Data modeling and interpretation of interferometric data

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Tone of the talk

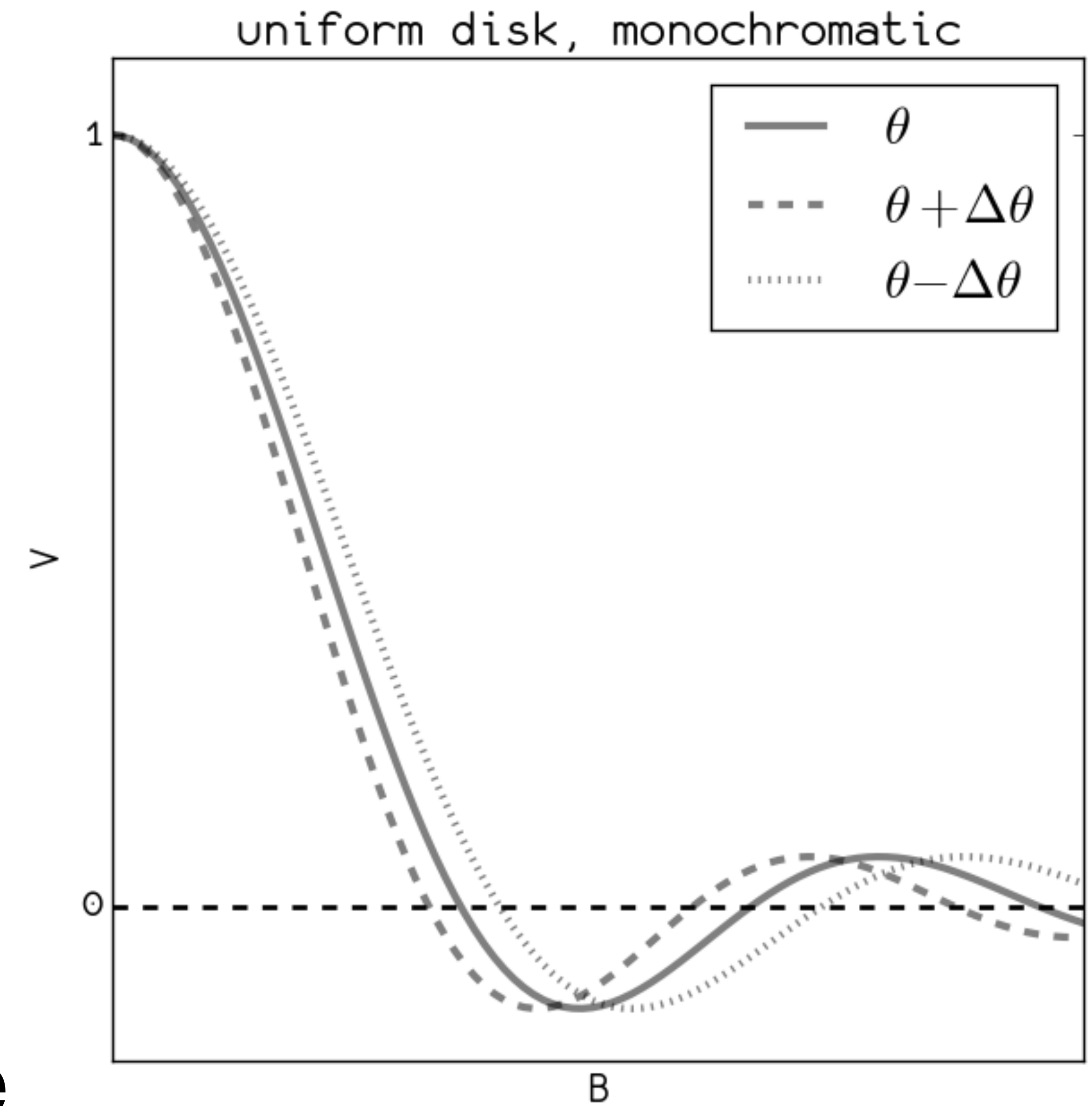
- Oriented toward instrumental aspects with important effects on the science data
- Not exhaustive
- Through different examples, gives general methods

Bandwidth smearing

- The effects of the wavelength bandpass on the interferometric observables
- Most beam combiners have crude spectral resolution to favor sensitivity
- e.g. whole K band, $R = \lambda/\Delta\lambda = 2.2/(2.4-2.0) = 4.4$

our setting

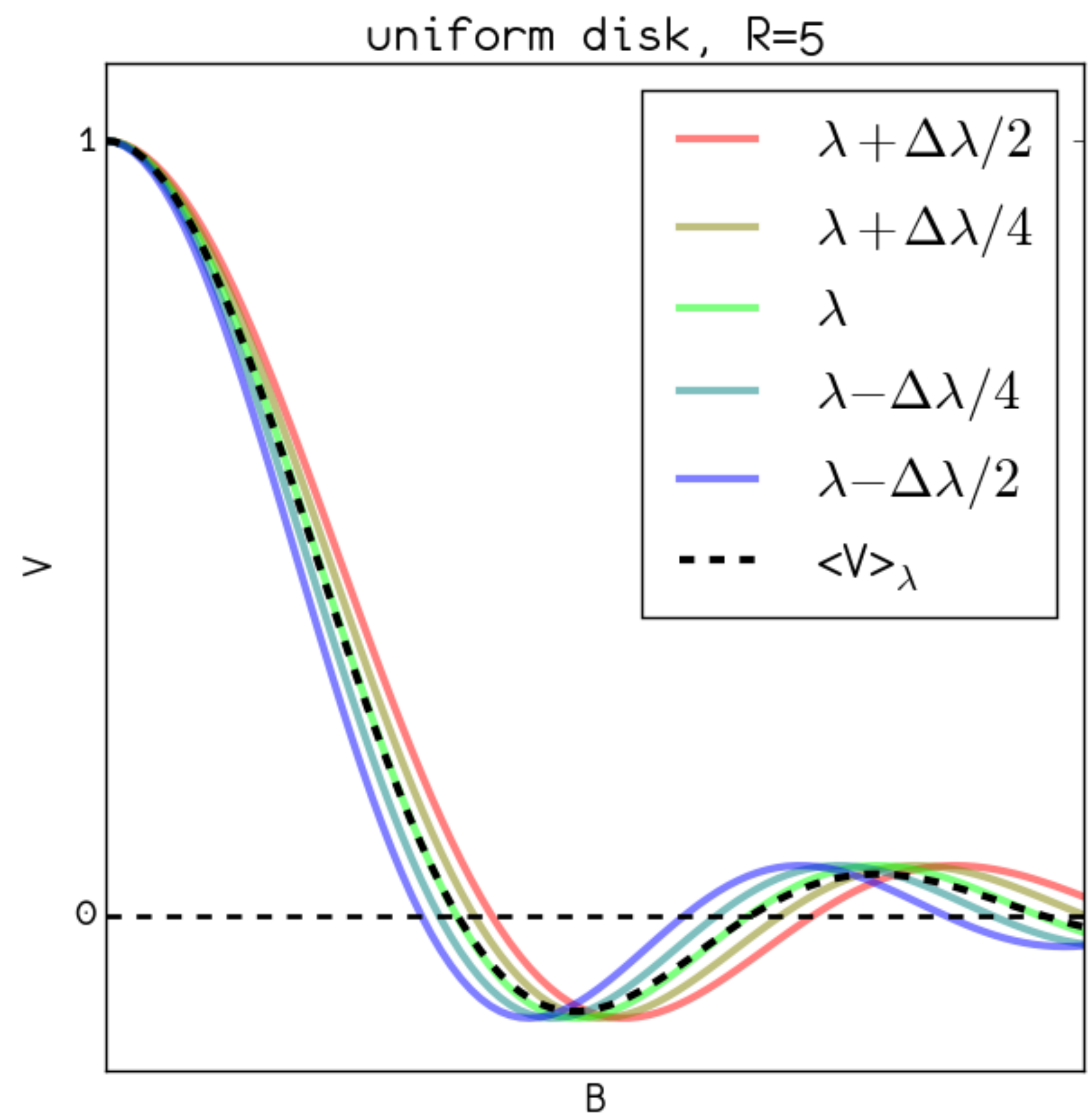
- single baseline interferometer
- we measure only the visibility (amplitude of the fringes)
- we want to measure stellar angular diameters using the first null of the visibility curve



$$V = 2\frac{J_1(\pi B\theta/\lambda)}{\pi B\theta/\lambda}$$

in broad band

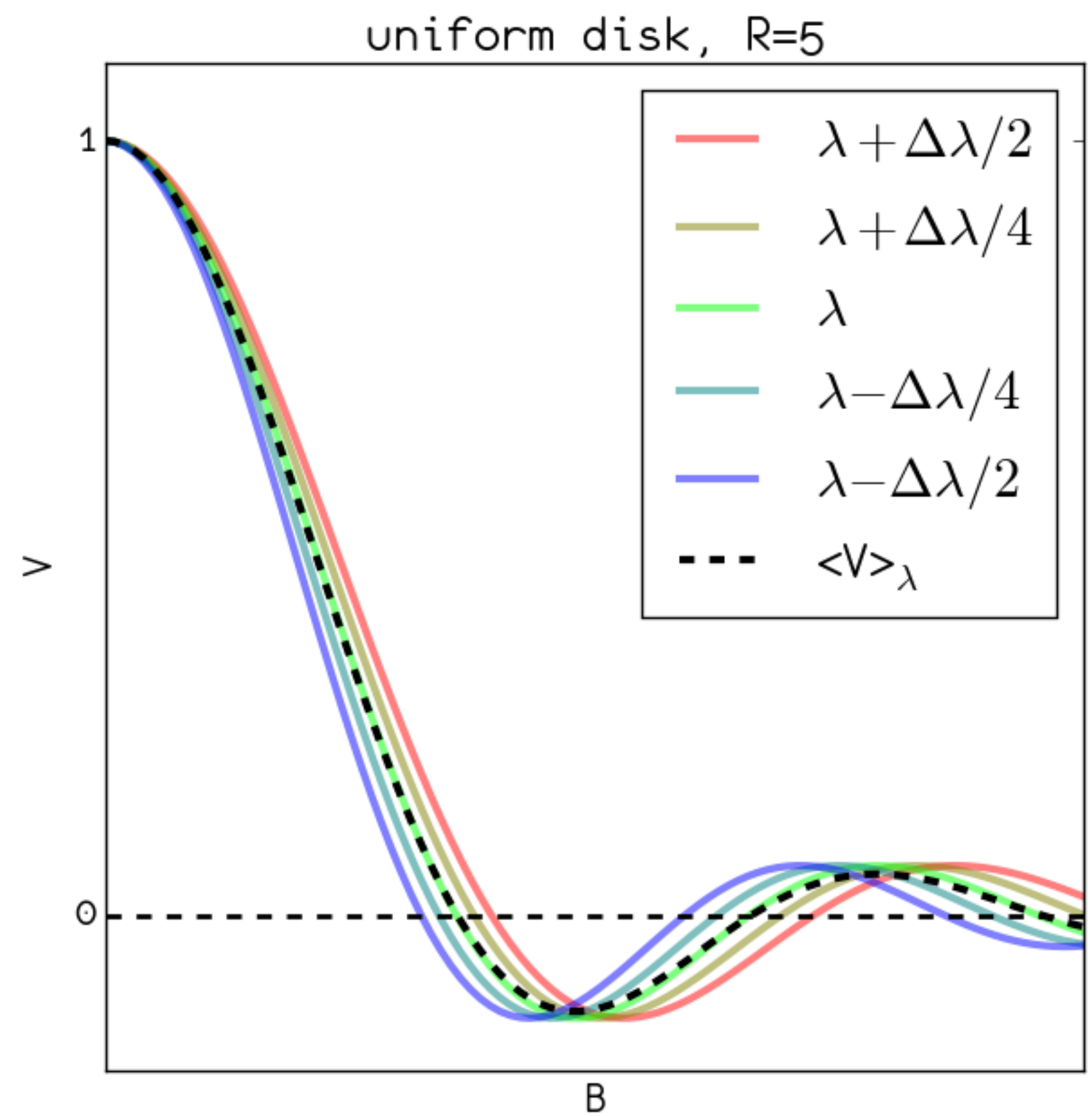
- our beam combiner has R= λ/Δλ = 5
- What is the observed visibility in broad band?

