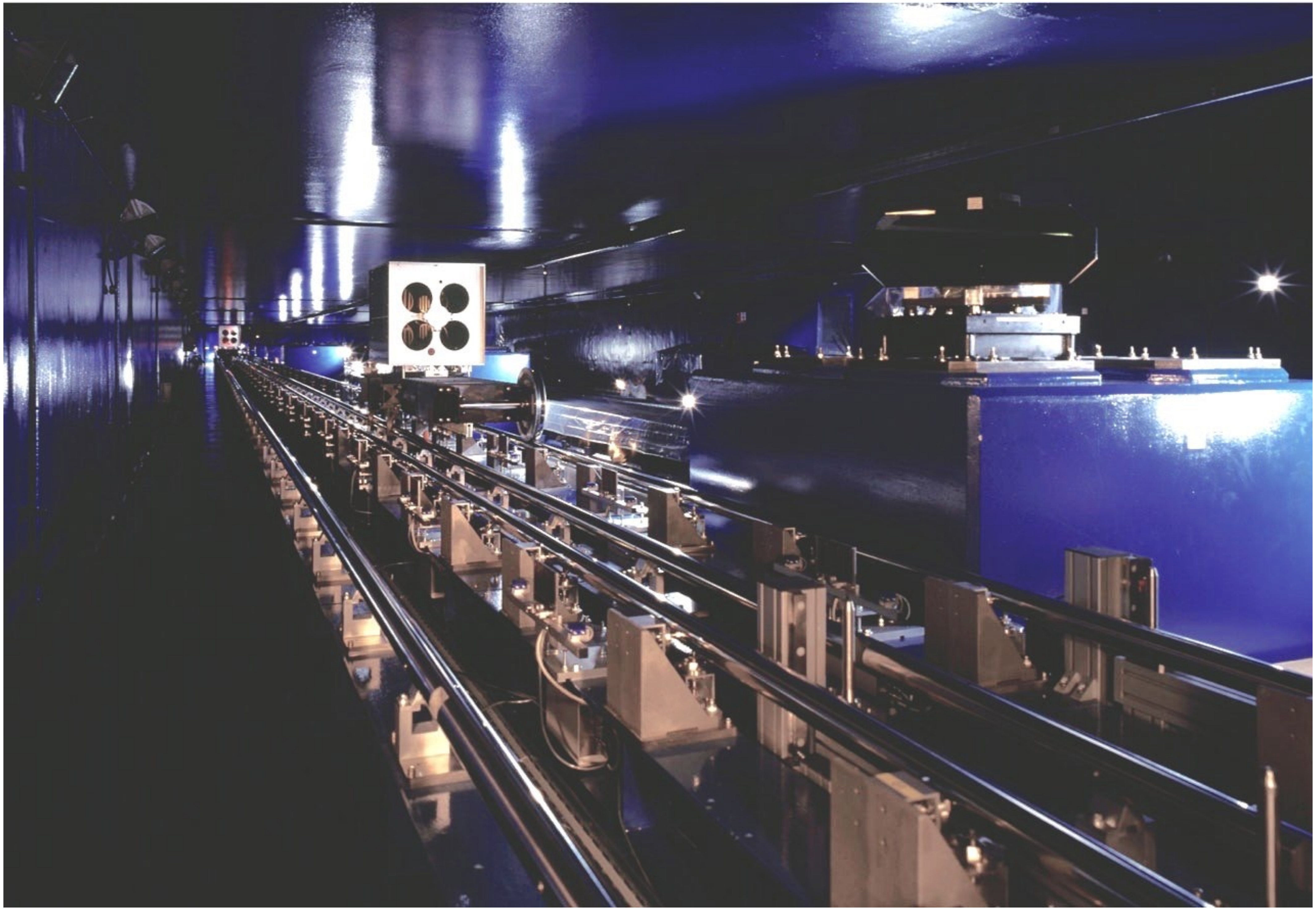

Telescope 1
Telescope 2
Nasmyth Focus
Nasmyth focus
Coudé
Optical
Train
Interferometric Laboratory
Coudé Focus
Coudé Focus
Interferometry fringes
Delay line
Cat's Eye Retro Reflector
Delay line
Optical delay

Zero-OPD

- Non-monochromatic light has a finite **coherence length**
- Real instrument observes **fringe packets**
- Delay lines are used to maintain **zero-OPD**

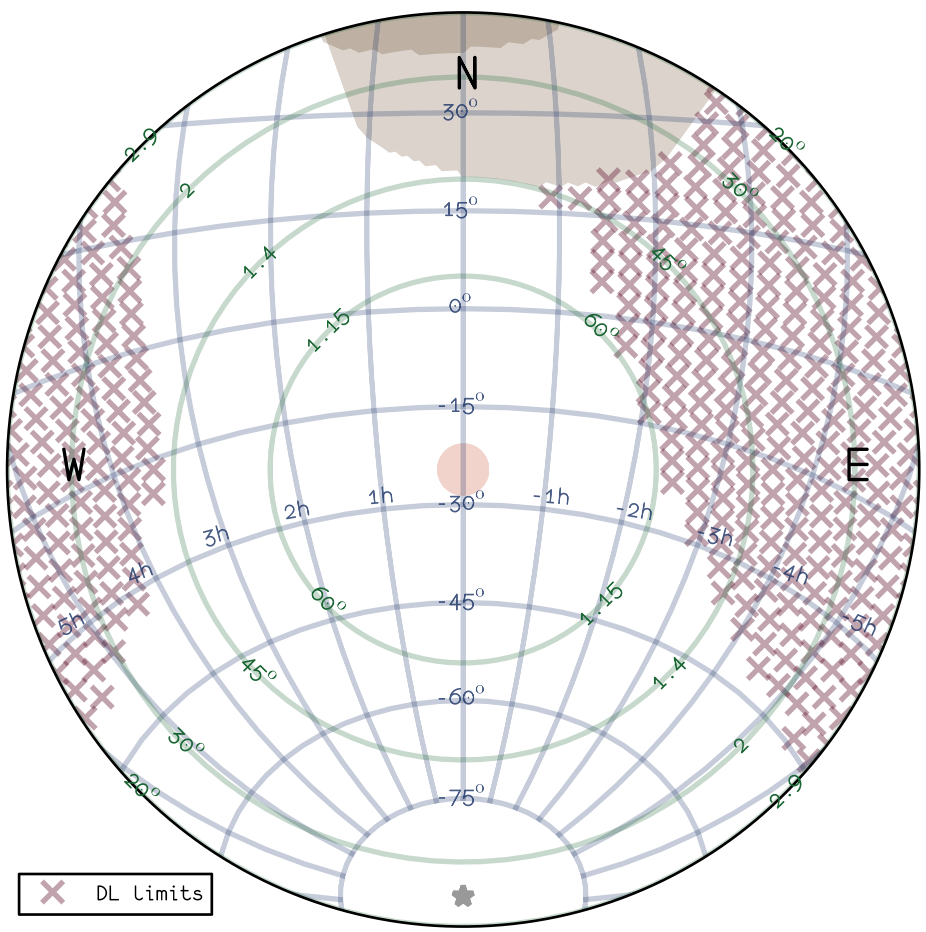


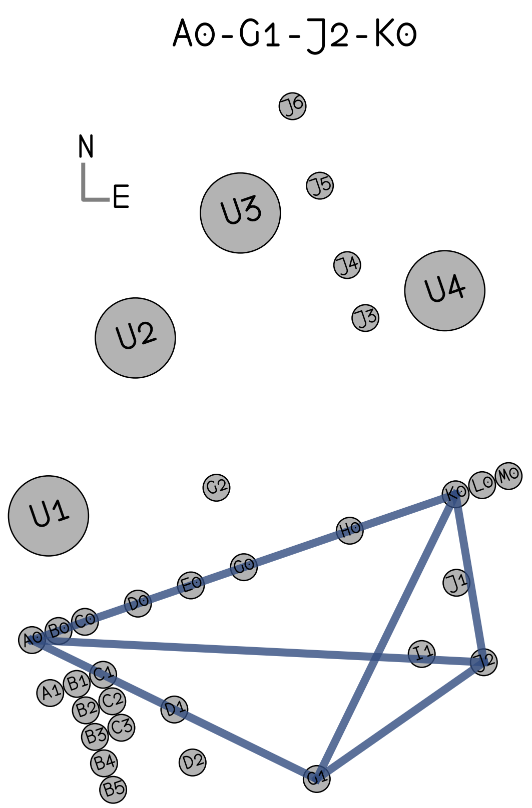
Delay Lines



Sky access

- Full sky access requires delay lines length ~ 2B
- Delay Line have limited stroke