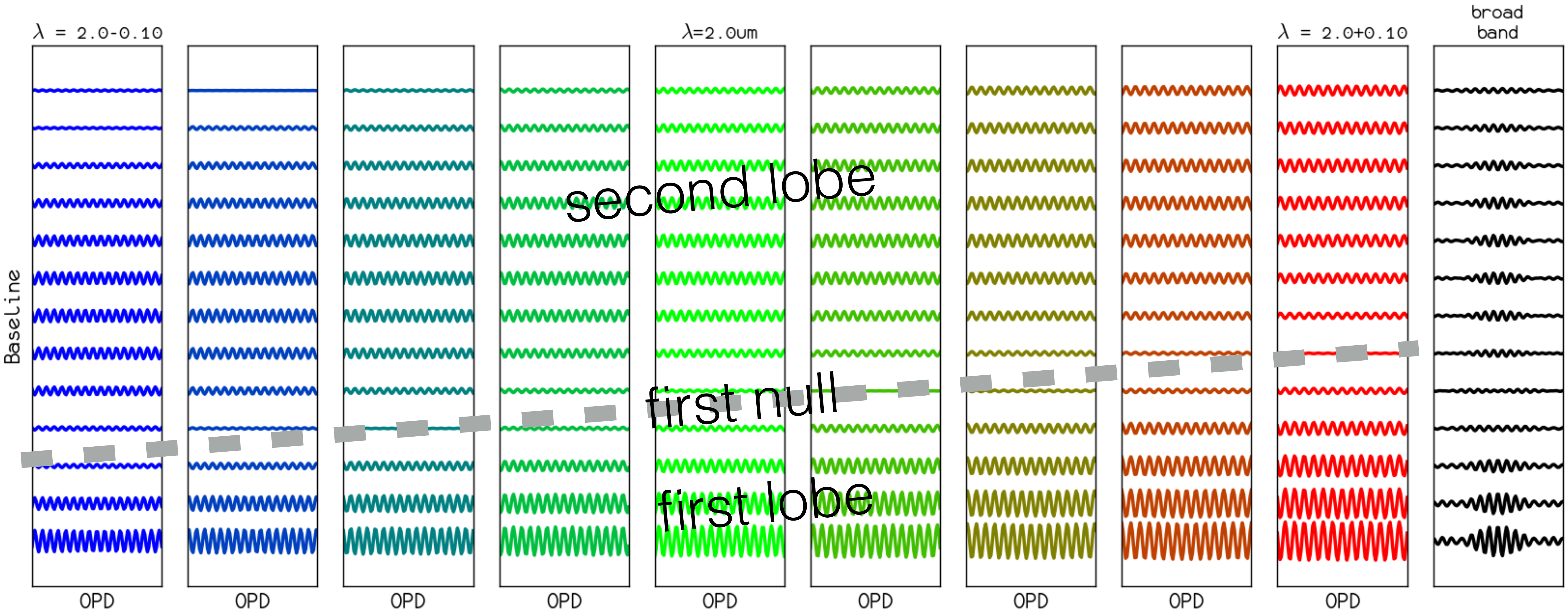


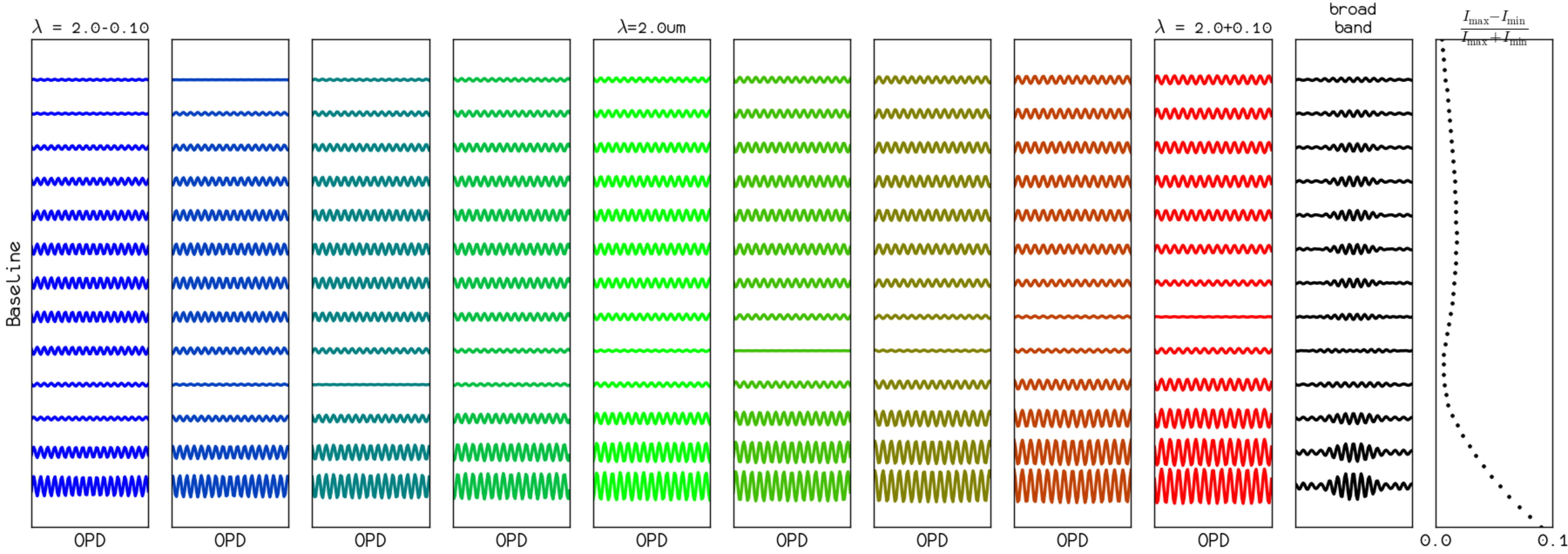
Reasoning

- Beam combiner sees the **sum** of fringes for each wavelengths inside the band

- The observed visibility must be the **average** of the visibility in the band

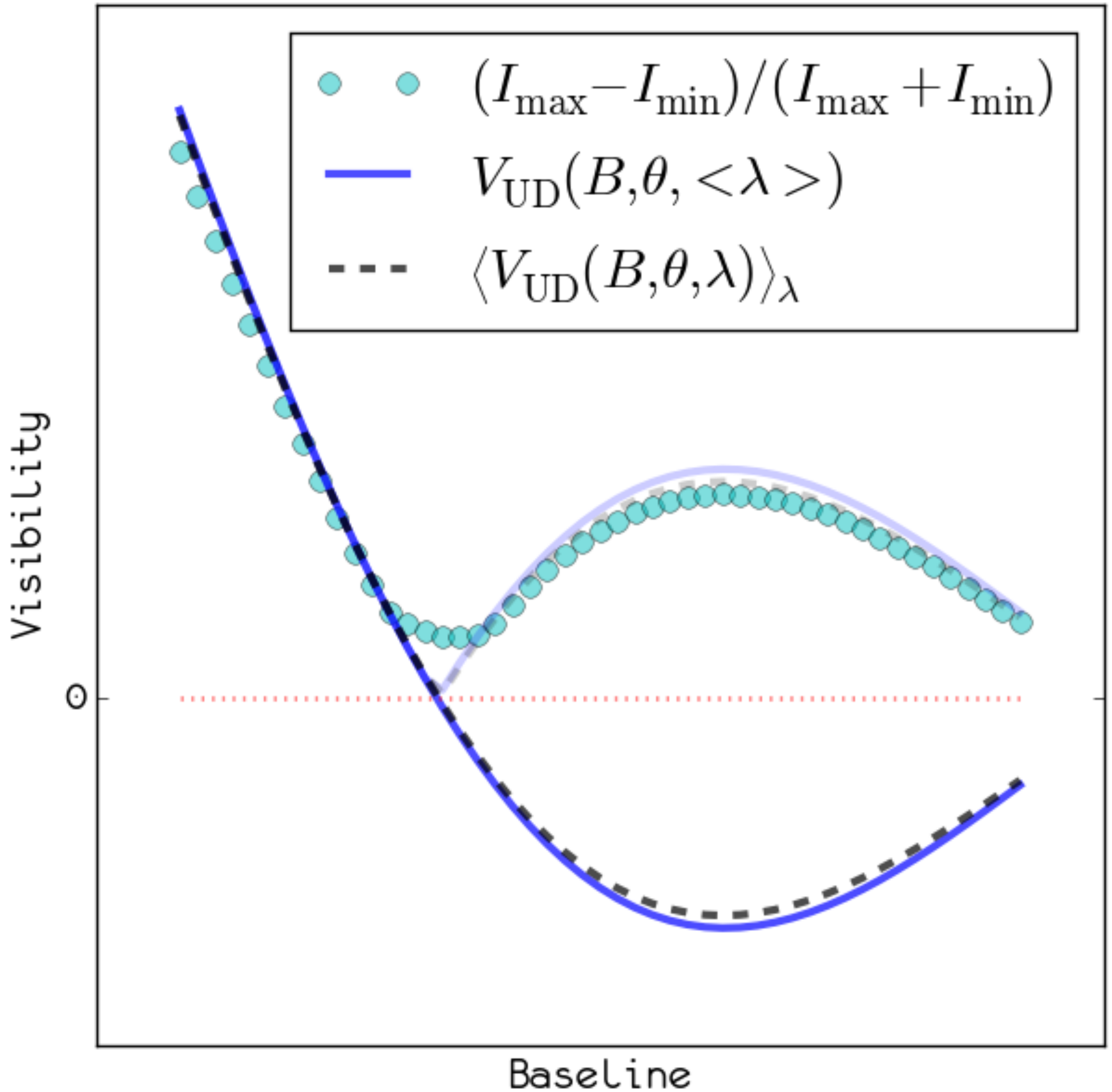



λ = 2.0-0.10
λ=2.0um
λ = 2.0+0.10
broad band
Baseline
second lobe
first null
first lobe
OPD
OPD
OPD
OPD
OPD
OPD
OPD
OPD
OPD



λ = 2.0-0.10
λ=2.0um
λ = 2.0+0.10
broad
band
$\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$
Baseline
OPD
0.0
0.1

What is going on ?!?

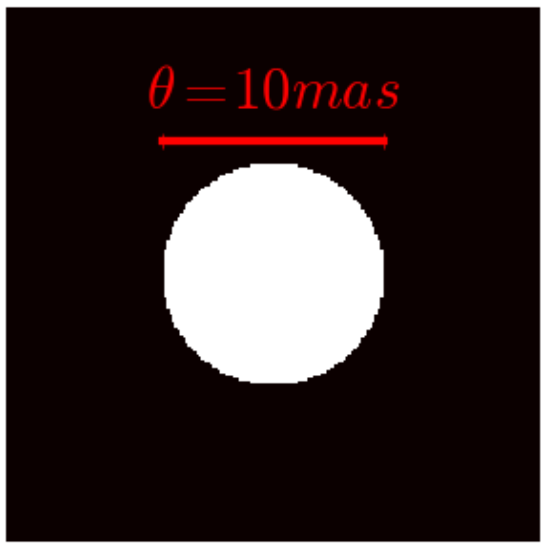


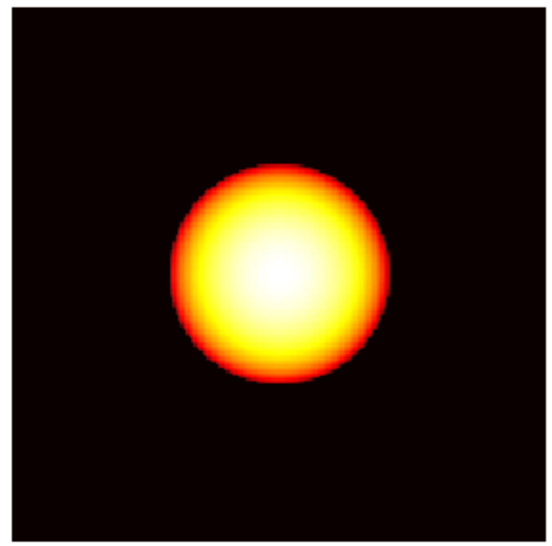
There is information in the V(B) curve

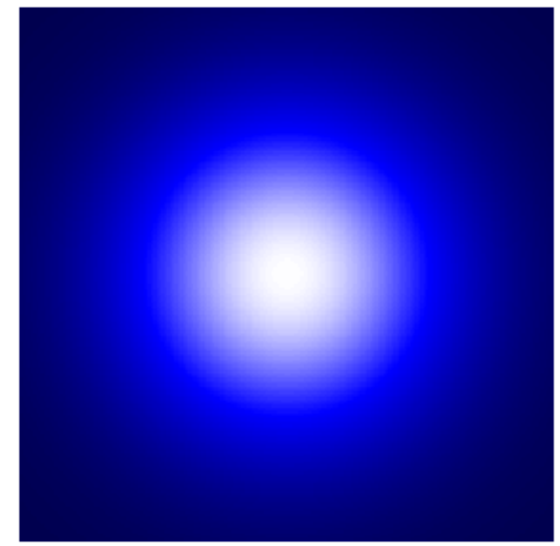
$$V_{\mathrm{UD}}(B,\lambda) = \left| 2\frac{J_1(x = \pi B\theta/\lambda)}{x} \right|$$

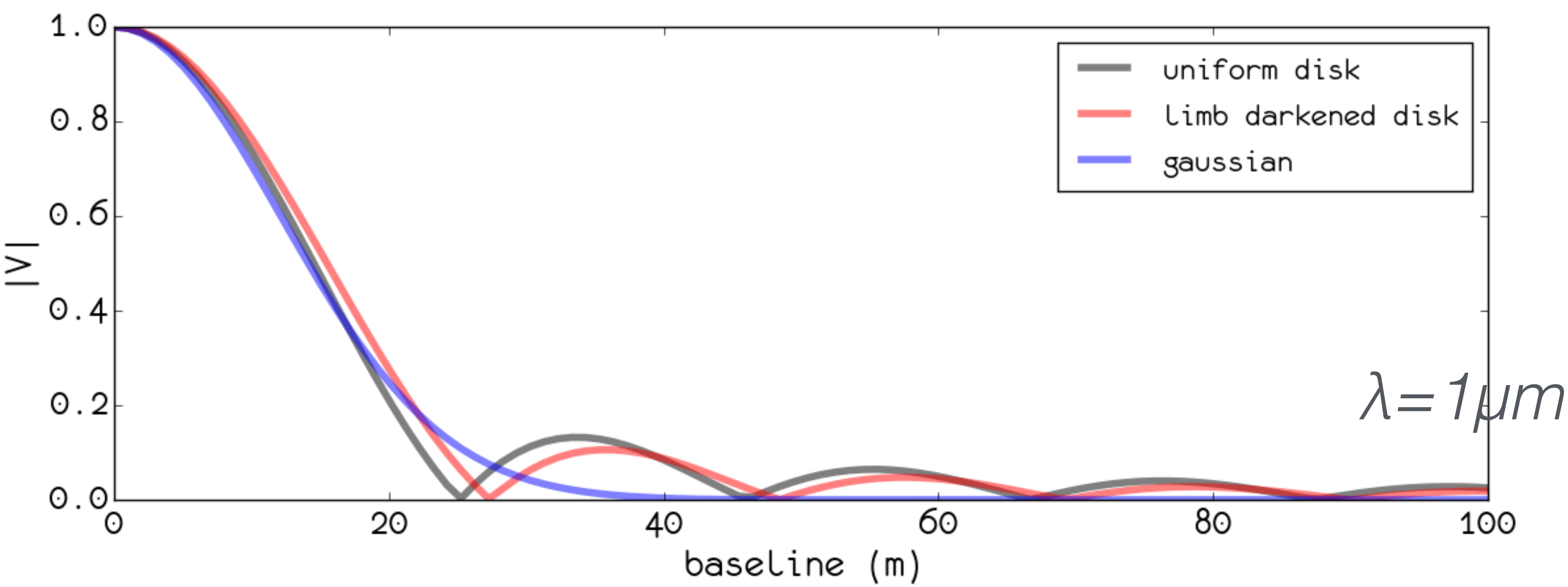
centro-symetric:
Hankel transform

$$V(B,\lambda) = \frac{\int I(r,\lambda) J_0(rB/\lambda) r dr}{\int I(r,\lambda) dr}$$

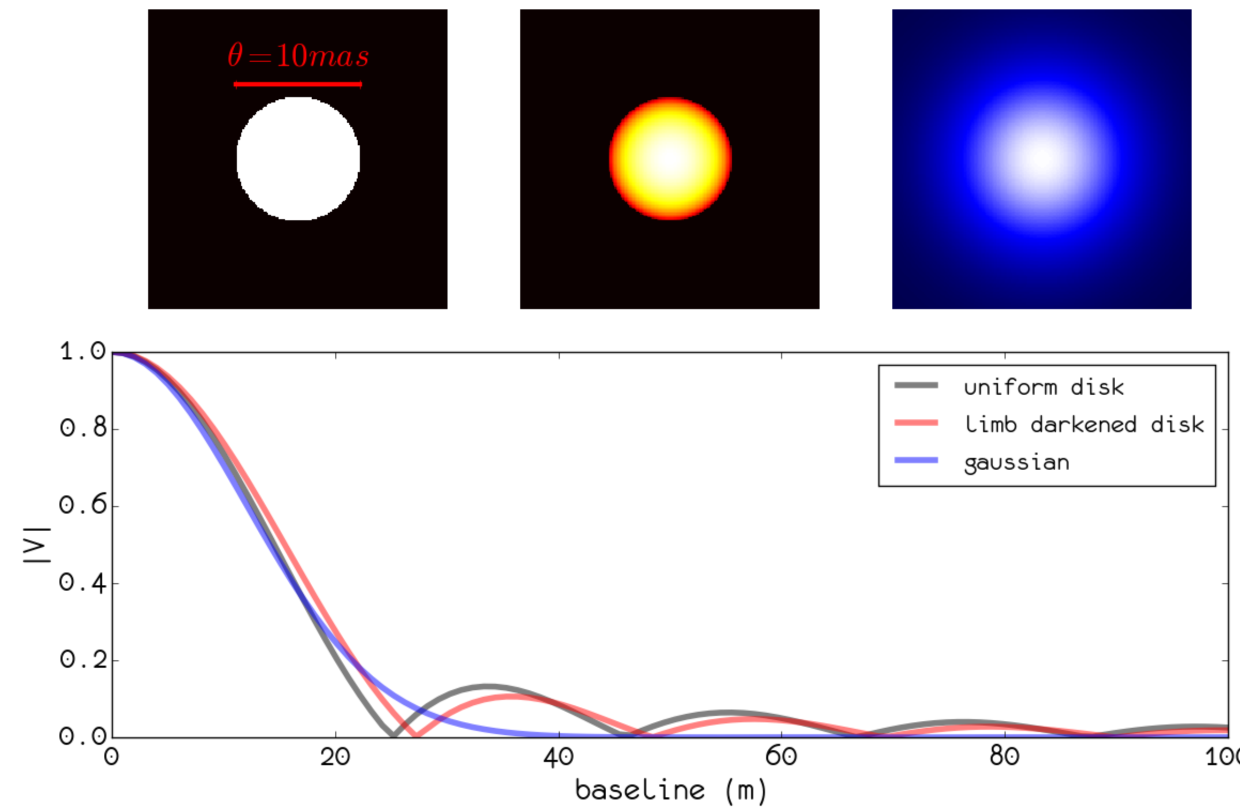


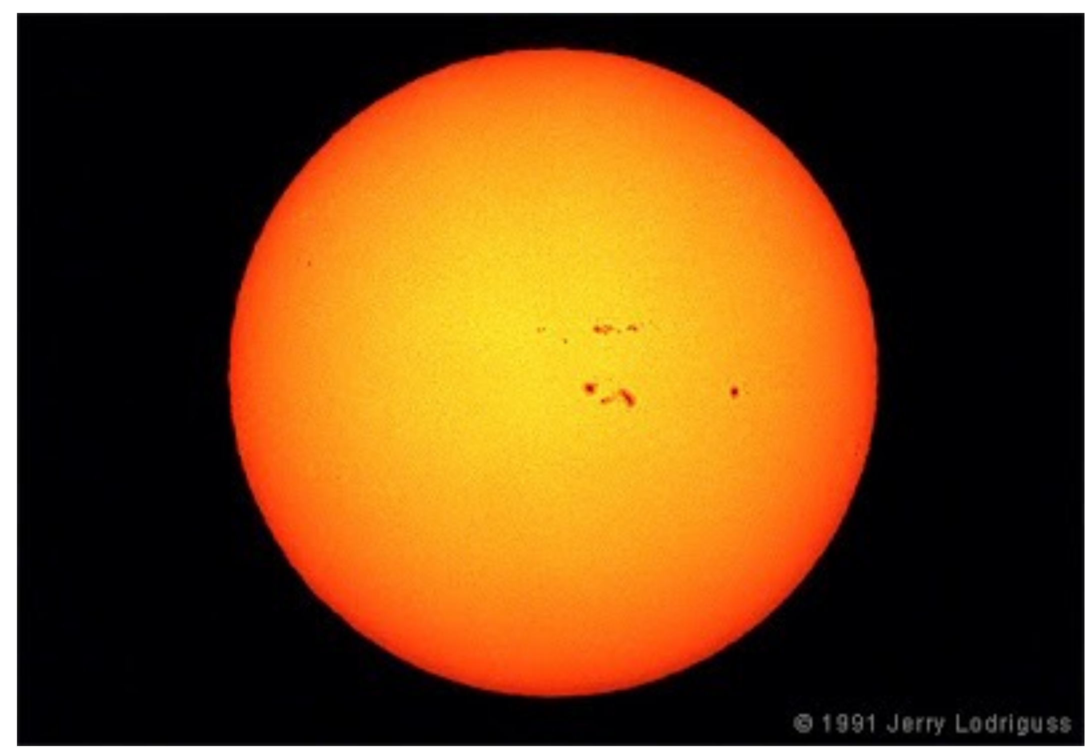






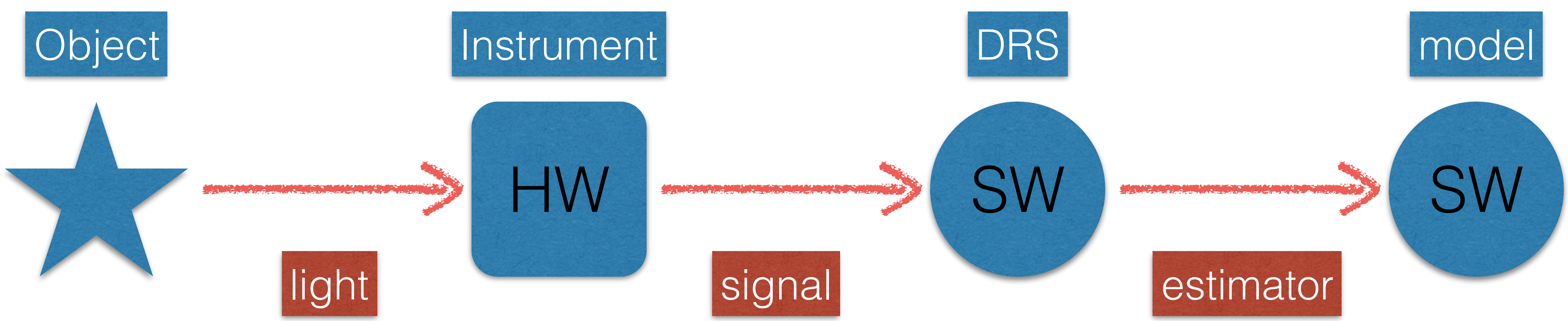
- Measuring diameters == inverting V(B,Θ,λ)
- True stars are NOT uniform disks
- limb darkening
  - lowers the visibility lobes
  - bias the diameter measurements



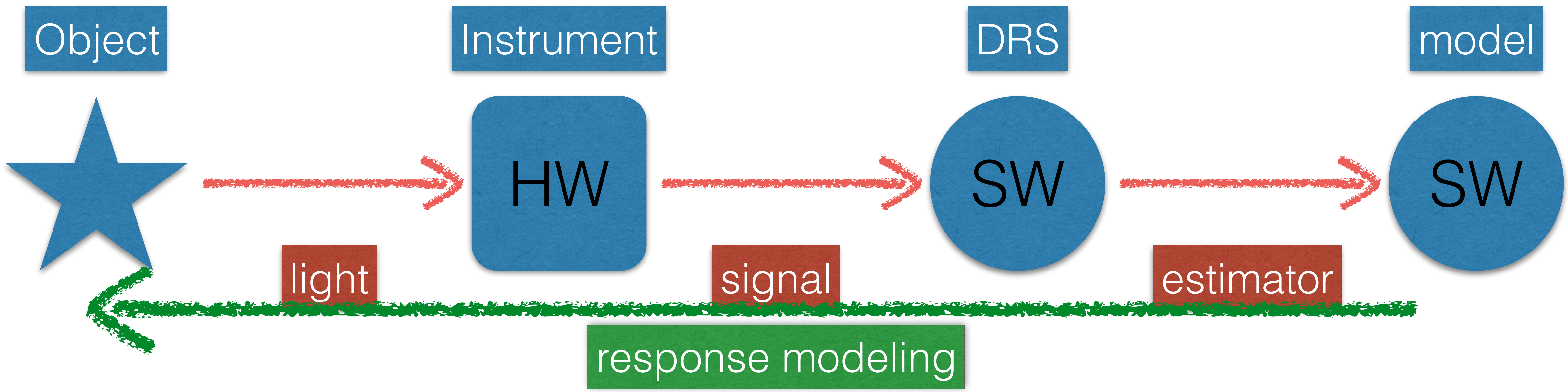


General considerations

- The instrument does not observe visibility, it observes **fringes**
- An **estimator** is used to derived the visibility from the fringes using a data reduction software (DRS)



The correct approach



The numerical model should match

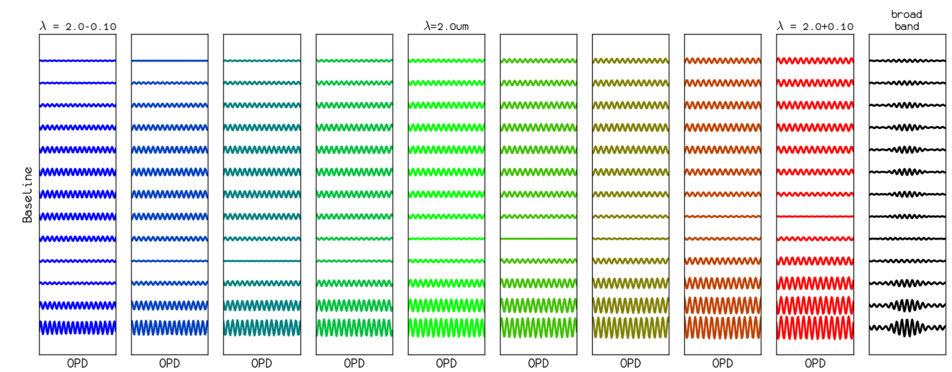
- the estimator
- the instrumental characteristics
- the object characteristics

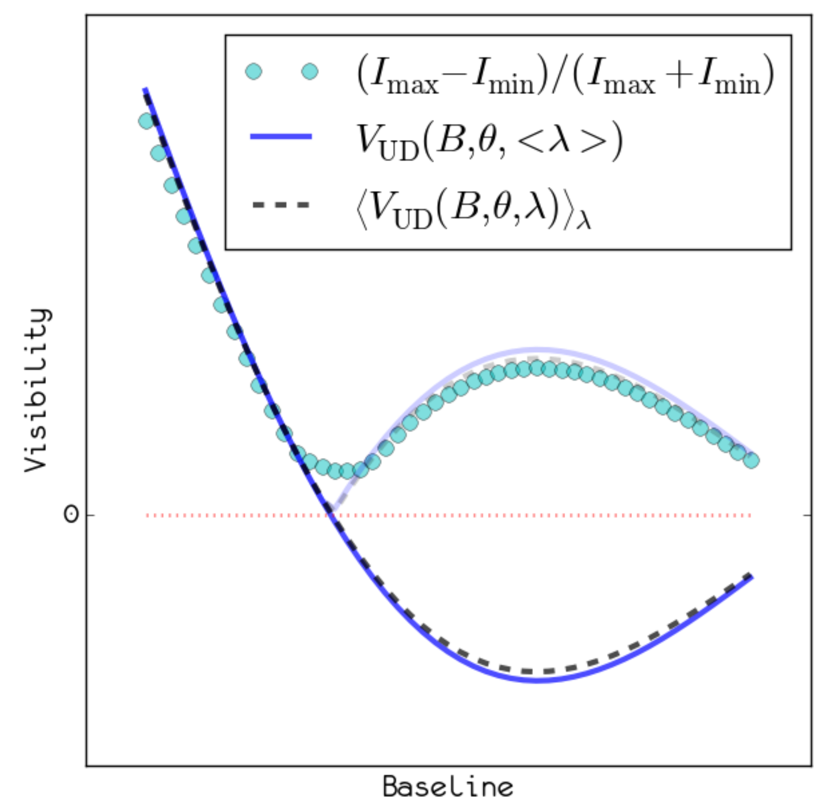
we were on the right direction…

- we **synthesized** a signal using an instrumental characteristic: bandwidth smearing

- we used an **estimator** of the visibility:

  ➡ $V \sim (I_{max} - I_{min}) / (I_{max} + I_{min})$





better yet, do it analytically

Let's start all over again…

- What is the visibility estimator?
- What instrument's characteristics should I take into account?
- What object's characteristics should I take into account?

Visibility estimator

- visibility has additive noise: V + n
- we measure fringe's contrast $\mu = |V+n|$
- averaging:
- **$<\mu> = <|V+n|>$**
- **$<\mu>$ is biased**

- what about $<\mu^2>$?
- $<\mu^2> = <|V+n|^2>$
- $<\mu^2> = <|V|^2>+<2Re\{Vn\}>+<|n|^2>$
- assuming V and n are uncorrelated: $<Re\{Vn\}>=0$
- **$<\mu^2> = <|V|^2> + <|n|^2>$**
- **$<\mu^2>$ is biased but can be unbiased if $<|n|^2>$ is estimated**

Fourier estimator

- Remember Parseval's identity?
- $\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |TF(f)(\sigma)|^2 d\sigma$
- The average squared amplitude of the signal == The average PSD

Analytical fringe signal

Fringes signal *F(δ)*:

function of monochromatic fringes *f(δ, λ)*,

function of OPD (δ)

and wavelength (λ)

$$F(\delta) = \frac{\int_0^\infty B(\lambda)T(\lambda)f(\delta,\lambda)\lambda d\lambda}{\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda}$$

$$B(\lambda) \rightarrow \text{stellar spectrum}$$

$$T(\lambda) \rightarrow \text{instrumental transmission}$$

$$f(\delta,\lambda) = 1 + Re\left(V(\lambda)e^{-2\pi\delta/\lambda}\right)$$

$$\lambda \rightarrow \text{photon detection}$$

$$PSD(\sigma) = |FT_\delta[F(\delta)]|^2$$

$$= \left| FT_\delta \left[\frac{\int_0^\infty B(\lambda)T(\lambda)f(\delta,\lambda)\lambda d\lambda}{\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda} \right] \right|^2$$

$$= \frac{\left|\int_0^\infty B(\lambda)T(\lambda)\tilde{f}(\delta,\lambda)\lambda d\lambda\right|^2}{\left|\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda\right|^2}$$

$$= \frac{\left|\int_0^\infty B(\lambda)T(\lambda)V(\lambda)\delta_{1/\sigma}(\lambda)\lambda d\lambda\right|^2}{\left|\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda\right|^2}$$

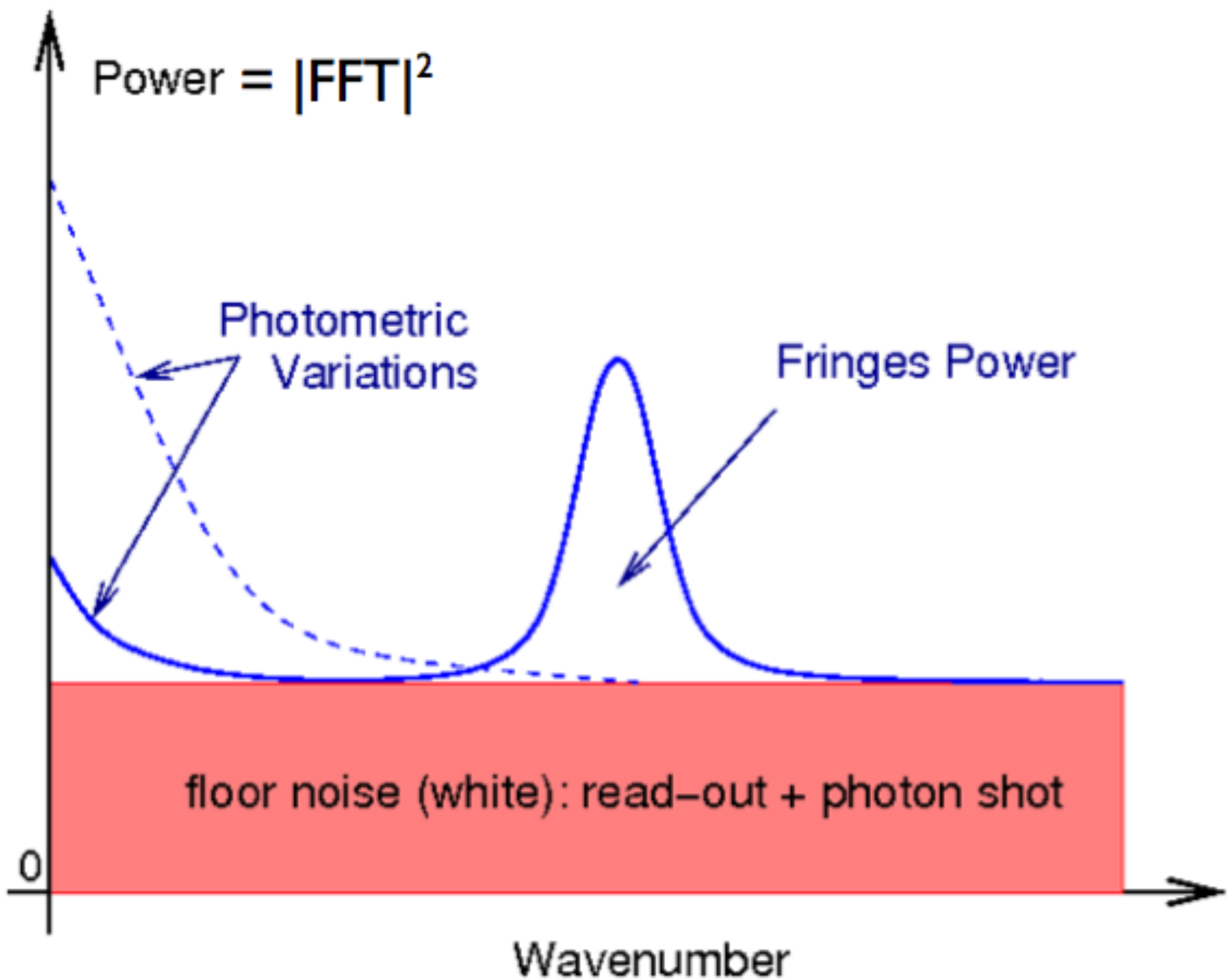
$$= \frac{[B(1/\sigma)T(1/\sigma)V(1/\sigma)1/\sigma]^2}{\left|\int_0^\infty B(\lambda)T(\lambda)\lambda d\lambda\right|^2}$$