Telescopes

Field of view?

Field of view 1

- Most instruments use **single mode fibers** to clean the incoming wave front of each telescope

- The field of view of the fiber matches the **diffraction pattern of the telescope**

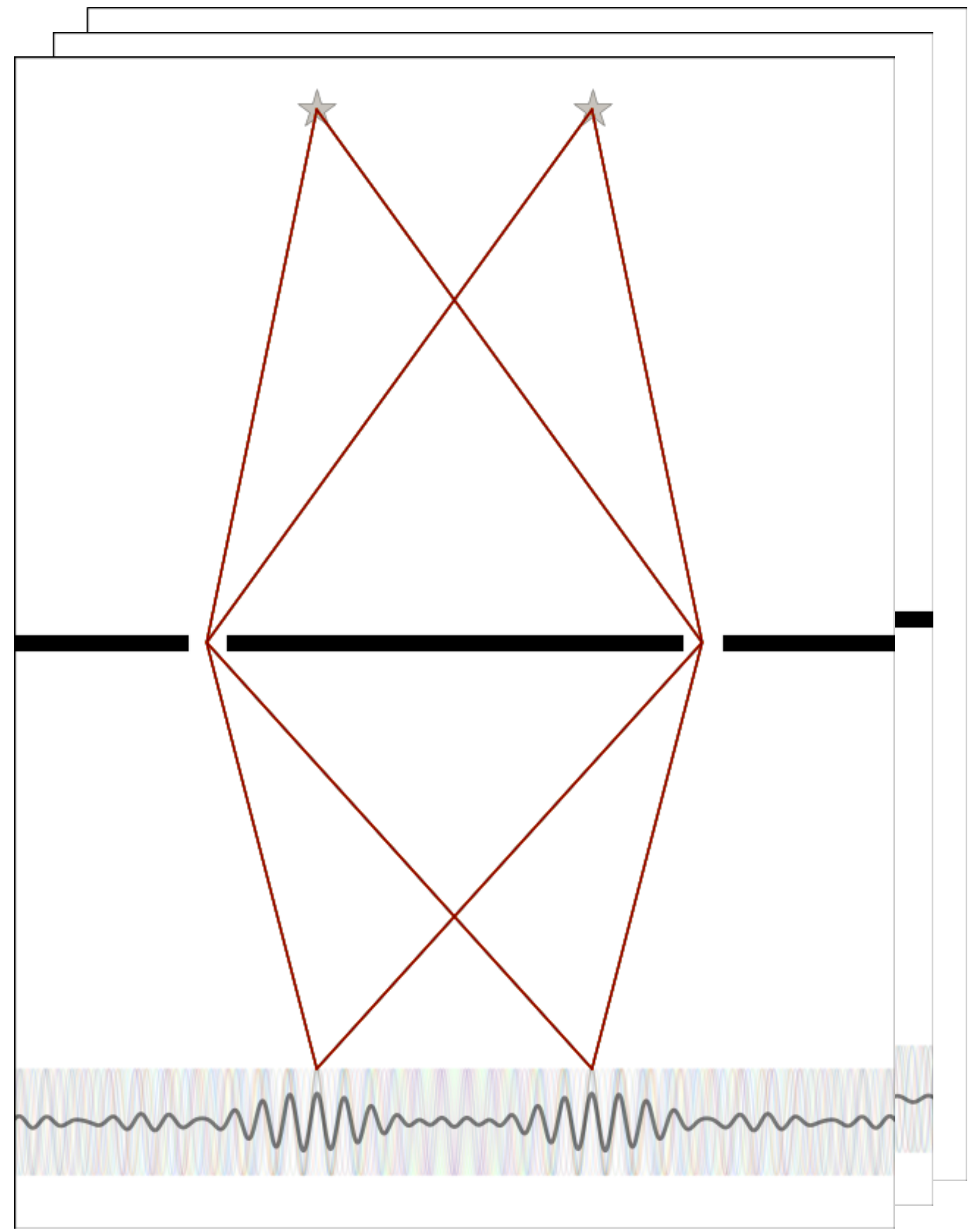
- In practice, the field of view of an interferometer is **λ/D**