linearity of FT

frequency selection

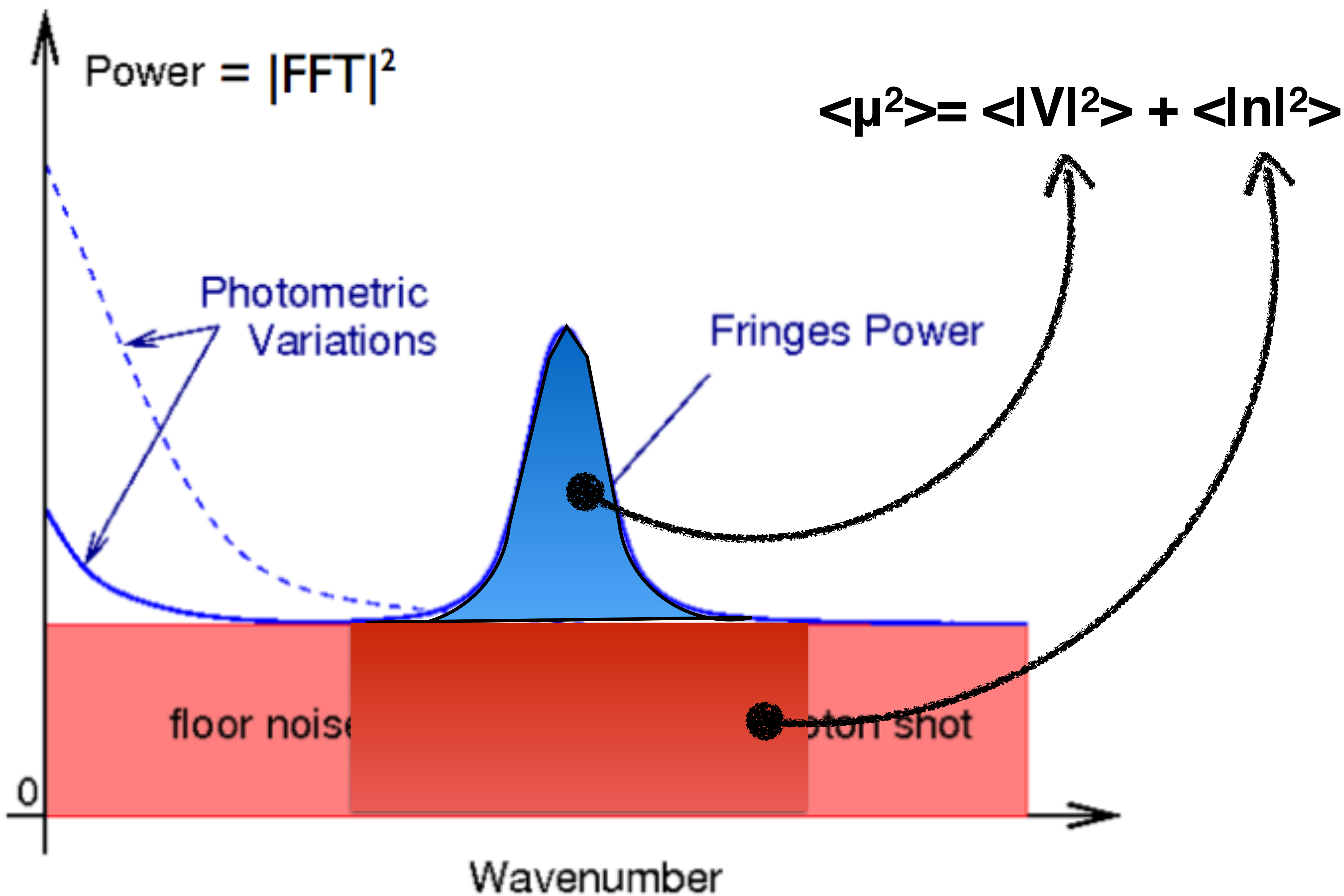
Normalised, frequency averaged object's visibility

Real signal



Power = |FFT|^2
Photometric Variations
Fringes Power
floor noise (white): read–out + photon shot
0
Wavenumber

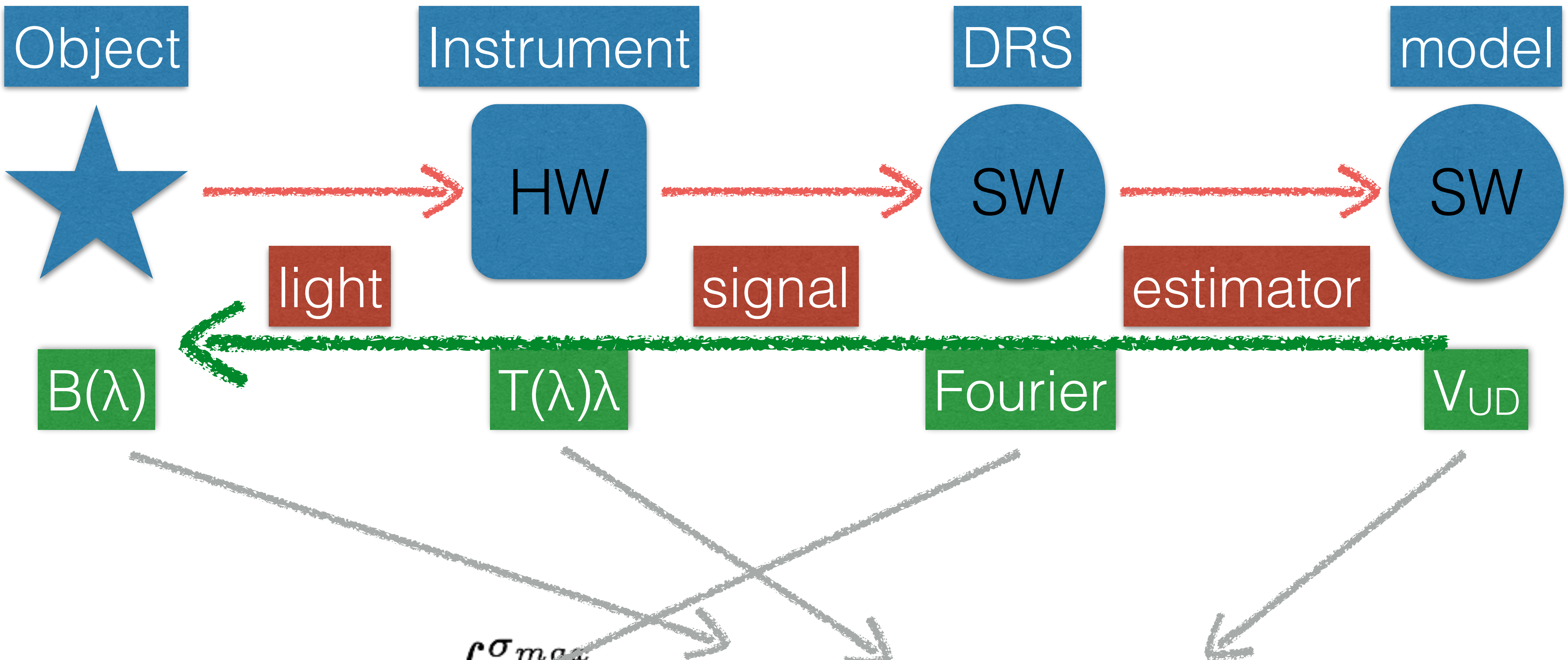
Real signal


Power = |FFT|²
<μ²>= <|V|²> + <|n|²>
Photometric Variations
Fringes Power
floor nois
oton shot
0
Wavenumber

Fourier estimator

$$V^2_{\text{measured}} \propto \int_{\sigma_{min}}^{\sigma_{max}} B(1/\sigma)^2 T(1/\sigma)^2 V(1/\sigma)^2 1/\sigma^2 d\sigma$$

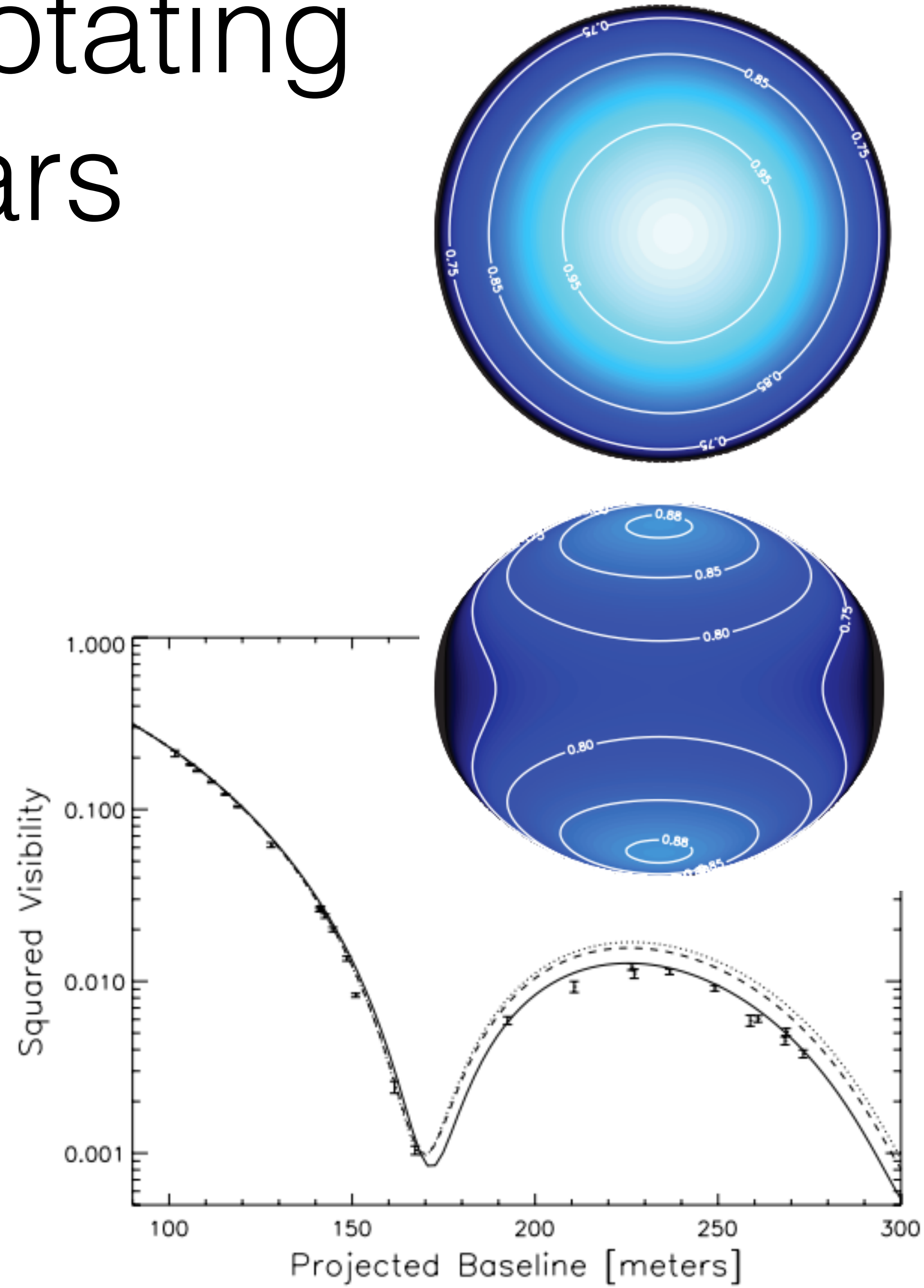
- weighted average of the squared visibilities
- function of the object spectrum
- function of the instrumental transmission



$$V^2_{\mathrm{measured}}(\theta, B) \propto \int_{\sigma_{min}}^{\sigma_{max}} B(1/\sigma)^2 T(1/\sigma)^2 V_{UD}(B, \theta, 1/\sigma)^2 1/\sigma^2 d\sigma$$

Fast Rotating stars

- Aufdenberg+ 2006
- Observations of the star Vega with FLUOR@CHARA
- Accurate modelling allowed to prove that the star is a rapid rotator seen pole-on
- **astrophysical effect ~ bandwidth smearing effects**



Why it is important

- Interferometric observations lead to visibilities, closure phases (+ differential quantities)

- **Images** can be reconstructed…

- … But the astrophysical **quantitative** results will always be derived from visibilities

When instrumental effects mimic astrophysical signal

- We have seen “obvious” effects: model disagree with the observations

- Some effects are more difficult to spot!

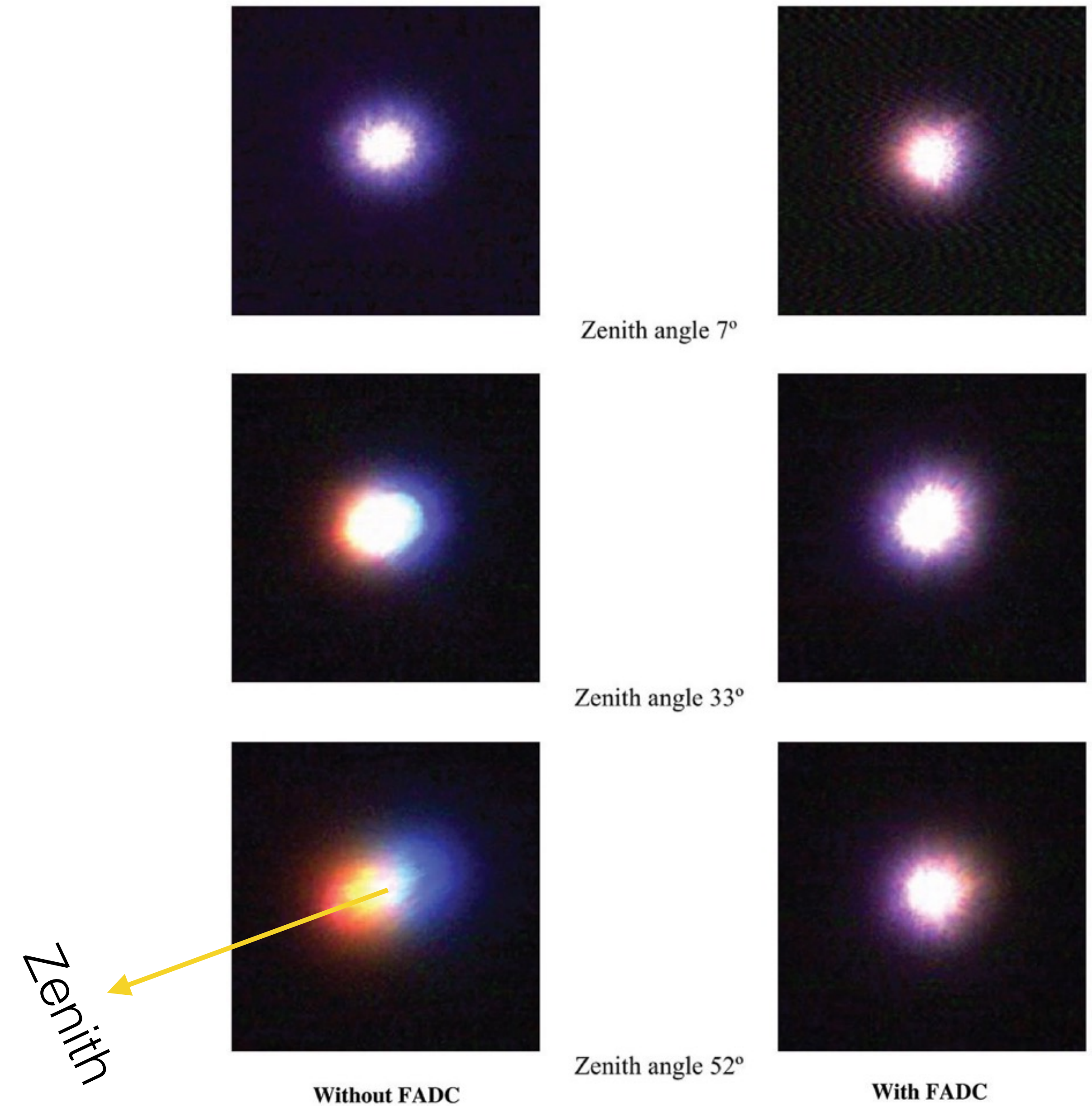
- Some signal:

  - Astrophysical phenomenon?

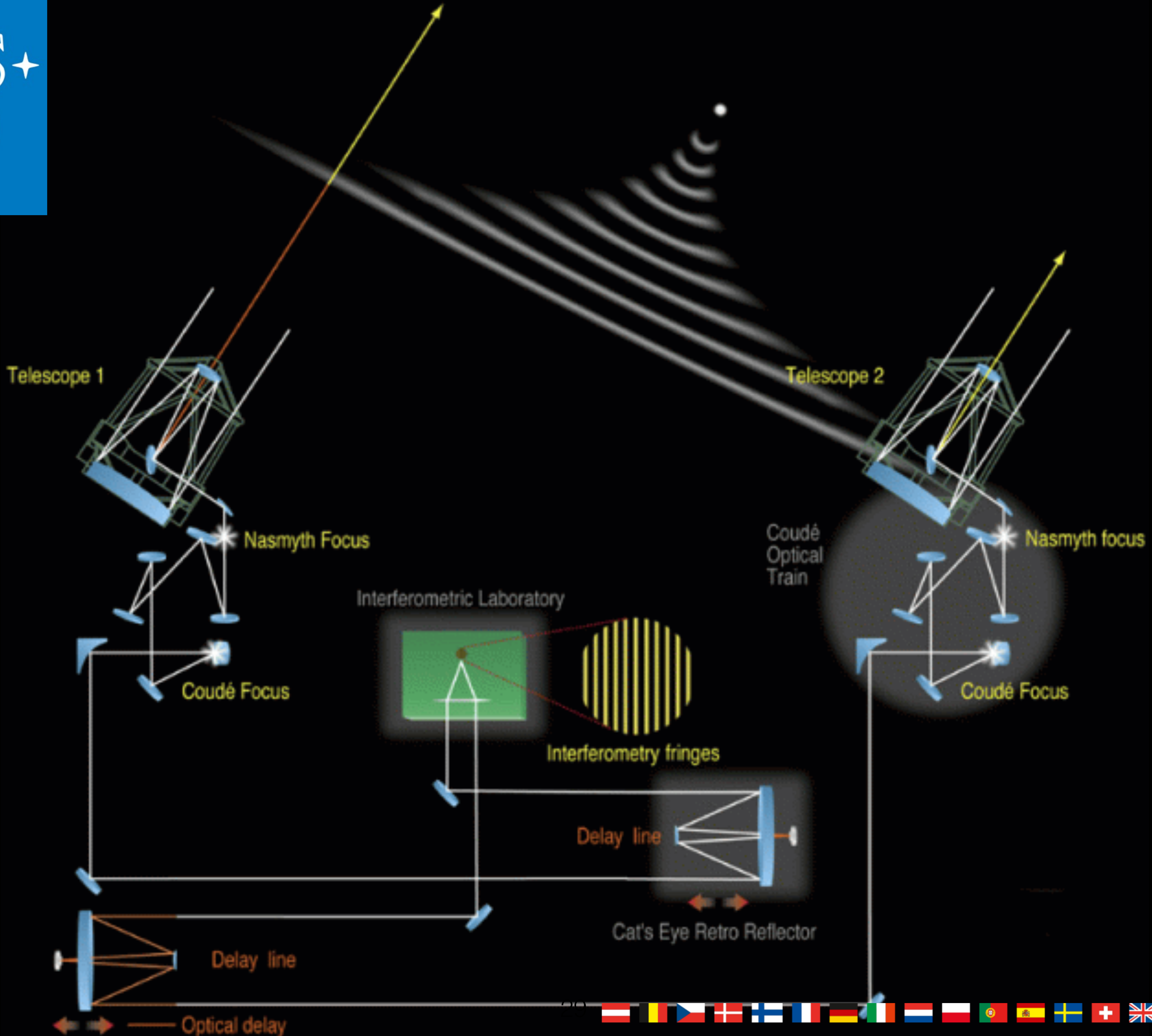
  - instrumental effect?

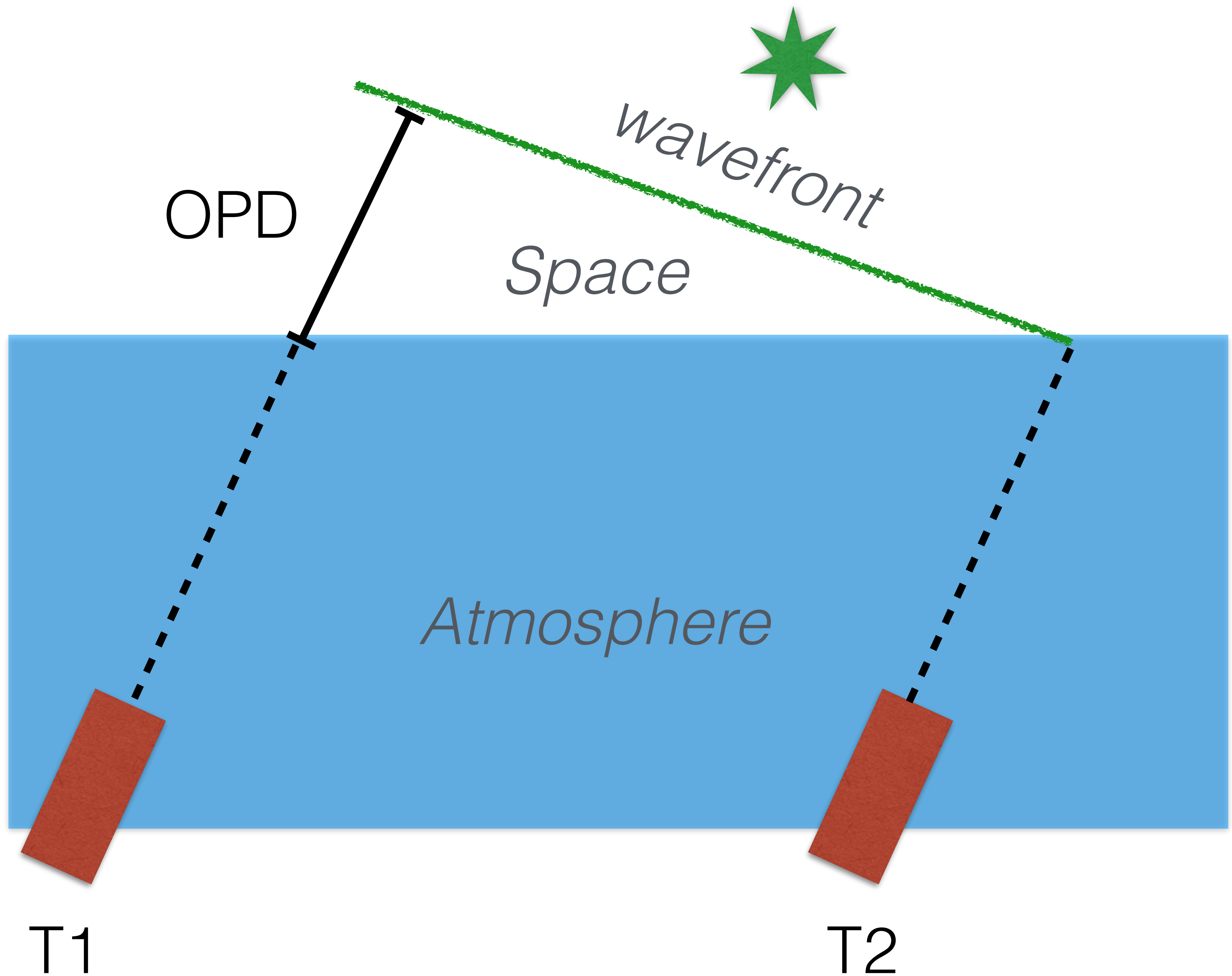
Atmospheric Dispersion

The refractive index of air is chromatic



Zheng+13

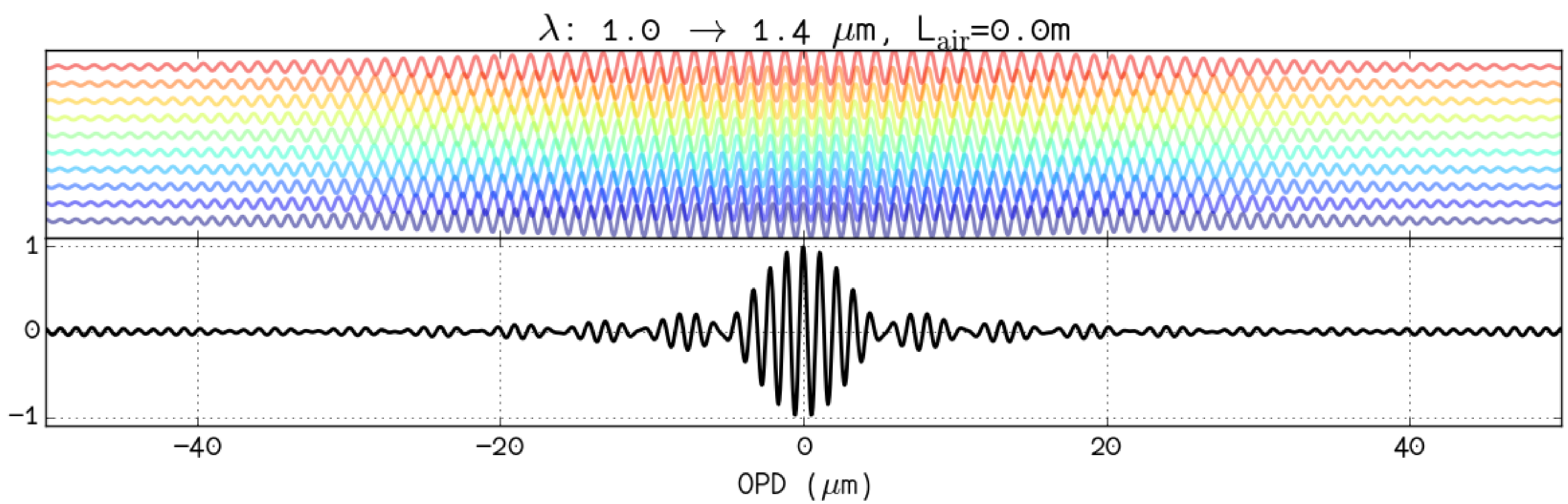

Telescope 1
Telescope 2
Nasmyth Focus
Nasmyth focus
Coudé Optical Train
Interferometric Laboratory
Coudé Focus
Coudé Focus
Interferometry fringes
Delay line
Cat's Eye Retro Reflector
Delay line
Optical delay

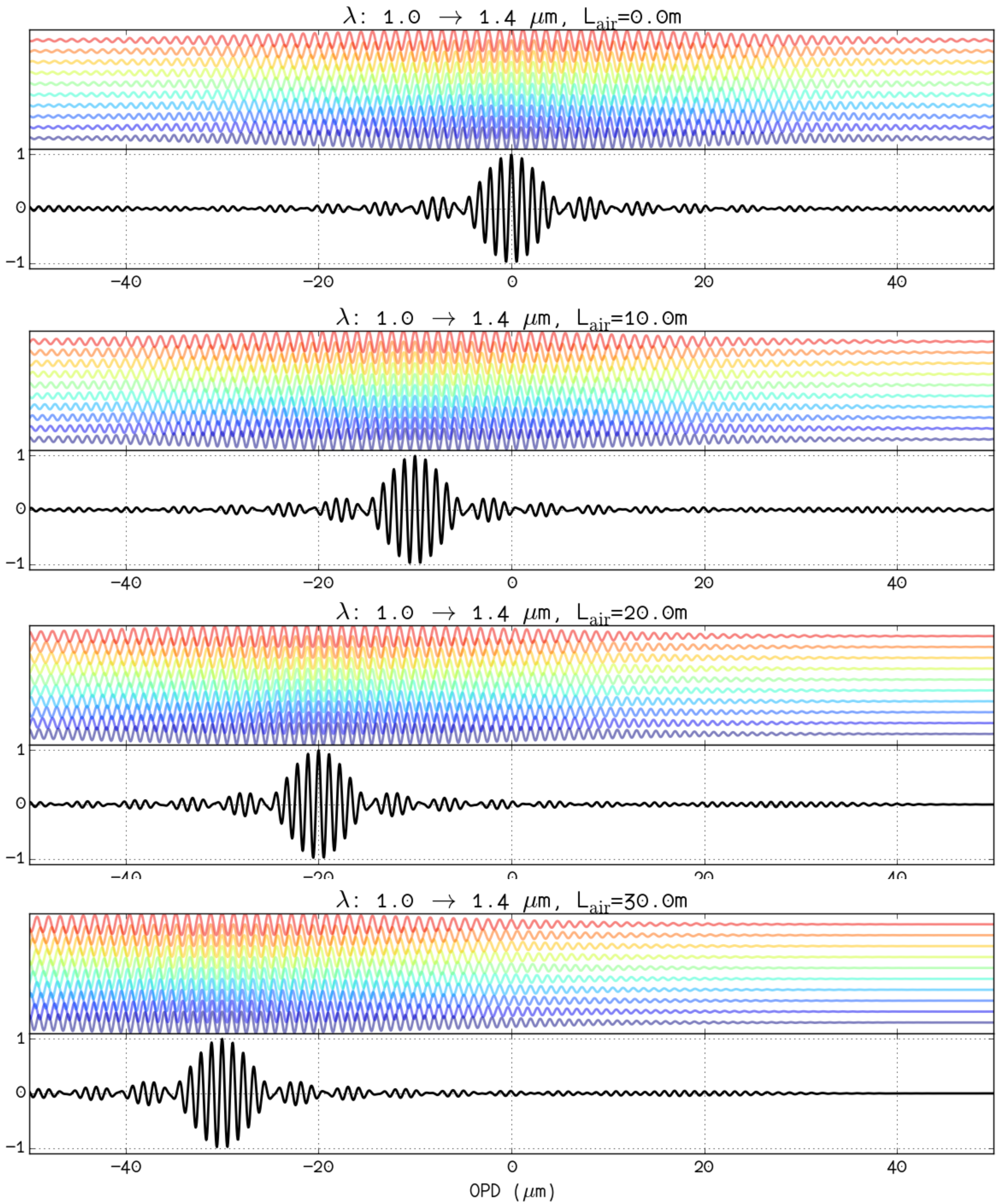


The OPD is in **vacuum**,
delay lines are in **air**

longitudinal dispersion

- The OPD in air is l x n(λ)
- The OPD chromatic




λ: 1.0 → 1.4 μm, L_air=0.0m
λ: 1.0 → 1.4 μm, L_air=10.0m
λ: 1.0 → 1.4 μm, L_air=20.0m
λ: 1.0 → 1.4 μm, L_air=30.0m
1
0
−1
−40
−20
0
20
40
OPD (μm)