Field of View 2

- Non-monochromatic light has a finite **coherence length**

- A binary target will produce **fringe amplitude modulation** if within the coherence length

- This limit depends on the **spectral resolution**

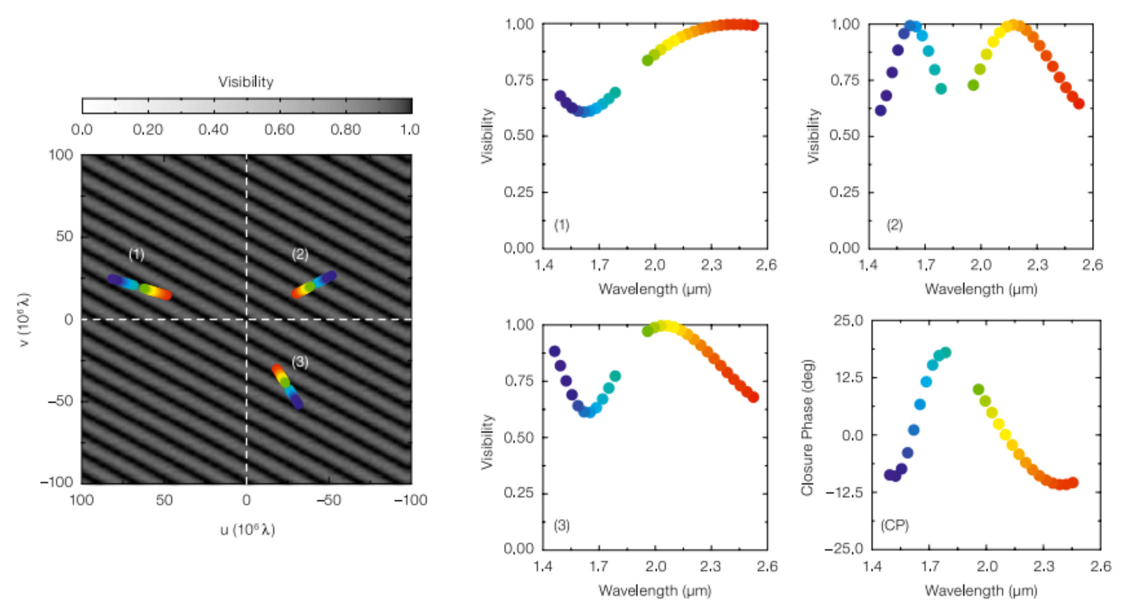


Take home message

- Interferometry is not a straightforward technique…
- Angular resolution of optical interferometry is unmatched
- Understanding the technique and its limitations leads to better (realistic) proposals
- A minimum of understanding is also required to interpret correctly the data you (will) have at hands

in a few words

- An interferometer is a band bass filter for angular frequencies
- “Imaging” requires numerical image reconstruction
- Astrophysical results are obtained using advanced modeling of the observed object and its effects on the interferometric observables



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