Effect on differential phase

ideal fringes

$$F(x,\lambda) = 1 + \cos(2\pi x/\lambda + \phi_\lambda)$$

OPD modulation

$$x = L_{\text{vac.}} - [\Delta l_{i,j} = (l_j - l_i)]n_{\text{air}} = 0$$

polynomial expansion

$$F(x,\lambda) = 1 + \cos\left[2\pi(L_{\text{vac.}} - \Delta l_{i,j} n_0 + \delta x)/\lambda + \phi_\lambda + 2\pi \Delta l_{i,j} n_1 + 2\pi \Delta l_{i,j}(n_2\lambda + n_3\lambda^2 + ...)\right]$$

actual fringes

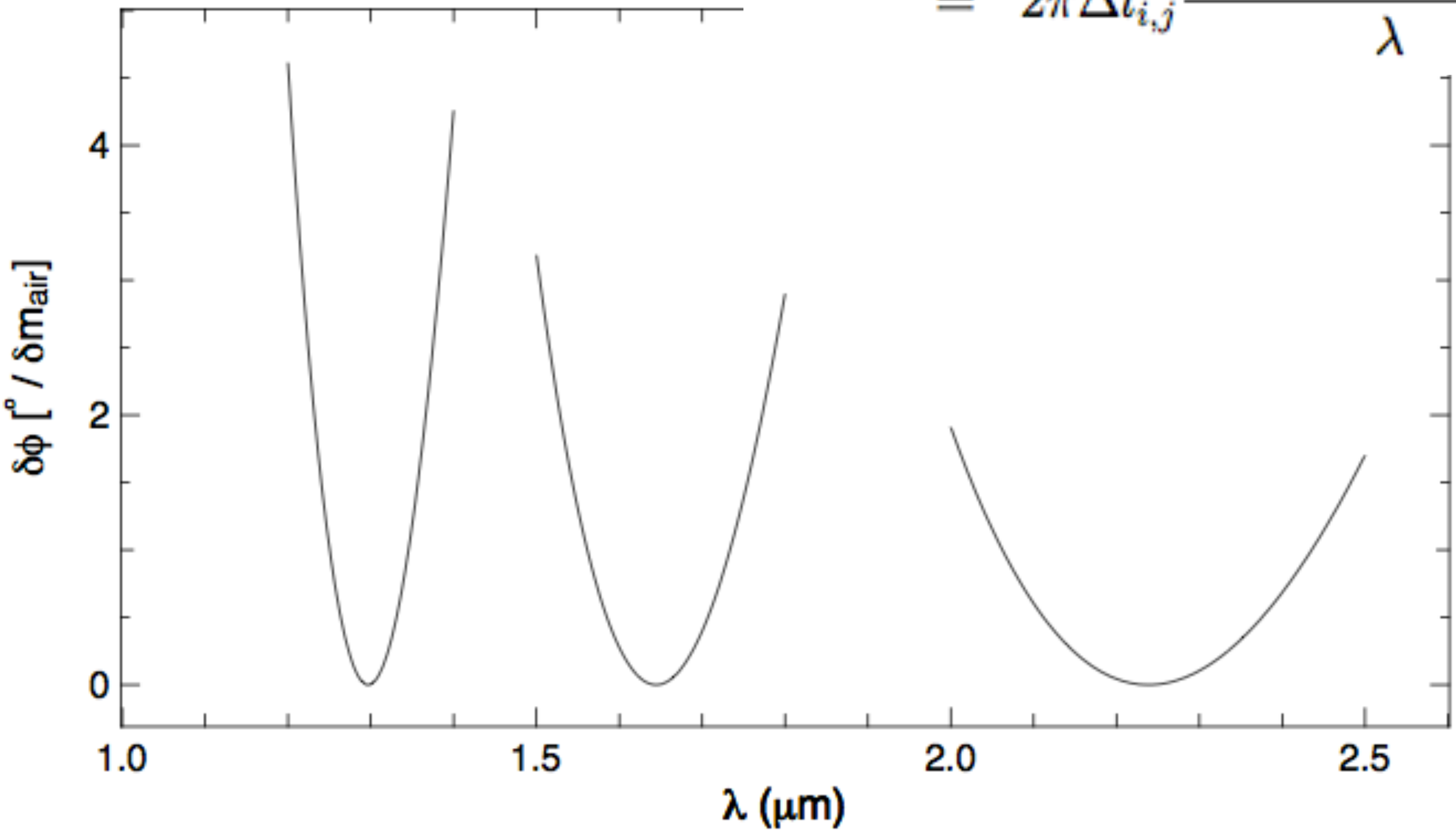
$$F(x,\lambda) = 1 + \cos\left[2\pi \delta x n_0/\lambda + \phi_\lambda + 2\pi \Delta l_{i,j}(n_2\lambda + n_3\lambda^2 + ...)\right]$$

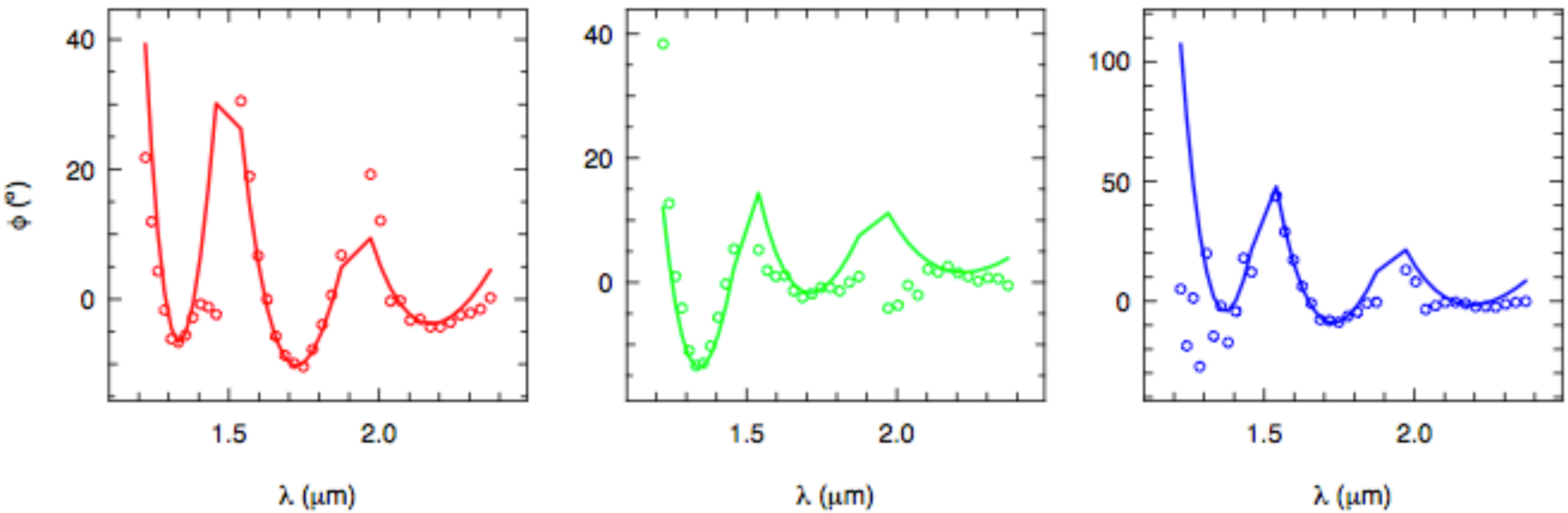
Chromatic phase

$$\begin{aligned}\phi_c(\lambda) &= 2\pi \Delta l_{i,j}\left(n_2\lambda + n_3\lambda^2 + ...\right) \\ &= 2\pi \Delta l_{i,j} \frac{n_{\text{air}}(\lambda) - n_0 - n_1\lambda}{\lambda}\end{aligned}$$

Example: AMBER

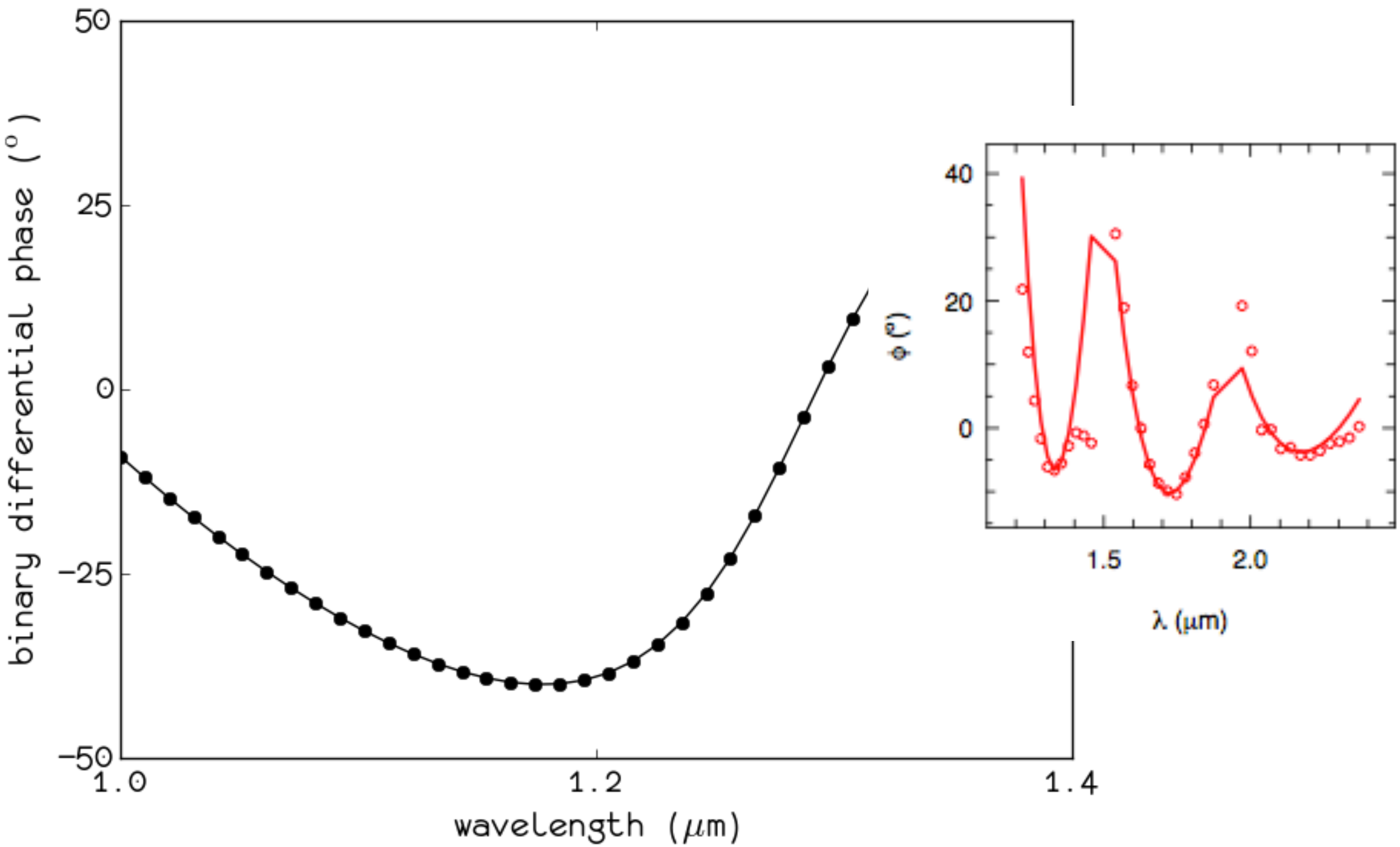
$$\phi_c(\lambda) = 2\pi\Delta l_{i,j}\left(n_2\lambda + n_3\lambda^2 + ...\right)$$
$$= 2\pi\Delta l_{i,j}\frac{n_{\rm air}(\lambda) - n_0 - n_1\lambda}{\lambda}$$





Observed differential phase
and model based on DL positions

There is information in the differential phase!

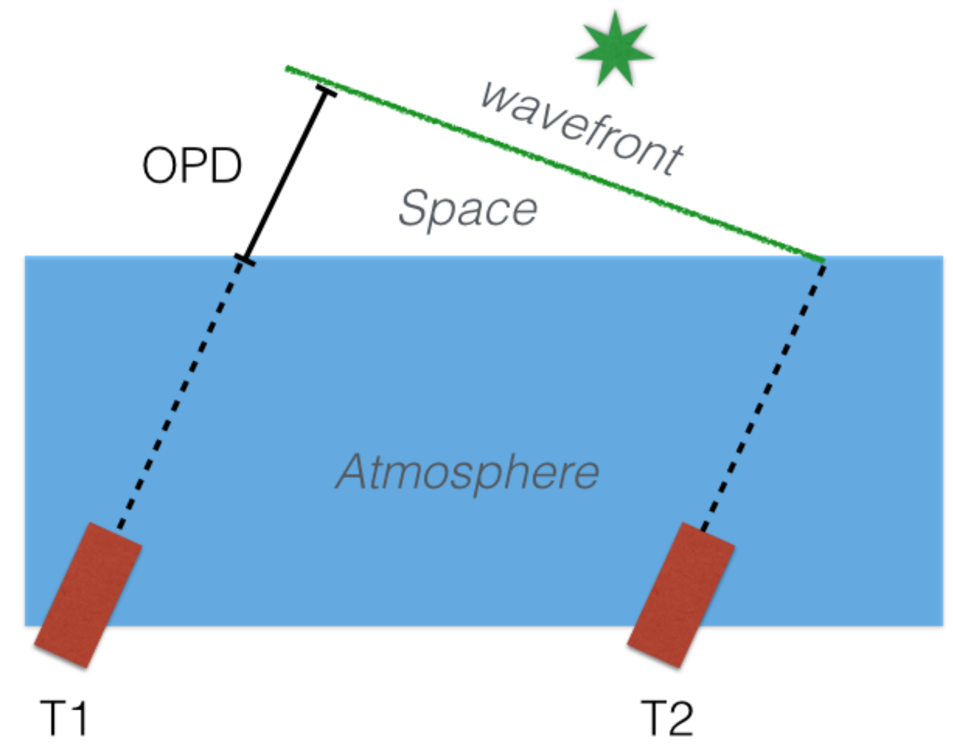


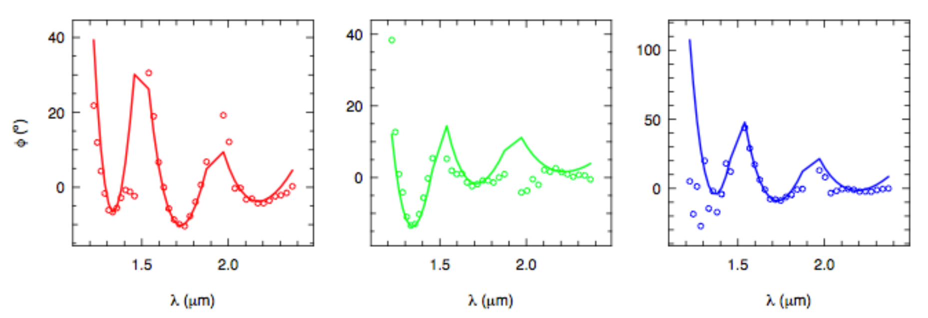
Use of differential phase?

- longitudinal air dispersion needs to be **accurately modeled** to extract the astrophysical signal

- **L_{air} is easy to estimate** (function of zenithal distance)

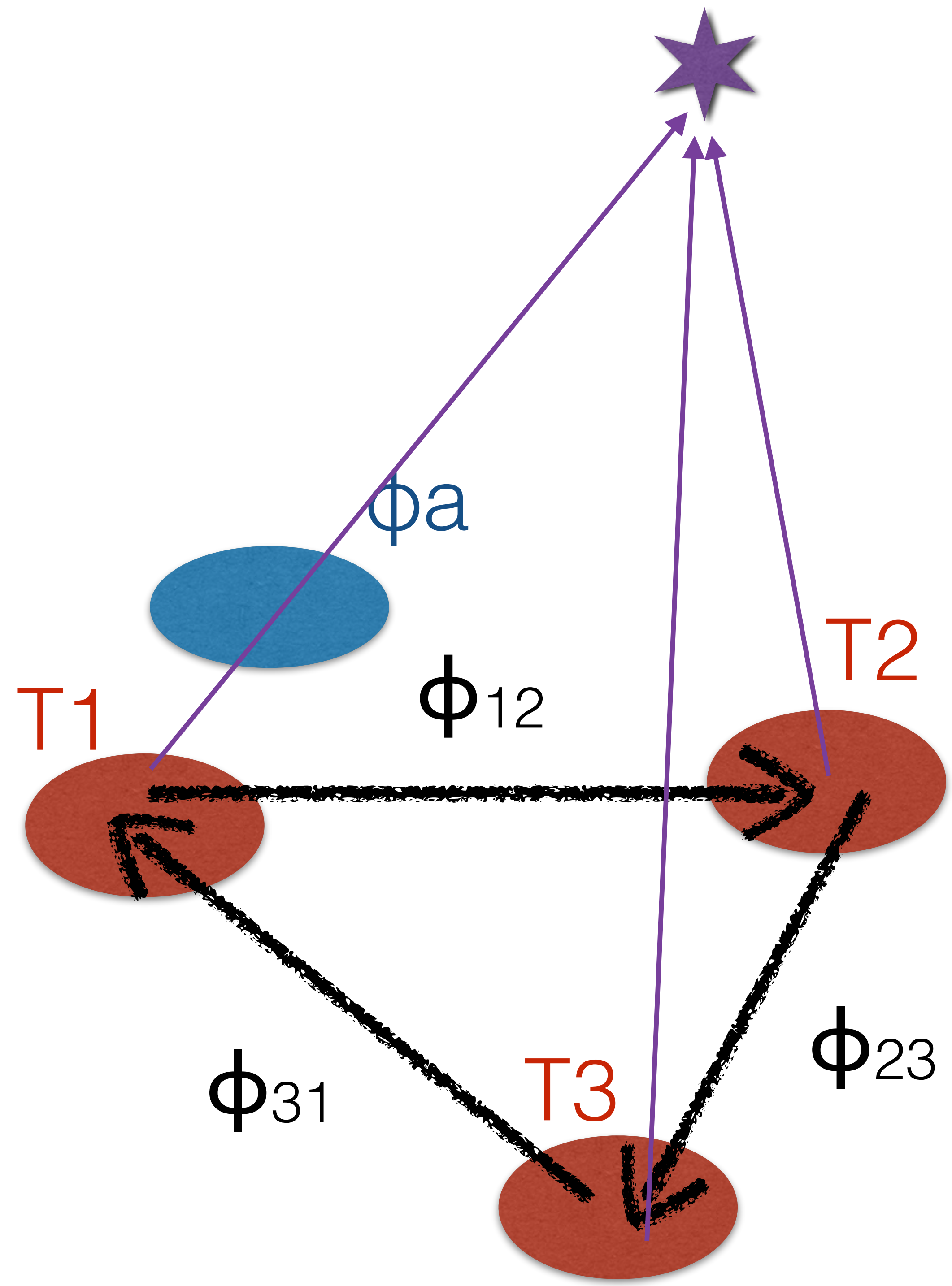
- n_{air} is a also function temperature, pressure, water content etc… **never perfect correction (≠ accurate)**





Closure Phase

- Measure phase sum in a **close triangle**
- CP = $(\phi_{12} + \phi_a) + (\phi_{23}) + (\phi_{31} - \phi_a) = \phi_{12}+\phi_{23}+\phi_{31}$
- <u>CP is insensitive to longitudinal dispersion!</u>



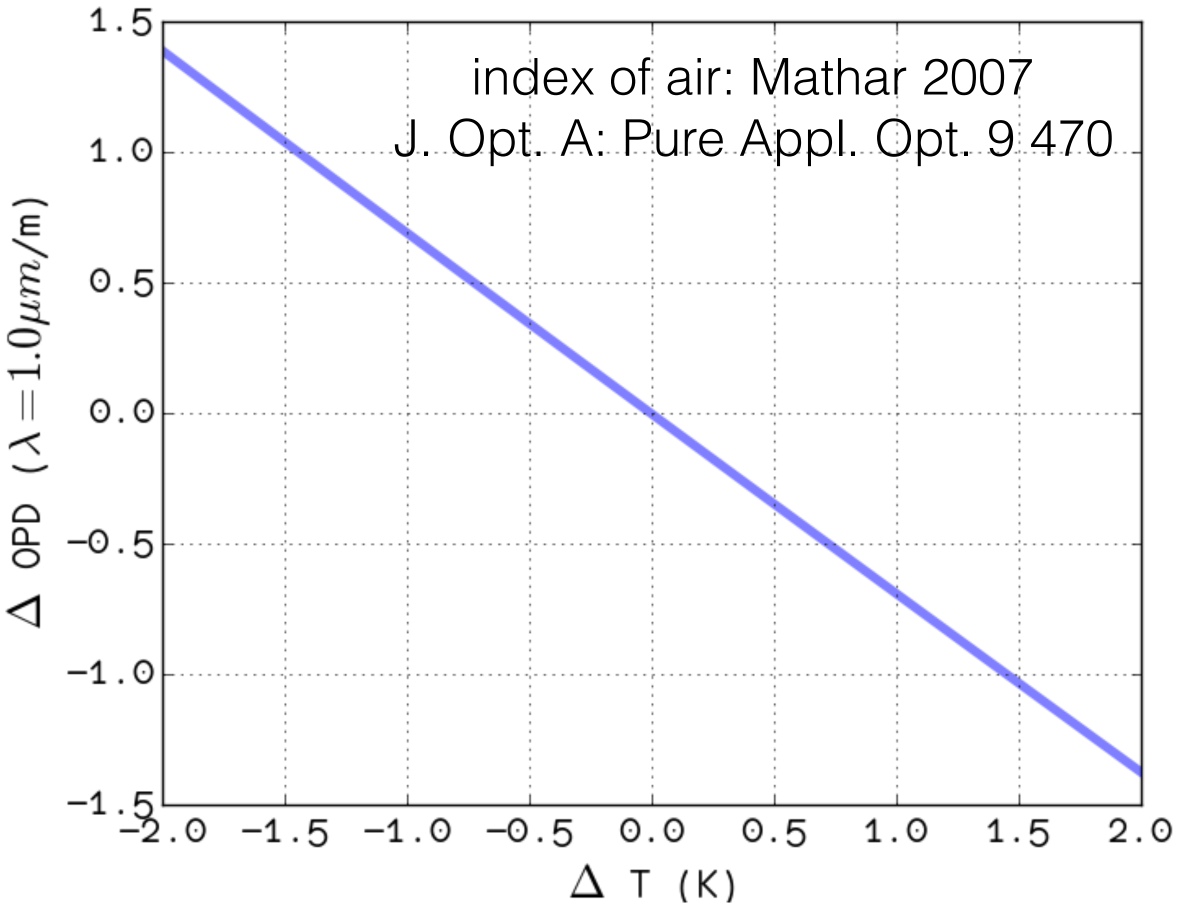
Differential Phase

- Differential phase has a strong instrumental bias (air dispersion)
- Bias is very large (many 10^{o})
- We have seen 2 solutions:
  1. model the effect
  2. use a robust estimator (closure phase)
- *Alternate solution: correct with glass with refractive chromaticism inverse to air (hard to get accurate)*

Phase jitter correction

1m bubble of air with 1.5°K difference produces a 1μm OPD difference

same for 10m bubble of air with 0.15°K difference



How long can we integrate?

- Reminder: for photon shot and readout noises, the longer integration the better
- How about the turbulent piston?
- loss of contrast:

$$V_{\text{loss}}^2 = e^{-\phi^2} = e^{-(2\pi \frac{\sigma_{\text{OPD}}}{\lambda})^2}$$

Interferometric SNR

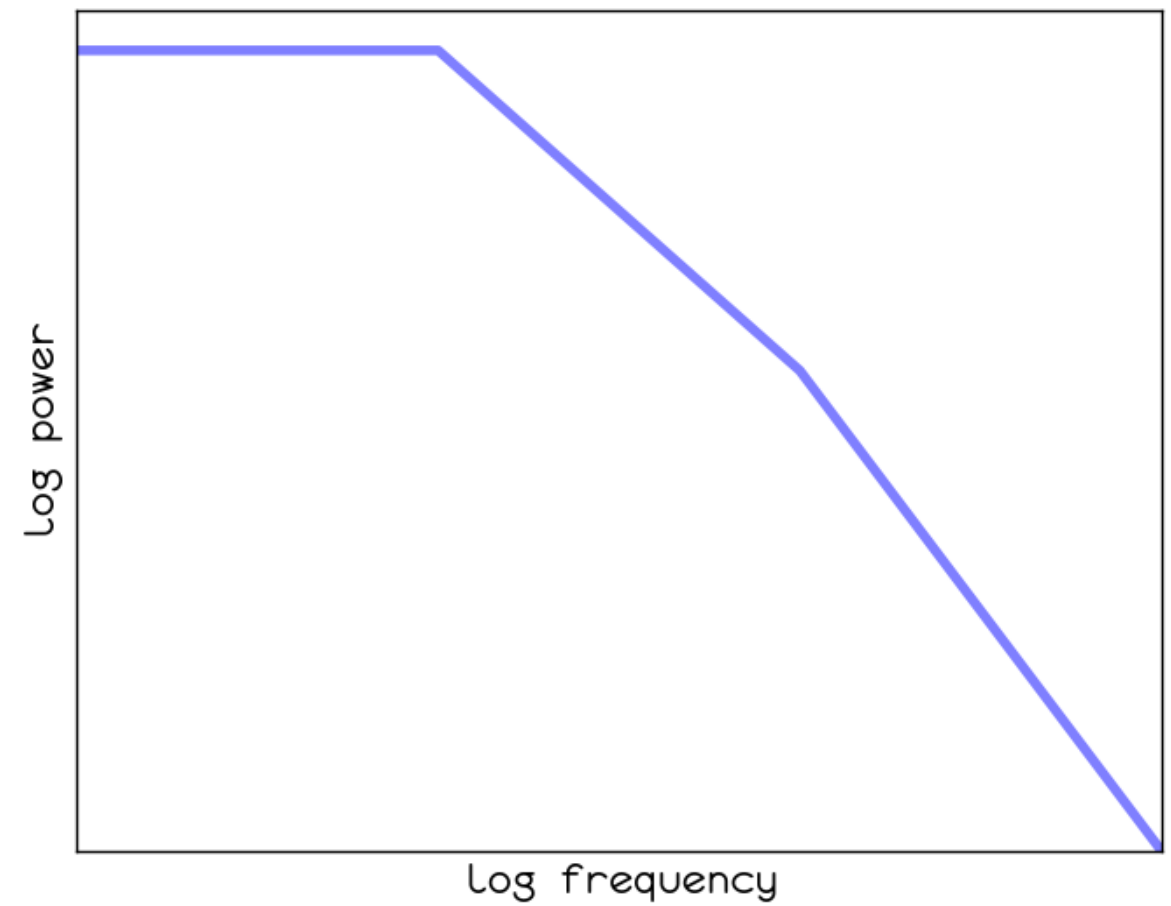
- signal: coherent flux ~ $N_{phot} \times V_{obs}$
- Noises: read-out and photon noises

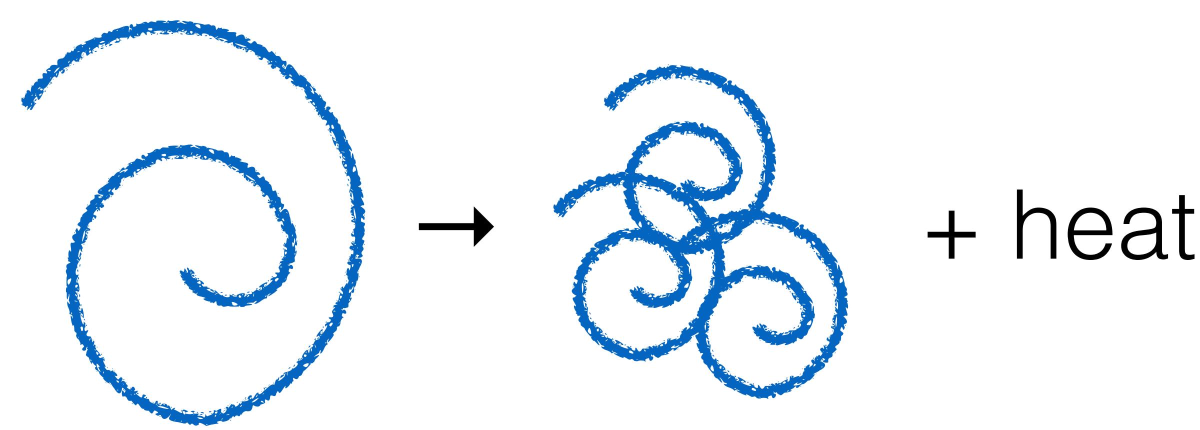
$$SNR \sim \frac{N_{phot}(t)e^{-\pi\frac{\sigma^2_{opd}(t)}{\lambda}}}{\sqrt{N_{phot}(t)+ron^2}}$$

Turbulent PSD

Typical of cascading energy phenomena

- energy injected a low frequency (wind, gravity waves)
- breaks down in smaller and smaller scales, losing each time more energy





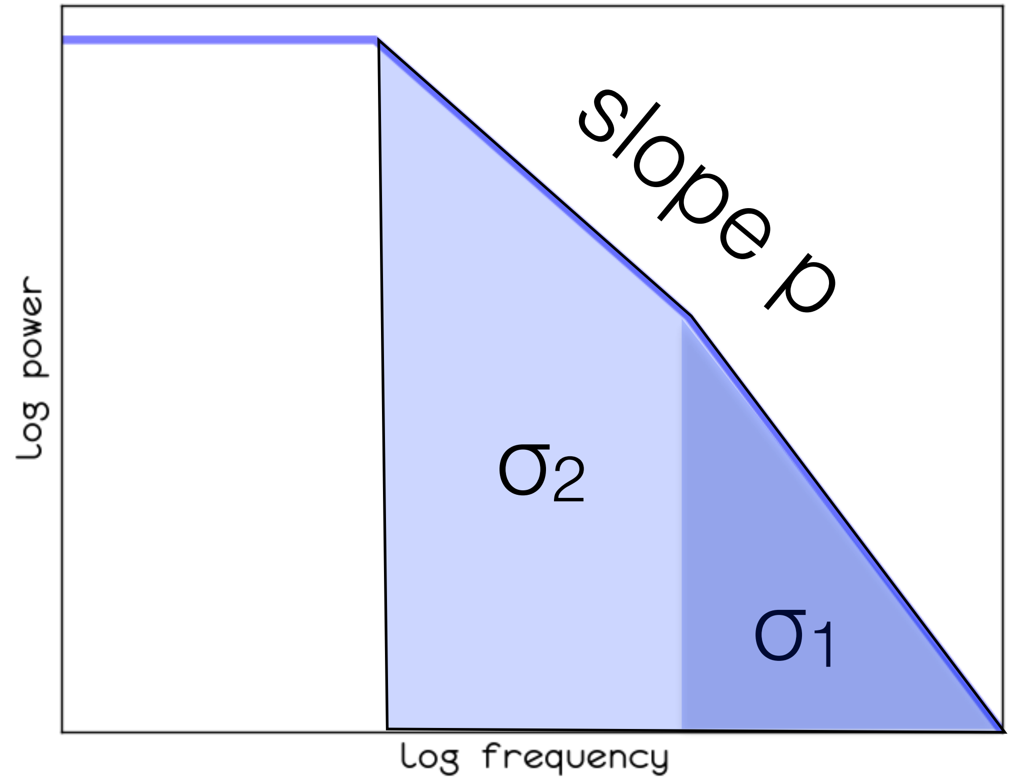
variance?

Parseval identity:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |TF(f)(\sigma)|^2 d\sigma$$

Variance ~ integral of PSD

$$\int_{0}^{T} |f(t)|^2 dt \sim \int_{1/T}^{\infty} |TF(f)(\sigma)|^2 d\sigma$$

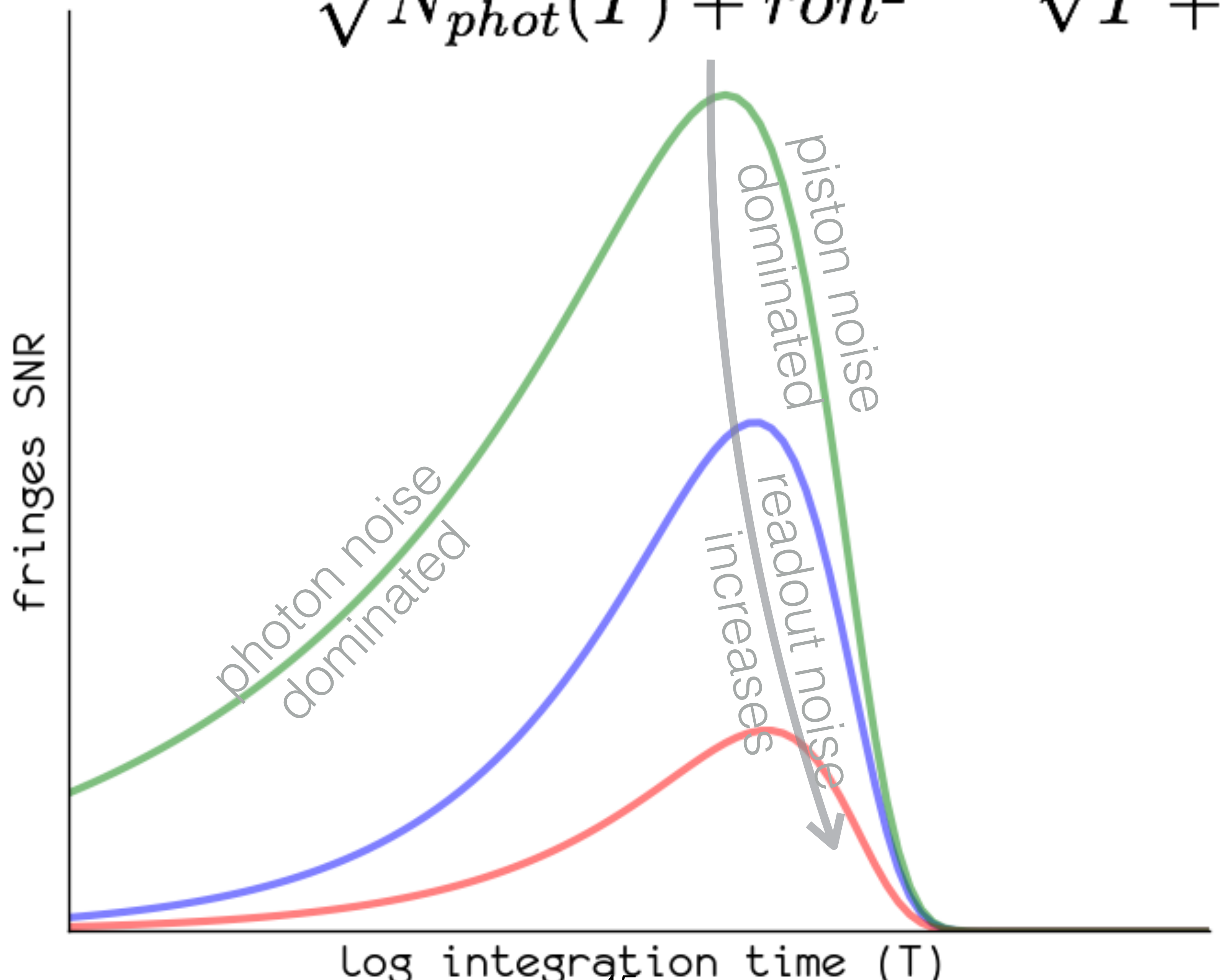


variance grows as $T^{(-p-1)}$

Kolmogorov p ~ -11/3 $\sigma_{OPD}^2(T) \sim T^{8/3}$

under turbulent atmosphere

$$SNR \sim \frac{N_{phot}(T)e^{-\pi\frac{\sigma^2_{opd}(T)}{\lambda}}}{\sqrt{N_{phot}(T)+ron^2}} \sim \frac{Te^{-T^{8/3}}}{\sqrt{T+ron^2}}$$

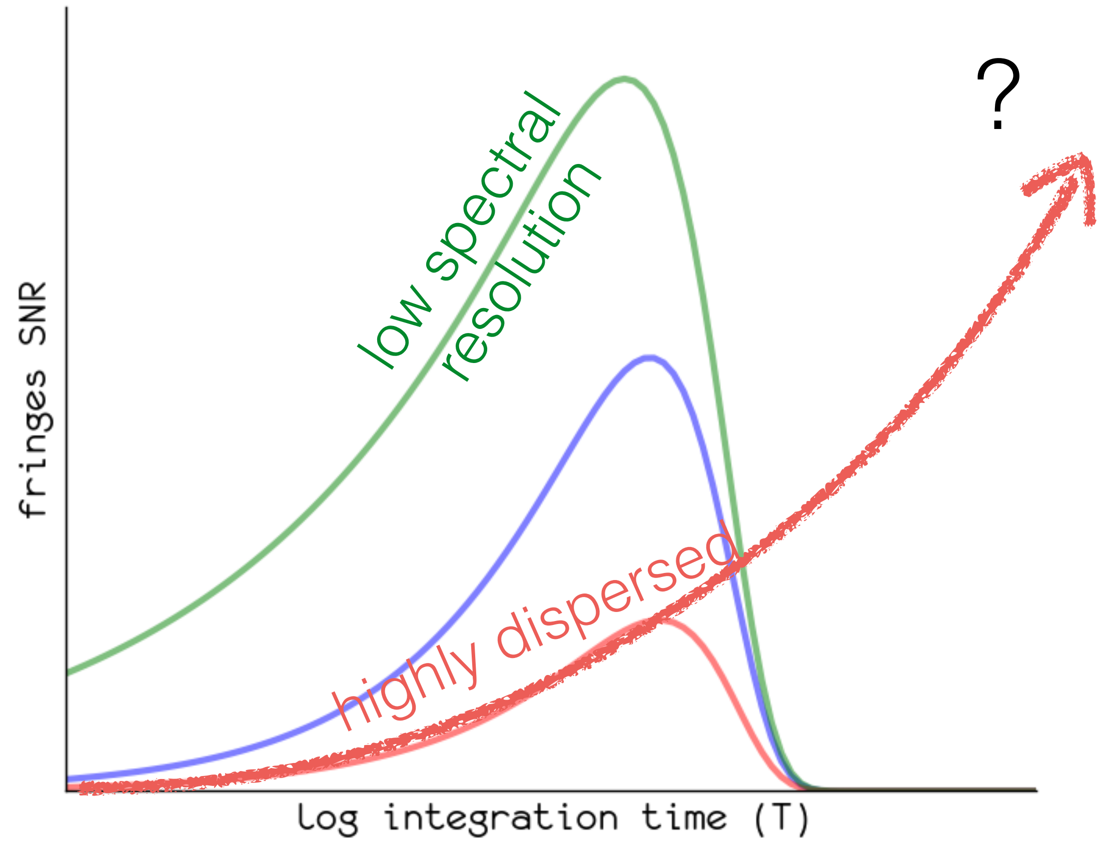


Fringe Tracking goal

Case of spectrally dispersed interferometer:

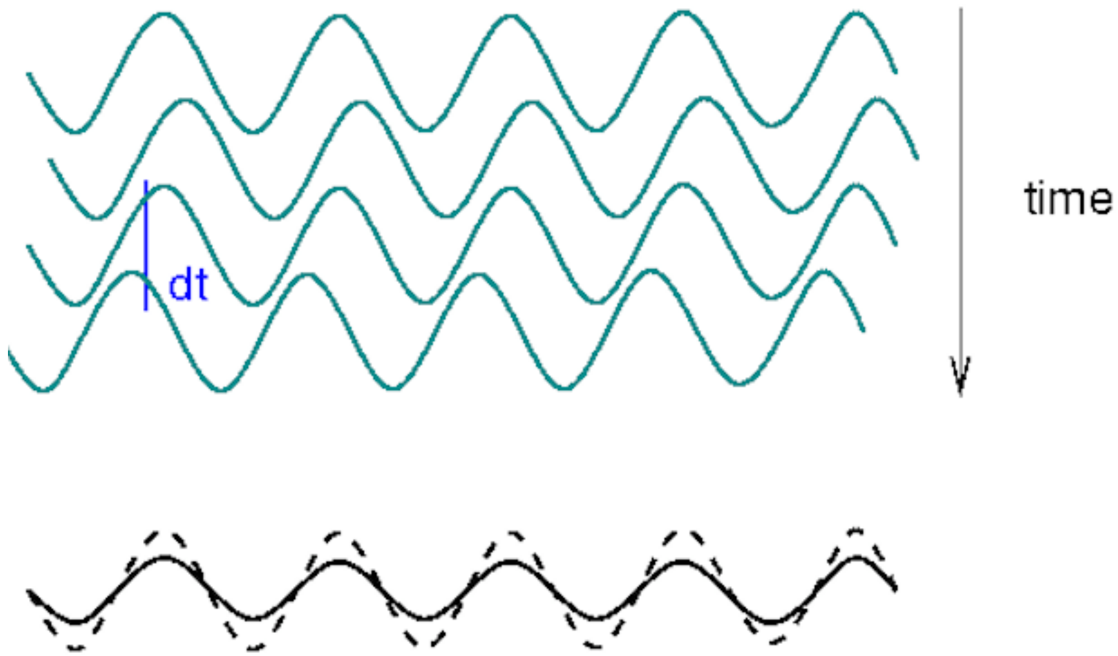
- Lack of sensitivity is a lack of photons, requiring long DIT

- everything being equal, low spectral resolution would have better SNR.

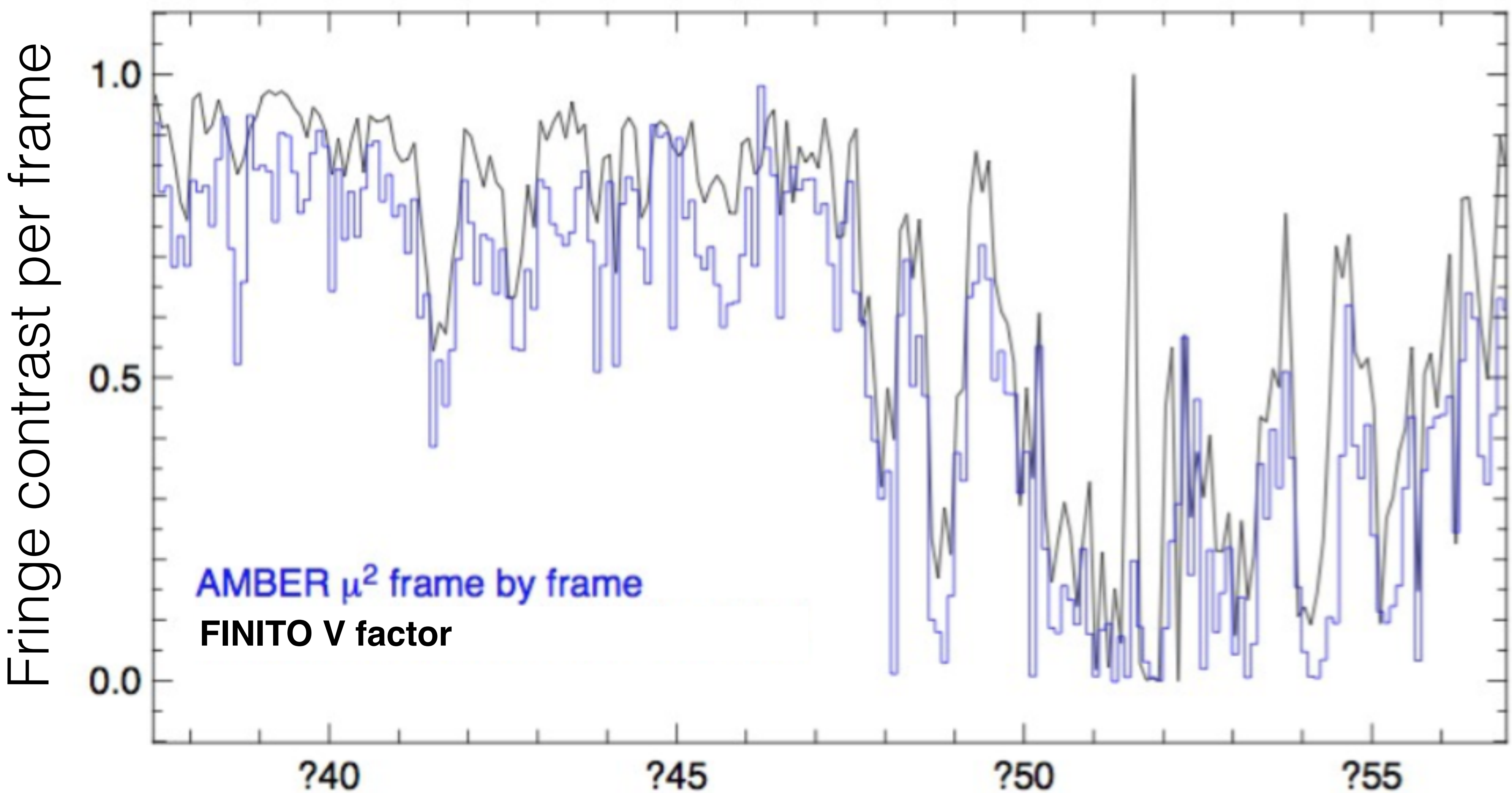


GRAVITY

- FT measures fringes every 0.001s
- SC integrates for ~10s
- FT has a tremendous amount of phase information during the SC integration
- post processing can assess the visibility loss due to FT residuals: **Gravity's V-Factor**



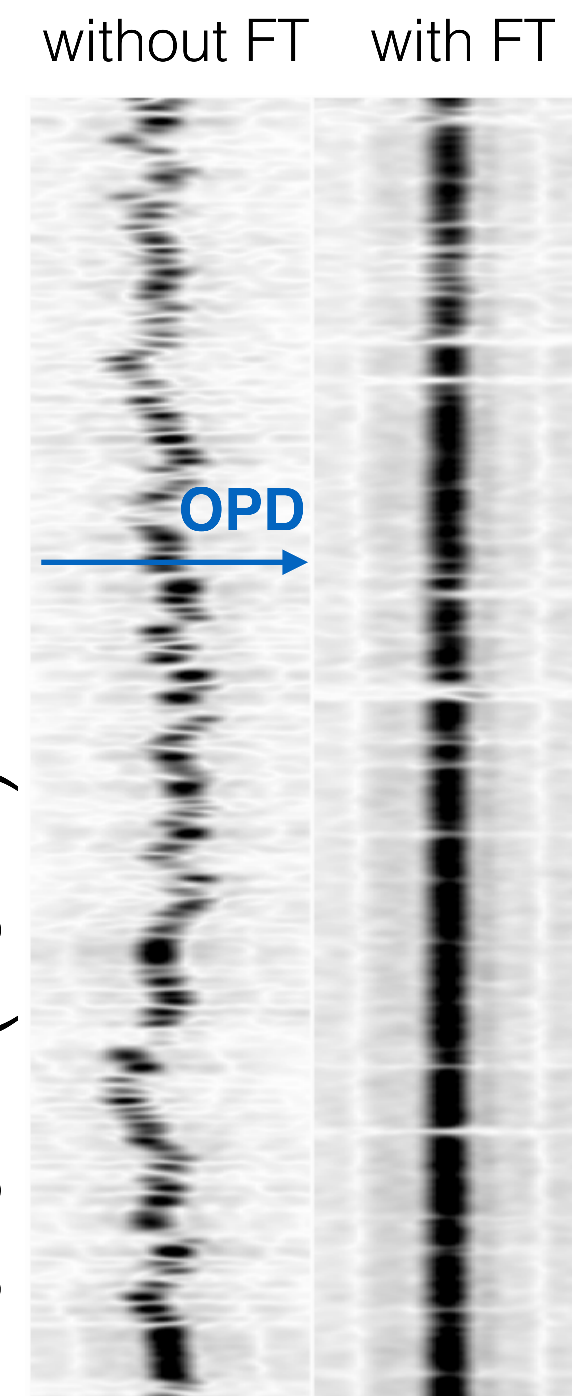
FINITO+AMBER



MIDI+FSU

- MIDI observed at 10μm, PRIMA FSU tracked at 2.2μm
- additional **post-processing correction** 2.2->10μm assumes to estimate water vapor (Koresko+ 2006)
- **Gain in sensitivity of MIDI 2.5mag:**
  - Observing mode unchanged
  - FT telemetry data recorded
  - Post processing

- Paves the way for “Gravity for MATISSE”



Müller+ (2014)

Recipe for Accuracy

Post processing and data modeling: