The University of Manchester
Jodrell Bank
Observatory

A gentle introduction to interferometry

Neal Jackson
ERIS 2015

Further reading

Principles of interferometry, Jackson 2007, LNP 742, 193
www.jb.man.ac.uk/~njj/int.pdf

Principles of interferometry and aperture synthesis
Thompson, Moran & Swenson

Synthesis imaging in radio astronomy
ASP, Proc NRAO summer school

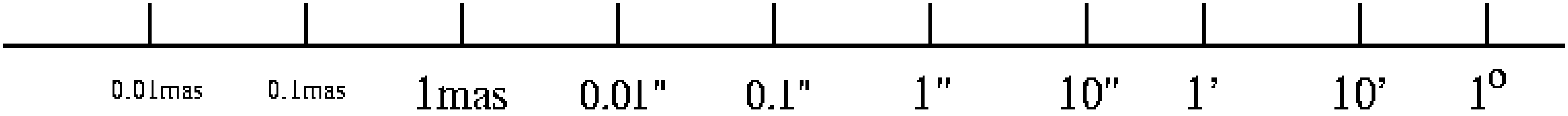
Optical interferometry in astronomy
Monnier, Rep. Prog. Phys, 66, 789, 2003

Outline of talk

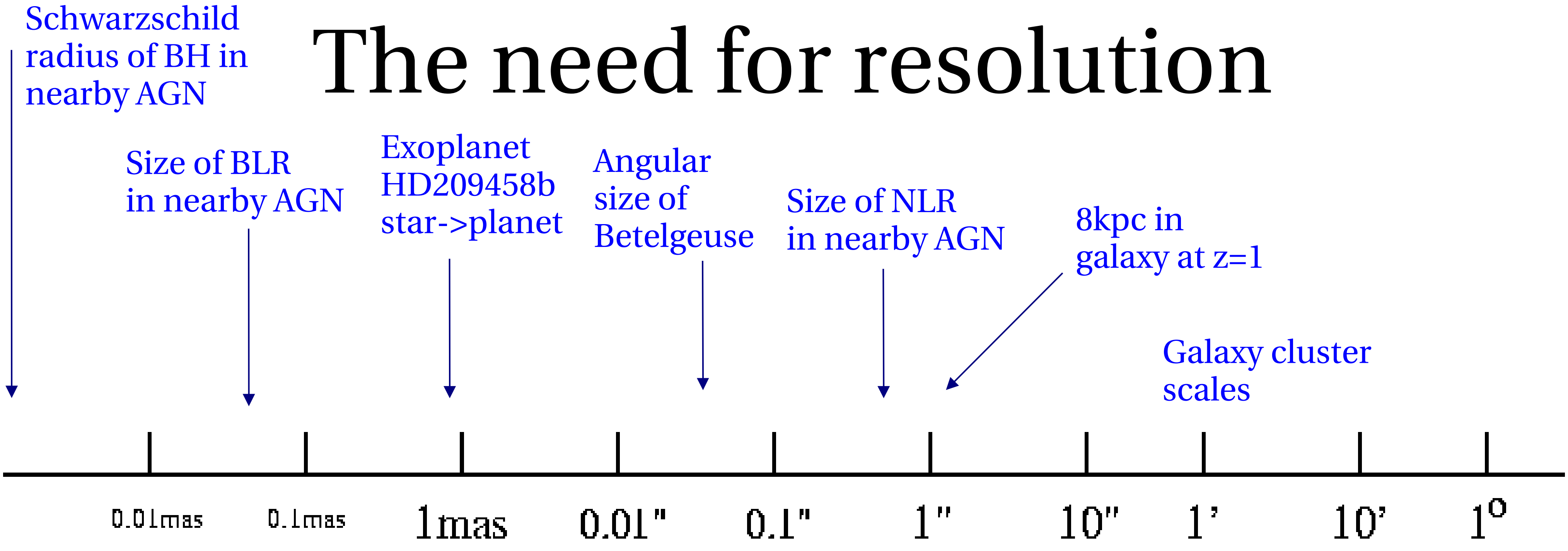
1. The need for resolution
2. Basic theory of interferometry
3. Some interferometers
4. Some practical details

1. The need for resolution

The need for resolution



The need for resolution



Schwarzschild radius of BH in nearby AGN

Size of BLR in nearby AGN

Exoplanet HD209458b star->planet

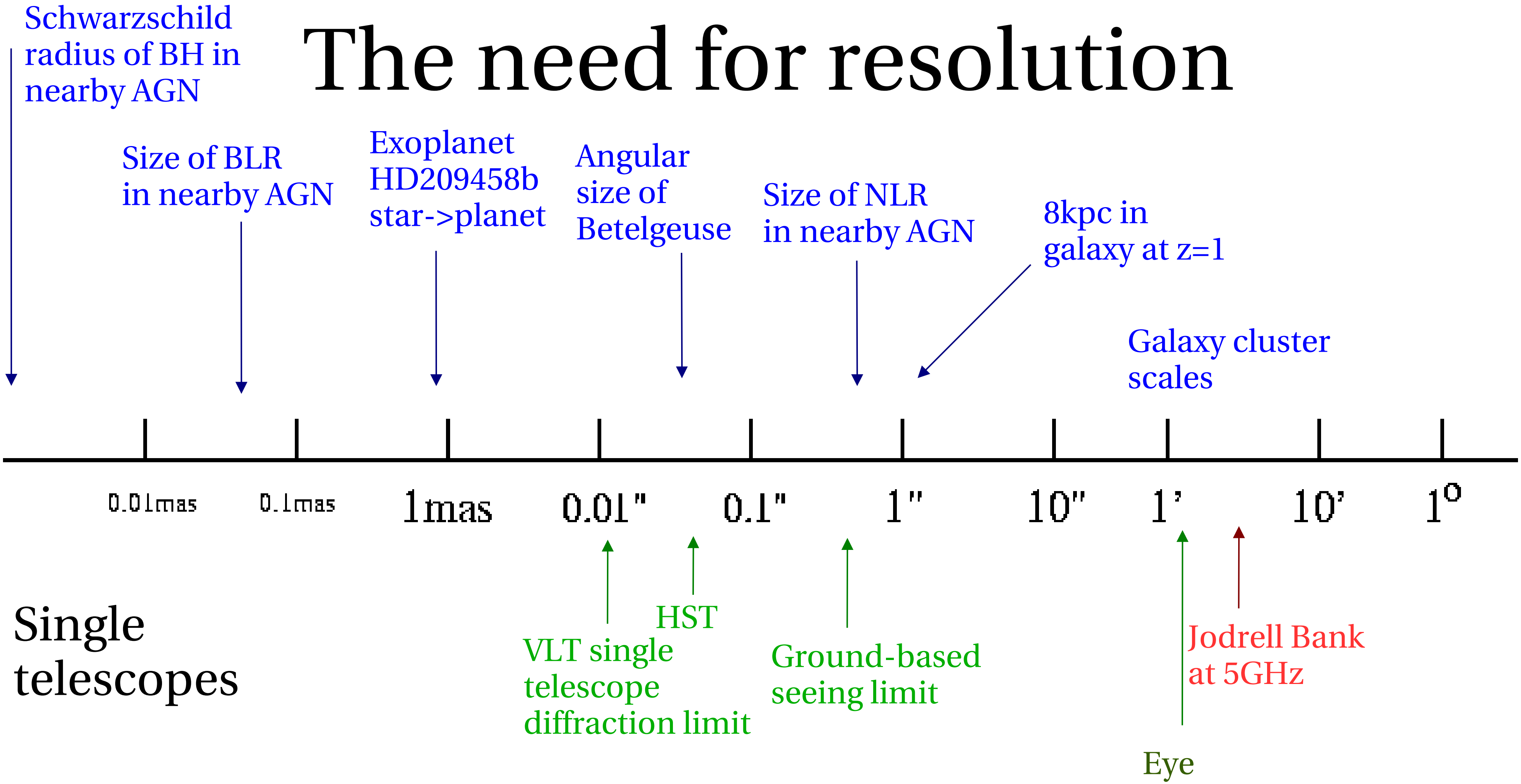
Angular size of Betelgeuse

Size of NLR in nearby AGN

8kpc in galaxy at z=1

Galaxy cluster scales

The need for resolution



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Galaxy cluster scales

VLT single telescope diffraction limit

HST

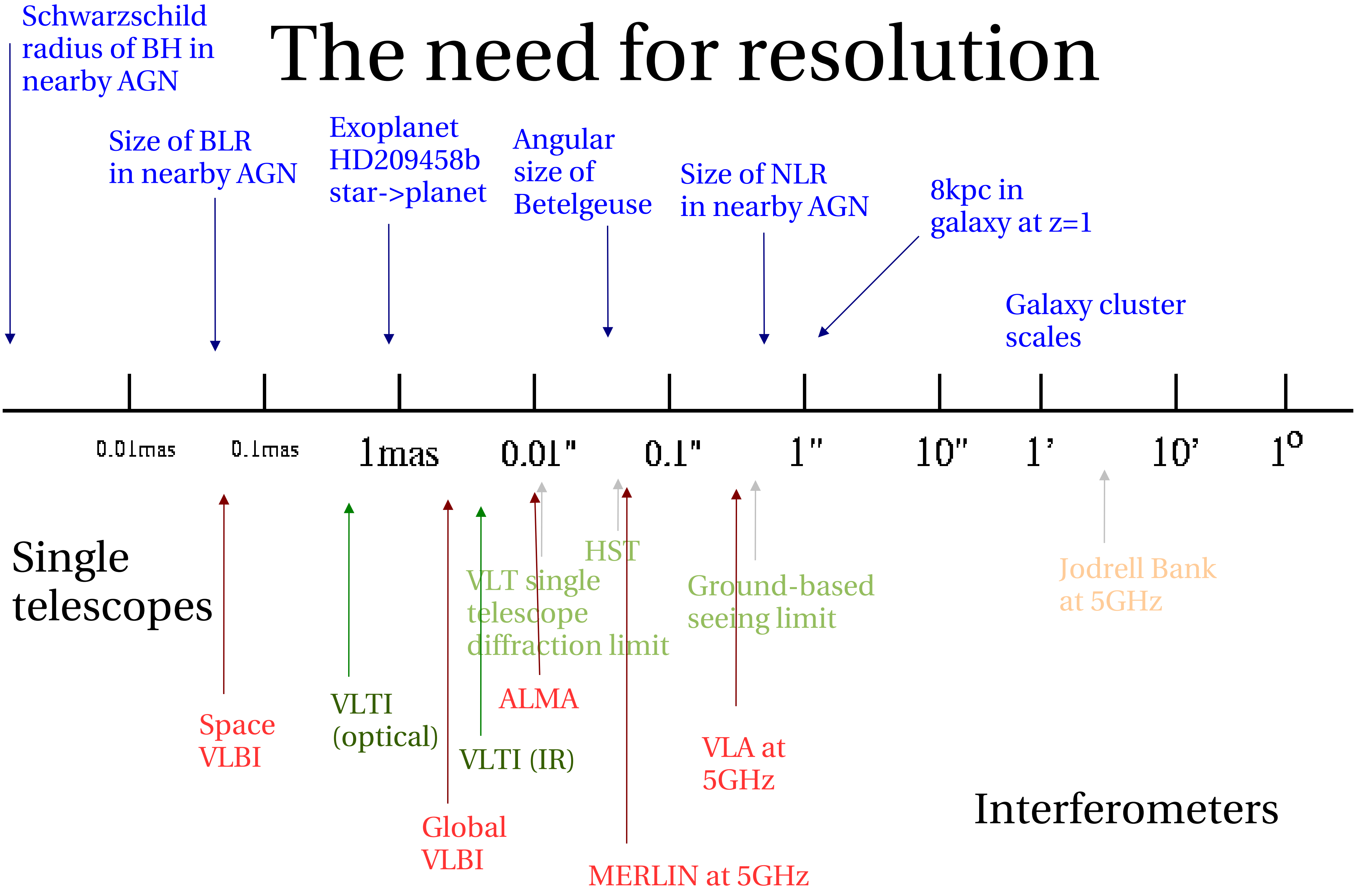
Ground-based seeing limit

Eye

Jodrell Bank at 5GHz

Single telescopes

The need for resolution



Schwarzschild
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Size of BLR
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Exoplanet
HD209458b
star->planet

Angular
size of
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Size of NLR
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8kpc in
galaxy at z=1

Galaxy cluster
scales

Single
telescopes

Space
VLBI

VLTI
(optical)

Global
VLBI

VLTI (IR)

VLT single
telescope
diffraction limit

ALMA

HST

MERLIN at 5GHz

Ground-based
seeing limit

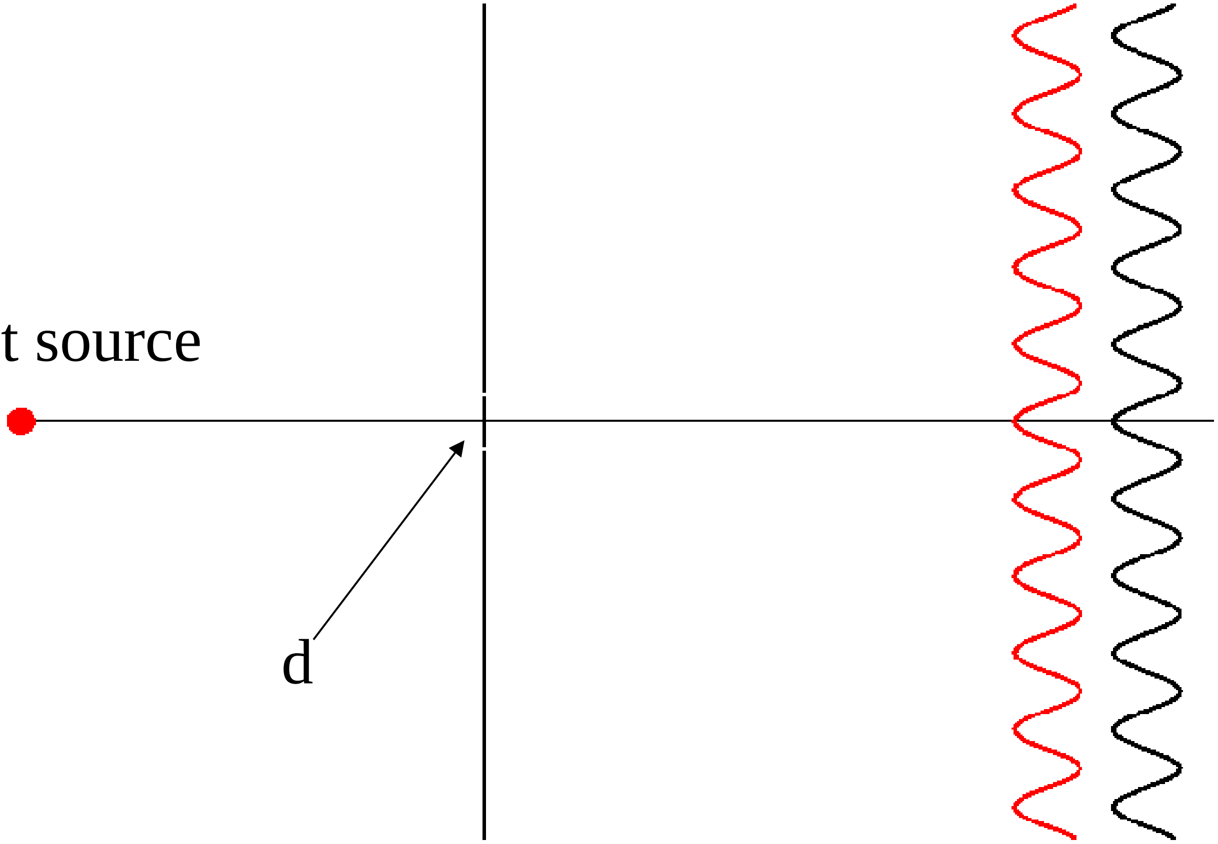
VLA at
5GHz

Jodrell Bank
at 5GHz

Interferometers

2. Basic theory of interferometry

Young's slits revisited

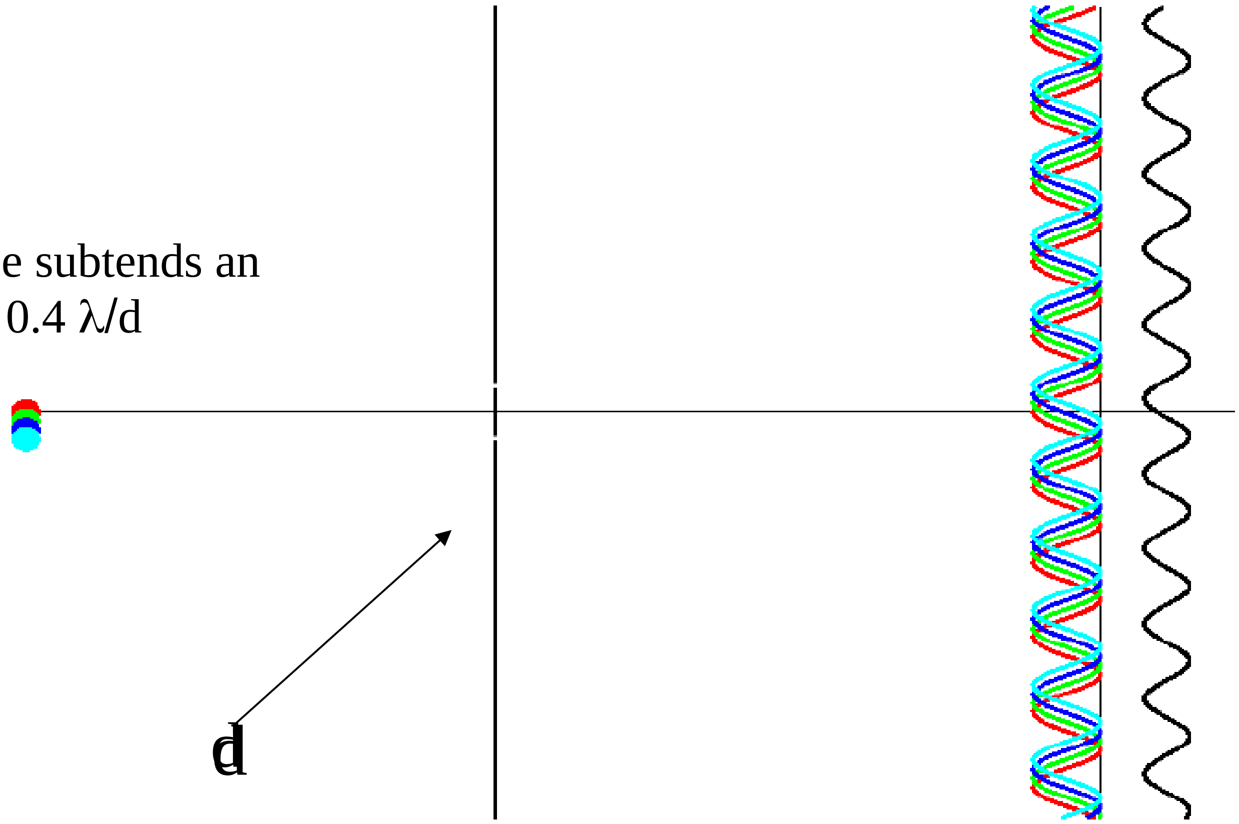


Point source

d

Fringes of separation λ/d

Larger source



Source subtends an angle 0.4 λ/d

Fringes move by 0.4 λ/d. Incoherent sources -> add intensities, fringes start to add out destructively

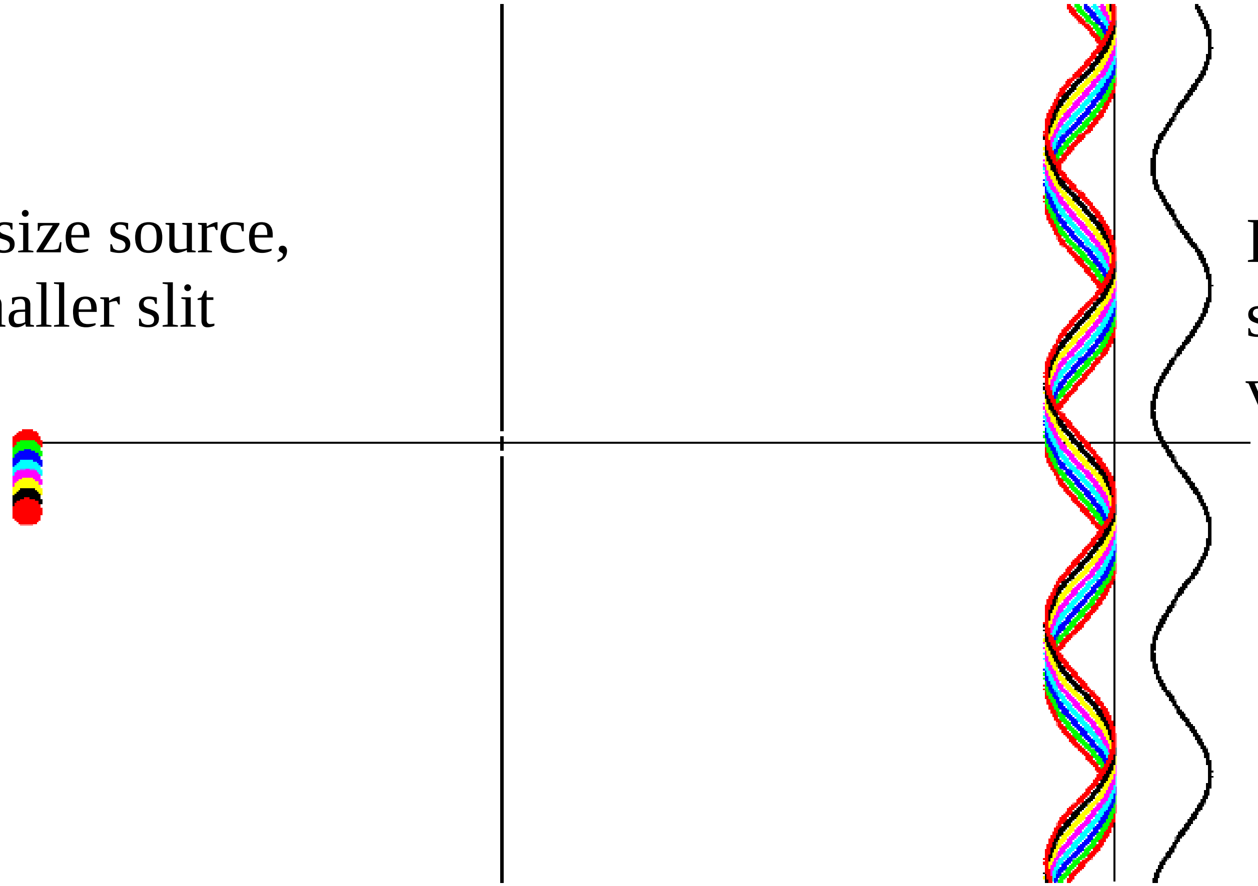
Define |fringe visibility| as $(I_{max}-I_{min})/(I_{max}+I_{min})$

Still larger source

Source size gets to λ/d

No fringes remain (cancellation). Little fringing seen for larger sources than λ/d either.

Effect of slit size



Same size source,
but smaller slit

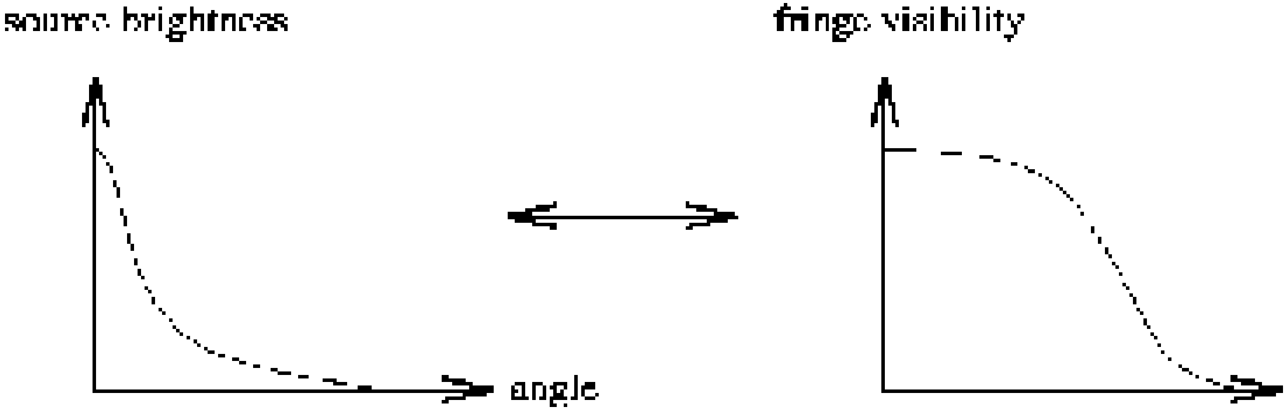
Increased fringe
spacing, so fringes
visible again

Young's slits: summary

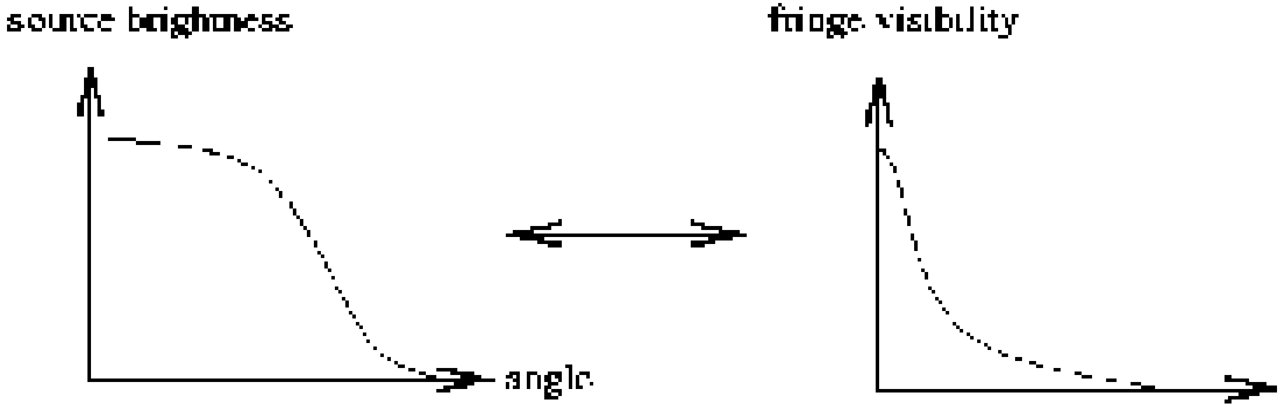
Visibility of interference fringes

- Decreases with increasing source size
- Goes to zero when source size goes to λ/d
- For given source size, increases for decreasing separation
- For given source size and separation, increases with λ

Summary in pictures



Baseline length



Baseline length

It's a Fourier transform!

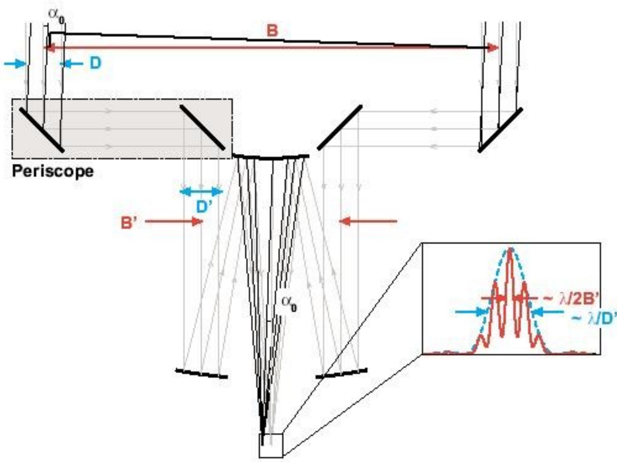
The fringe visibility of an interferometer gives information about the Fourier transform of the sky brightness distribution.

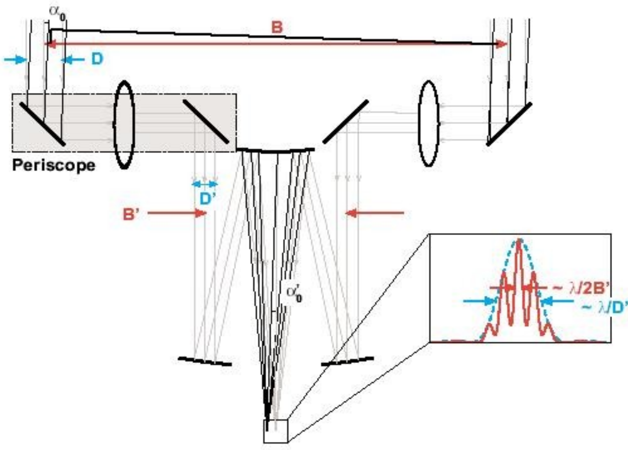
Long baselines record information about the small-scale structure of the source but are INSENSITIVE to large-scale structure (fringes wash out)

Short baselines record information about large-scale structure of the source but are INSENSITIVE to small-scale structure (resolution limit)

Combining the signals

Non-photon-limited: electronic, relatively straightforward
can clone and combine signals
“correlation” (multiplication+delay)
can even record signals and combine later
Photon-limited case: use classical Michelson/Fizeau arrangements
delay lines for manipulation
cannot clone photons



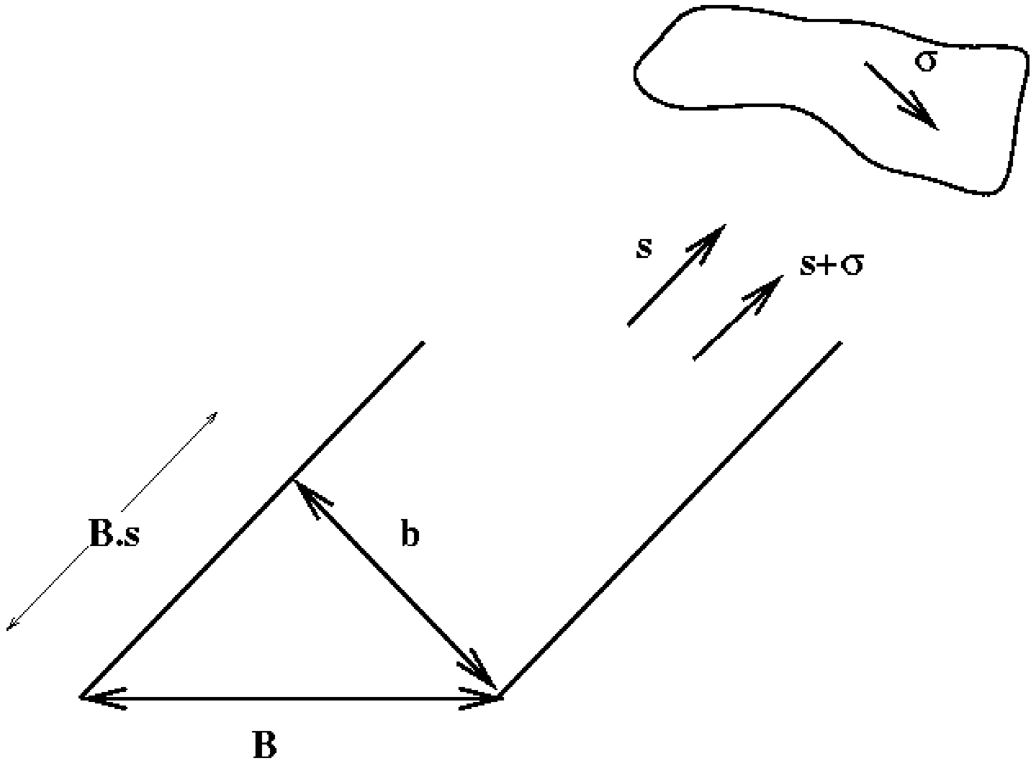


Images: A. Glindemann, Introduction to Stellar Interferometry,
VLTI website www.eso.org/projects/vlti

The same, with maths

for a multiplying interferometer i.e.

$R =< E_1^* E_2 >= E_1 E_2 e^{ikx}$



$$dR = dI(\sigma)e^{ik\mathbf{B}.(\mathbf{s}+\sigma)}, \text{but } \mathbf{B}.\sigma = \mathbf{b}.\sigma$$

$$R = e^{ik\mathbf{B}.\mathbf{s}} \int I(\sigma)e^{ik\mathbf{b}.\sigma}d\sigma$$

Can write $\sigma = \sigma(x, y)$, $\mathbf{b} = \mathbf{b}(u, v)$

$$R(u, v) = e^{ik\mathbf{B}.\mathbf{s}} \int I(x, y)e^{2\pi i(ux+vy)}dxdy$$

Fringes

$$R(u,v) = \underline{e^{ik\mathbf{B.s}}}\ \underline{\int I(x,y)e^{2\pi i(ux+vy)}dxdy}$$

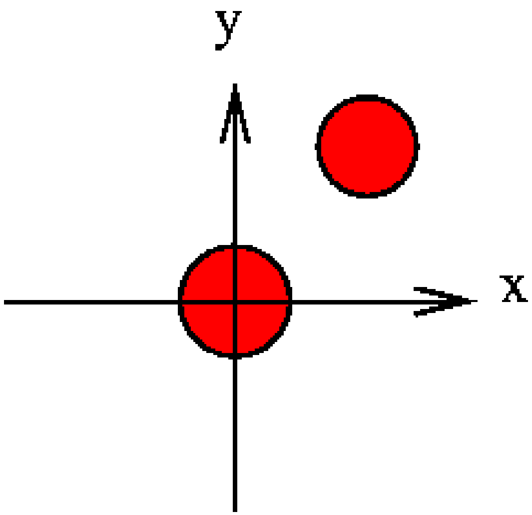
This is a series of fast fringes whose amplitude is the Fourier transform of the source brightness distribution.

May need to get rid of fringes before integrating (fringe stopping).

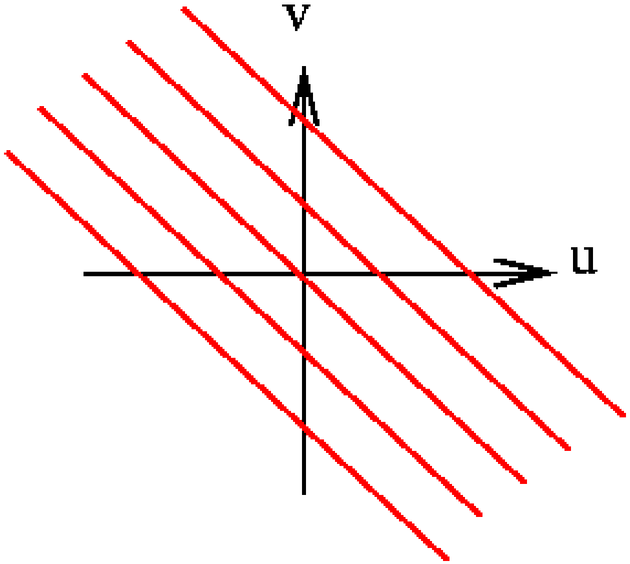
R(u,v) has an amplitude and phase; both are interesting!

The u-v plane

Source brightness as a function of angle ↔ Fringe visibility as a function of baseline length in wavelengths



Double source of separation a arcsecond



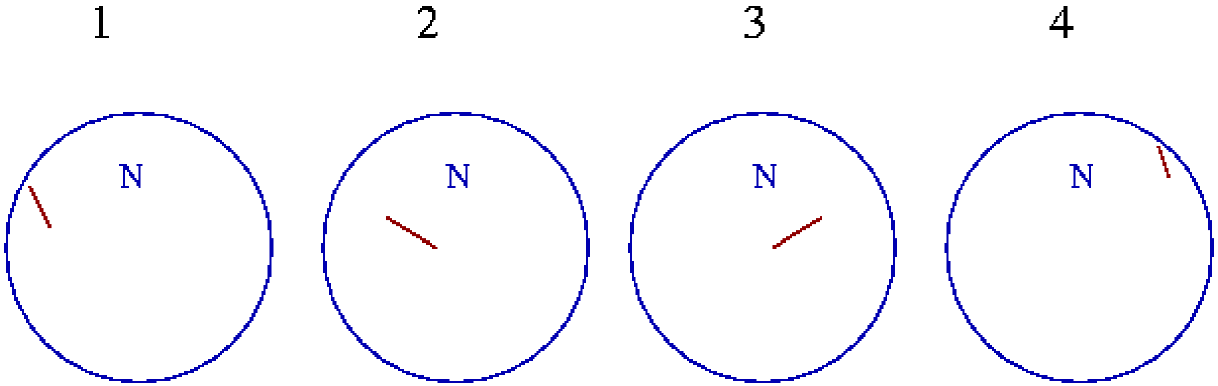
Stripes of separation 206265/a wavelengths

If we could measure FV for all u,v, transform -> image

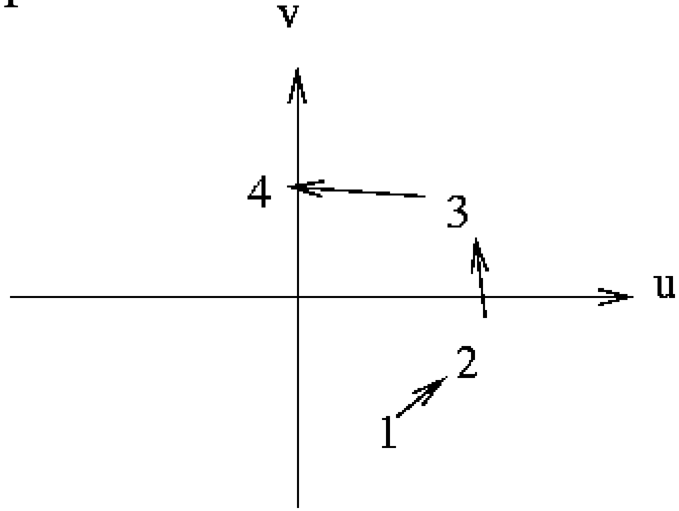
Earth rotation aperture synthesis

Image: A. Gunn/University of Manchester

Over a day,
can measure
many points in
u-v plane with
a single baseline

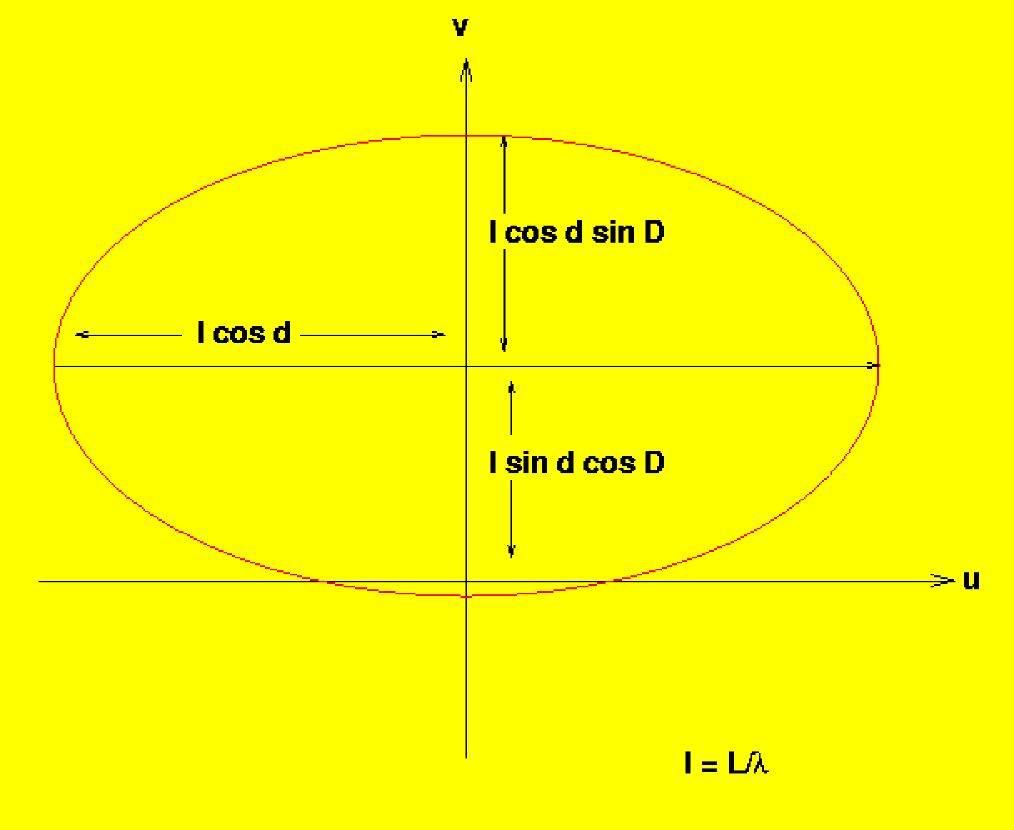


b-vector plotted in brown



Locus is an ellipse;
the longer the baseline,
the larger the u-v (higher
resolution)

Exact form of u-v track



D=declination of source
d=declination of point on
sky pointed to by baseline

Resolution given by maximum extent of tracks

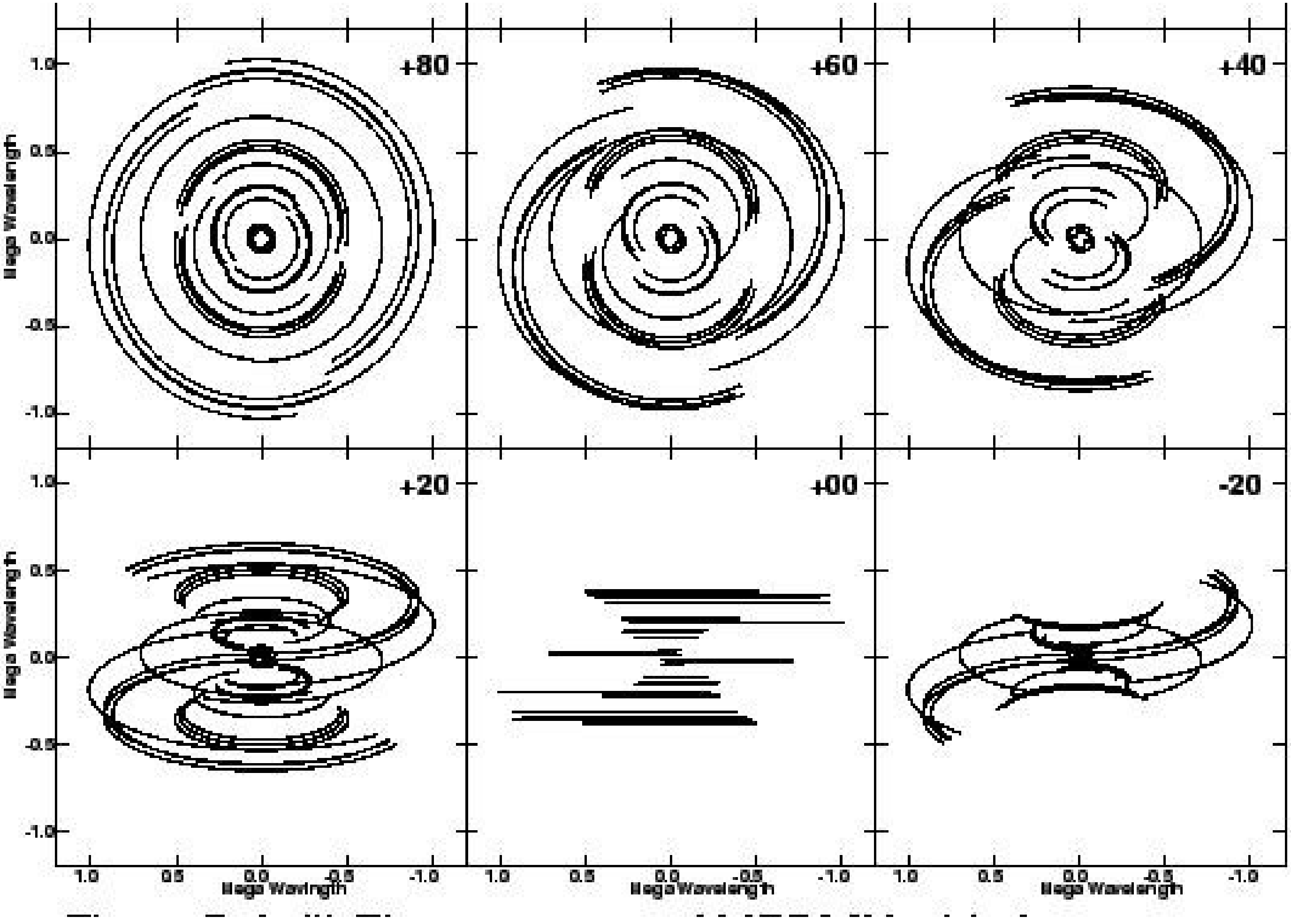
ERAS imaging of sources at D=0 is hard!

$$u = \frac{L}{\lambda}\cos d \sin(H-h),\, v = \frac{L}{\lambda}(\sin d \cos D - \cos d \sin D \cos(H-h))$$

$$\mathbf{B.s} = |B|(\sin d \sin D + \cos d \cos D \cos(H-h))$$

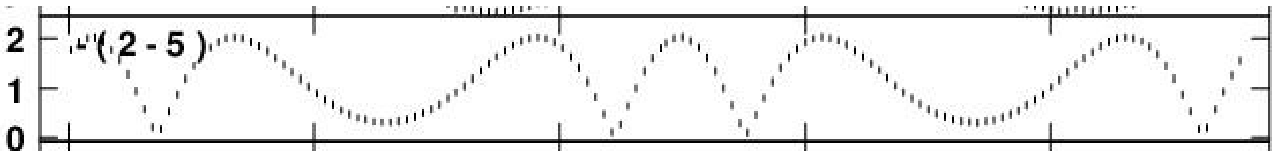
(just in case you ever need it!)

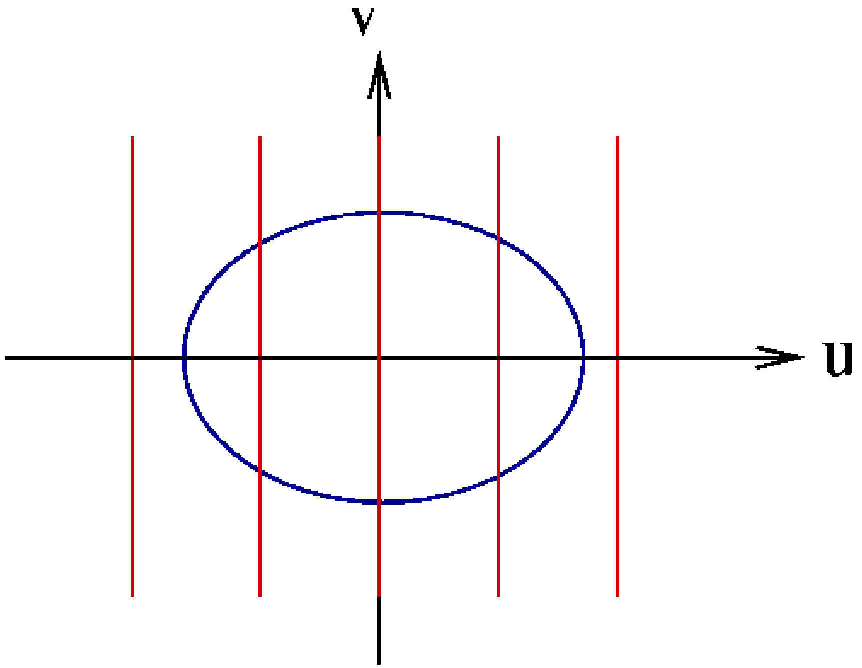
u-v tracks example: MERLIN



Low dec – elongated beam in x-y plane

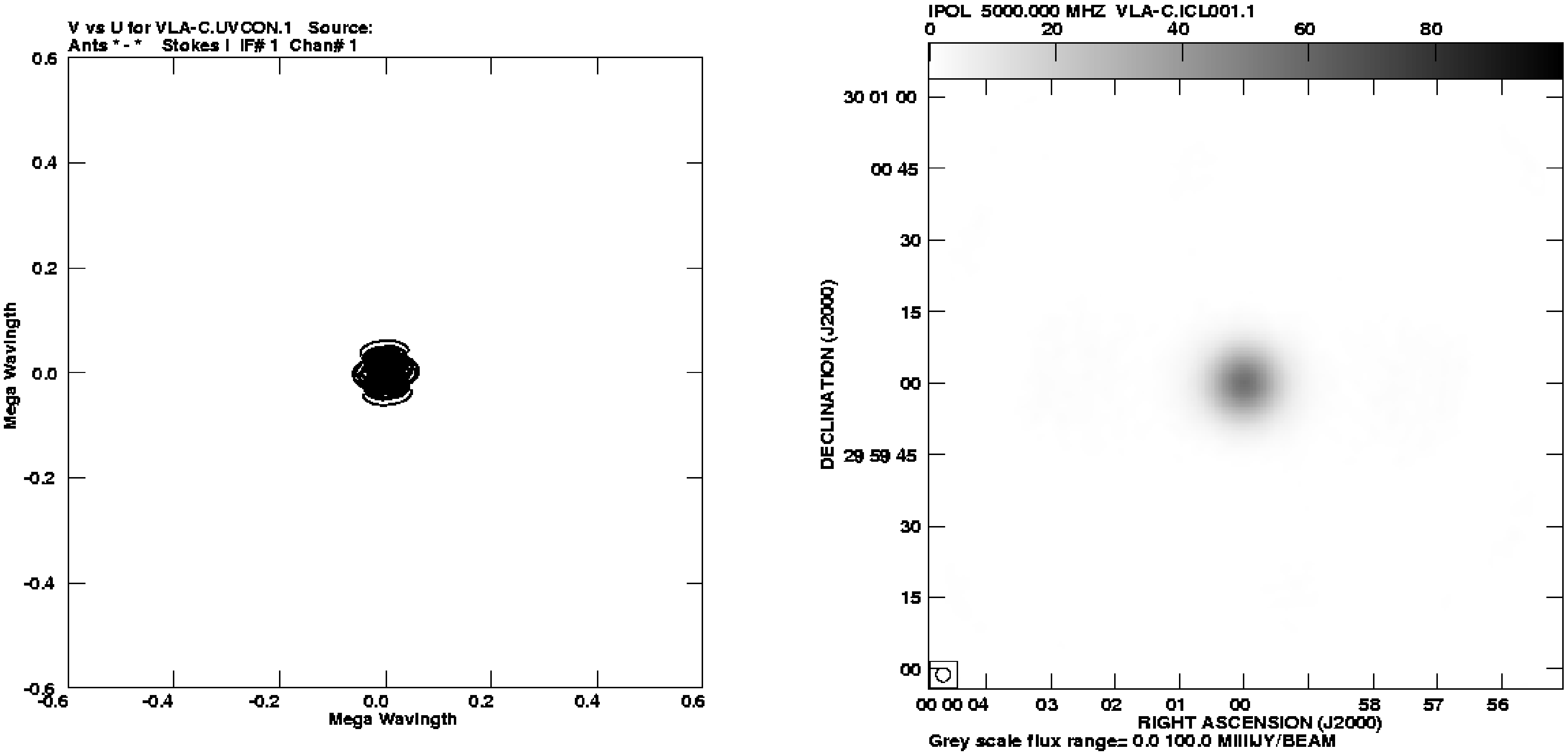
Actual fringe visibility





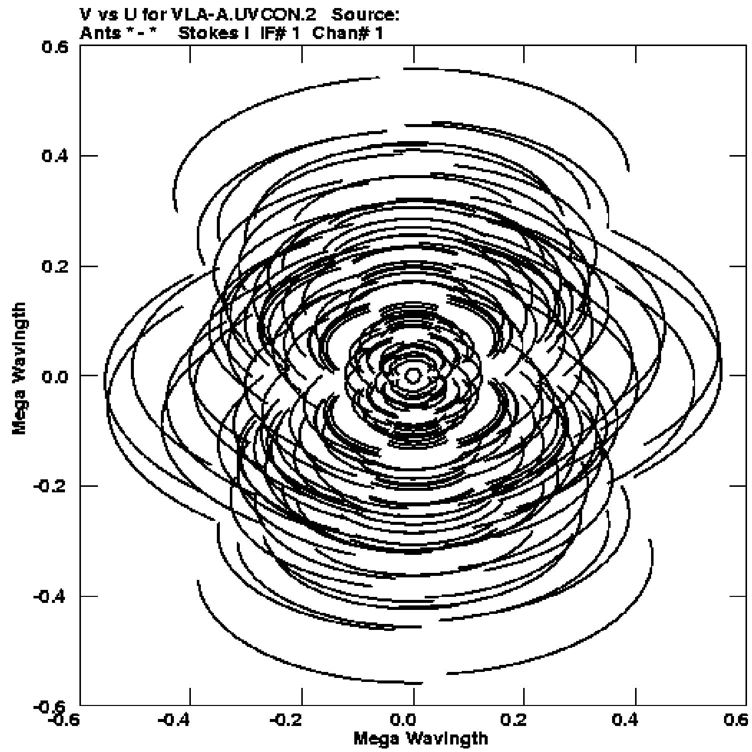
Double source
each component 1Jy
separation calculable
from baseline length

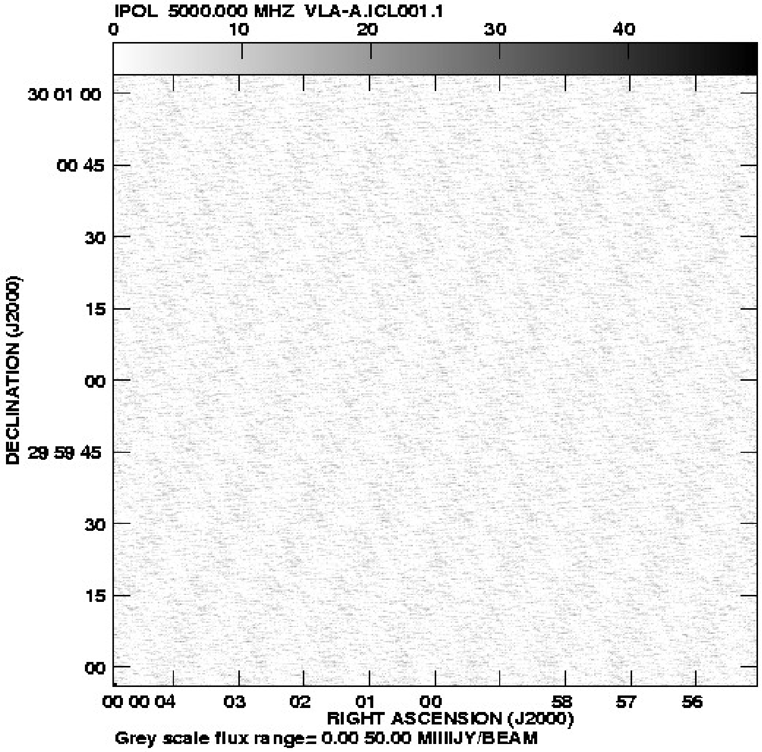
FT imaging is not like direct imaging!



12-arcsec source mapped with u-v coverage giving 3” resolution

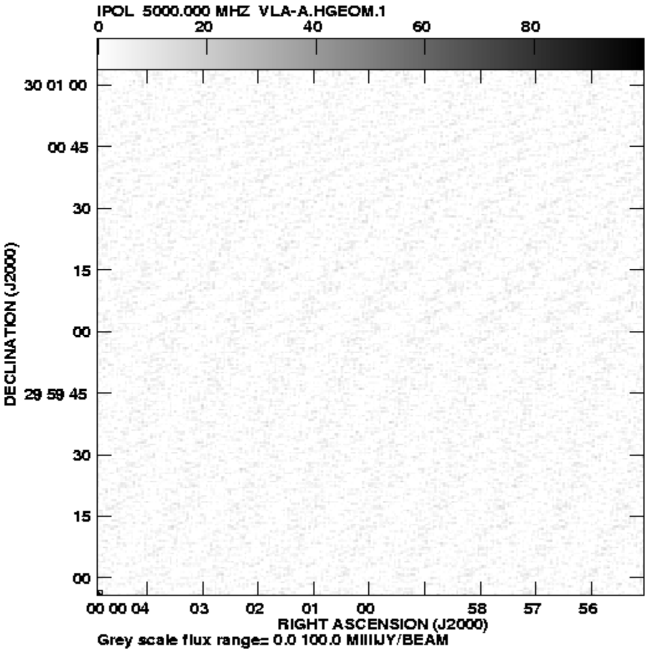
FT imaging is not like direct imaging!





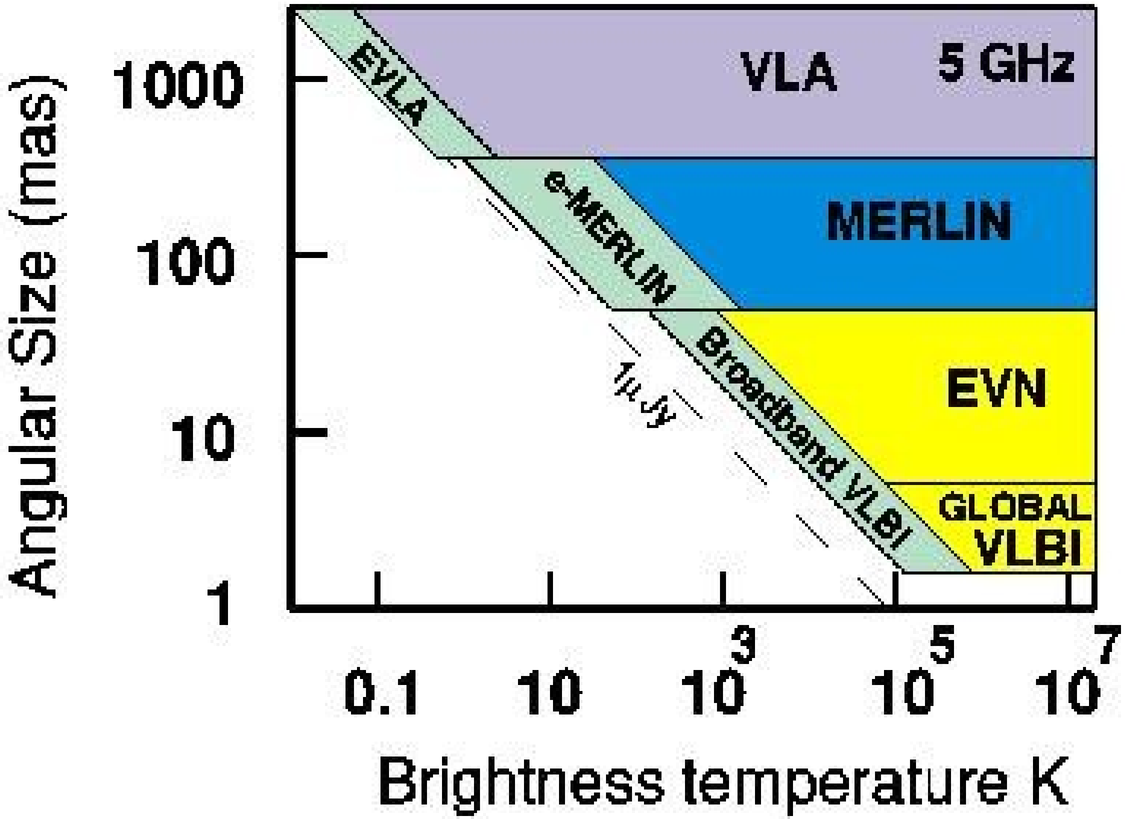
Multiply all baseline lengths by 10 -> higher resolution (0.3”).
No image! But you can get it back by smoothing, right?

FT imaging is not like direct imaging!



Wrong! Smoothed image to 3” shows no source.
Moral: longer baselines are INSENSITIVE to the large-scale structure – unlike direct imaging you lose it IRRETRIEVABLY. Use the range of baselines appropriate to the problem.

This is why you need >1 interferometer in the world…

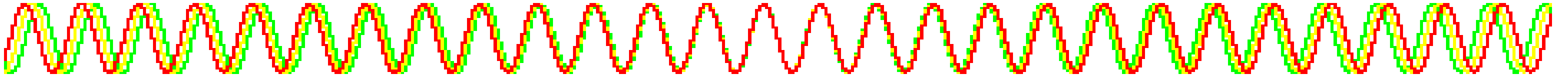


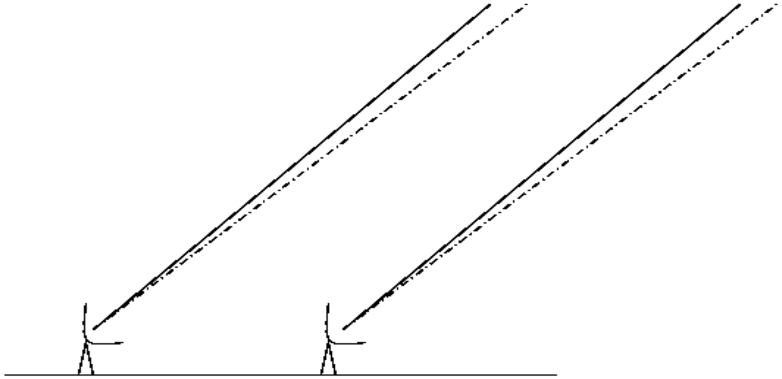
JVLA 30m-36km
e-MERLIN 6km-250km
EVN 250km-2300km
VLBA 250km-9000km
Global VLBI-12000km

Space VLBI-32000km

Limits on the field of view

1. Finite range of wavelengths





Fringe pattern OK at field centre
but different colours out of phase
at higher relative delay

Bigger range – smaller field of view (FT again)

FOV = (λ/Δλ) x (λ/L) i.e. λ/Δλ resolution elements

Limits on the field of view

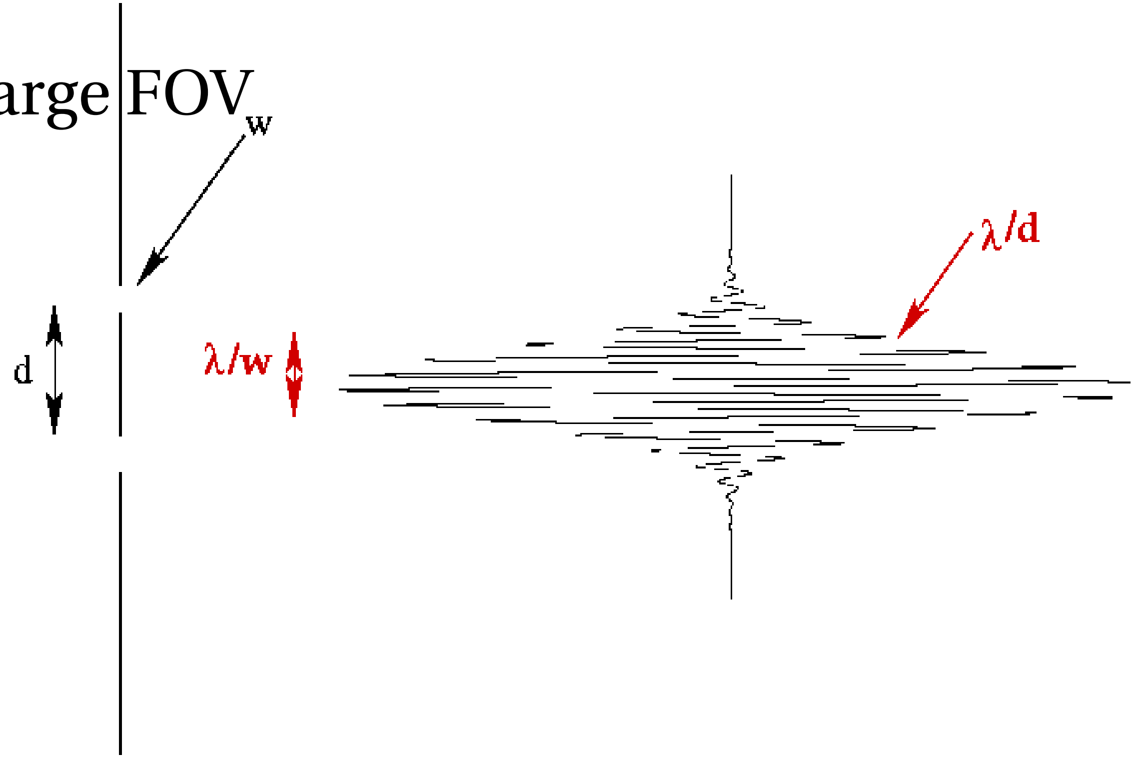
2. Too big integration time per data point
   (Limit: 13000/T times the resolution)

Rather technical, and only a problem for wide-field imaging

3. Non-flat sky over large FOV

4. Primary beam

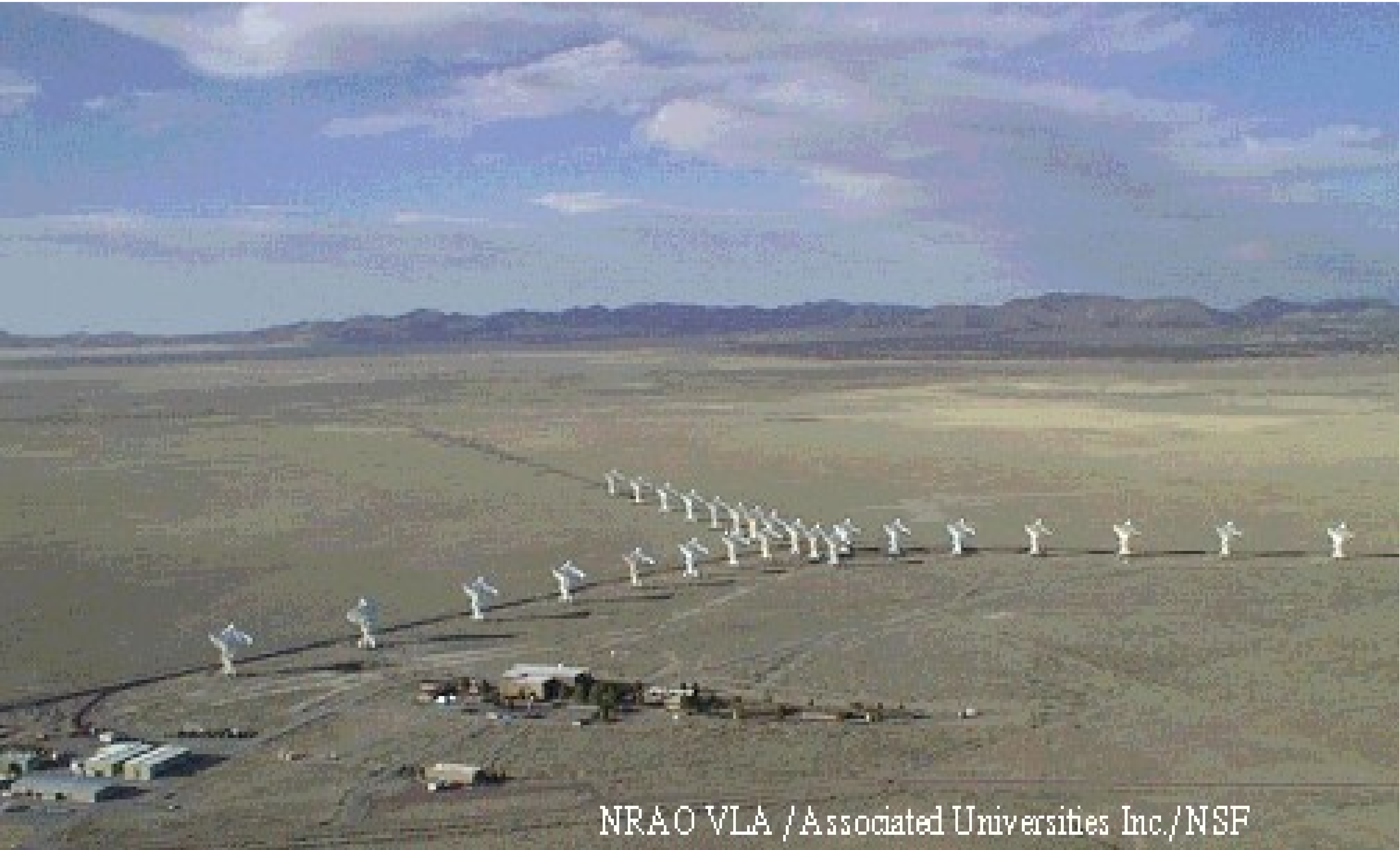
maximum field of view set by the telescope aperture



3. Some interferometers

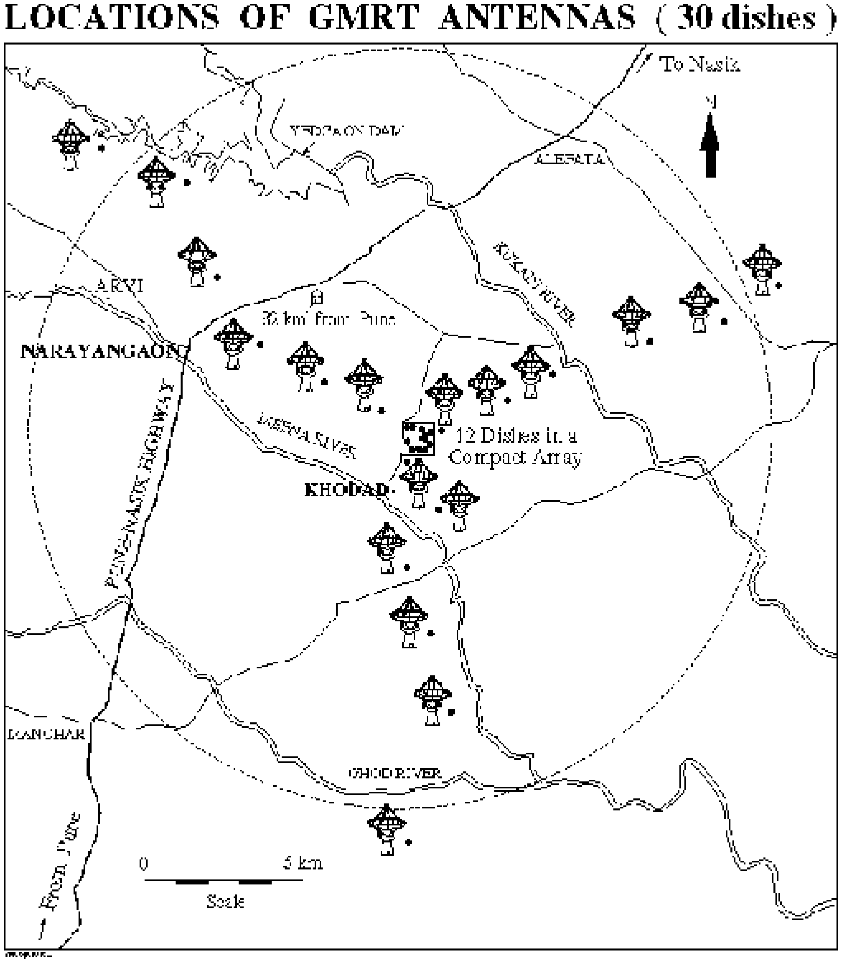
(mostly radio)

Very Large Array, NM, USA

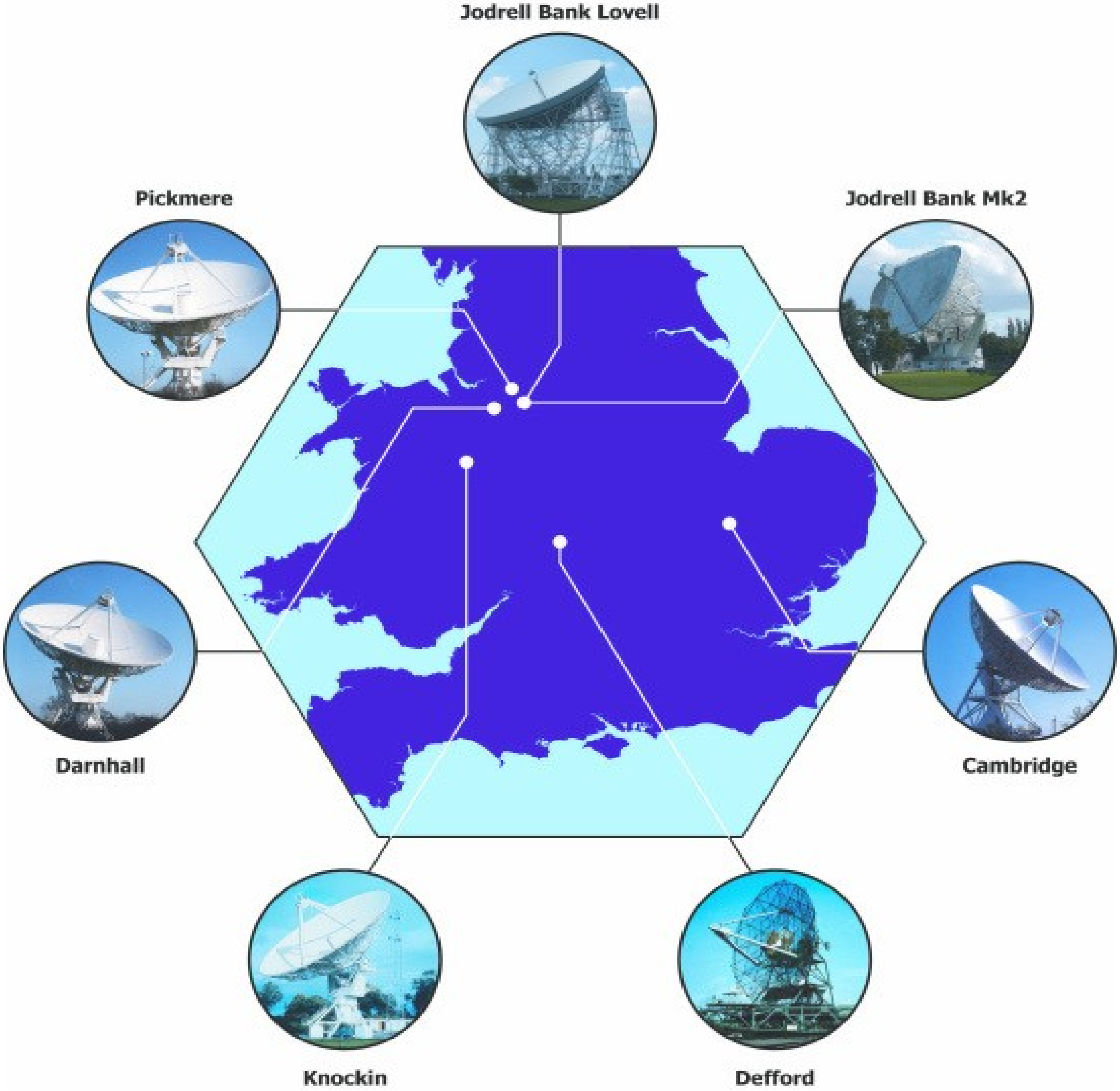


NRAO VLA / Associated Universities Inc./NSF

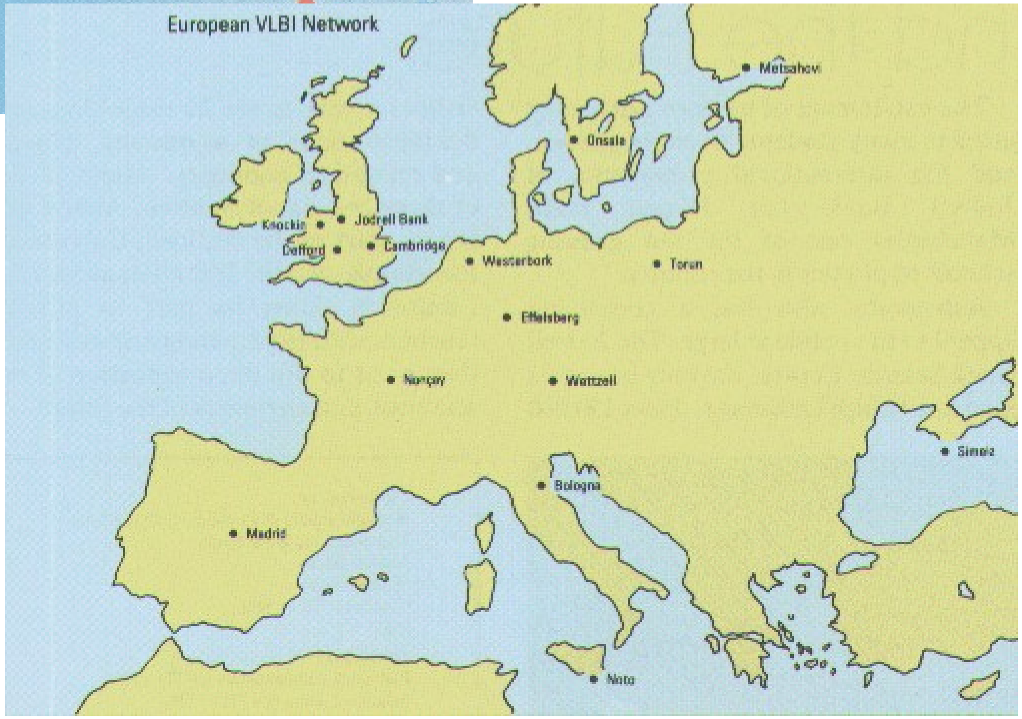
Giant Metrewave Radio Telescope, India



e-MERLIN. UK


Jodrell Bank Lovell
Pickmere
Jodrell Bank Mk2
Darnhall
Cambridge
Knockin
Defford

VLBI (Very Long Baseline Interferometry)

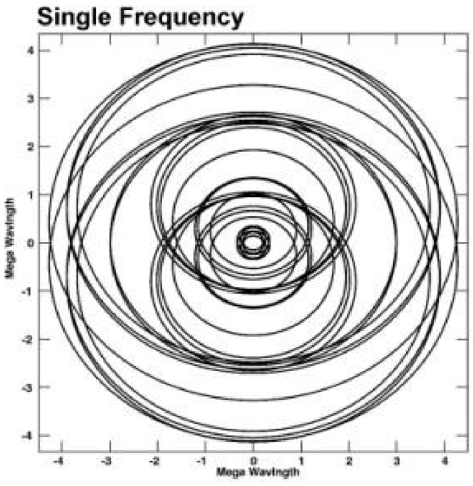


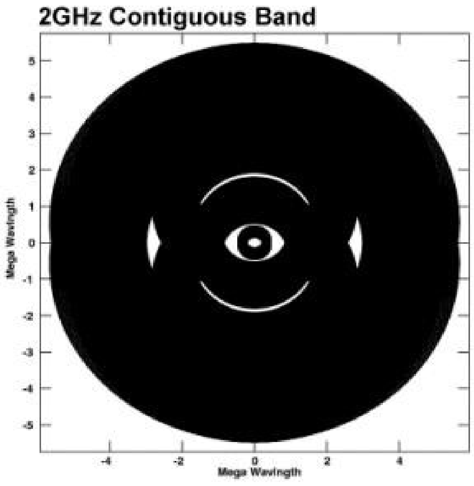
Limited only by Earth size
12000km baselines
-> mas resolution

EVLA and e-Merlin upgrades

2 effects:

1) optical fibres give higher sensitivity by $\Delta\nu^{1/2}$
2) because u-v plane coverage in wavelengths,
   images higher fidelity


Single Frequency


2GHz Contiguous Band

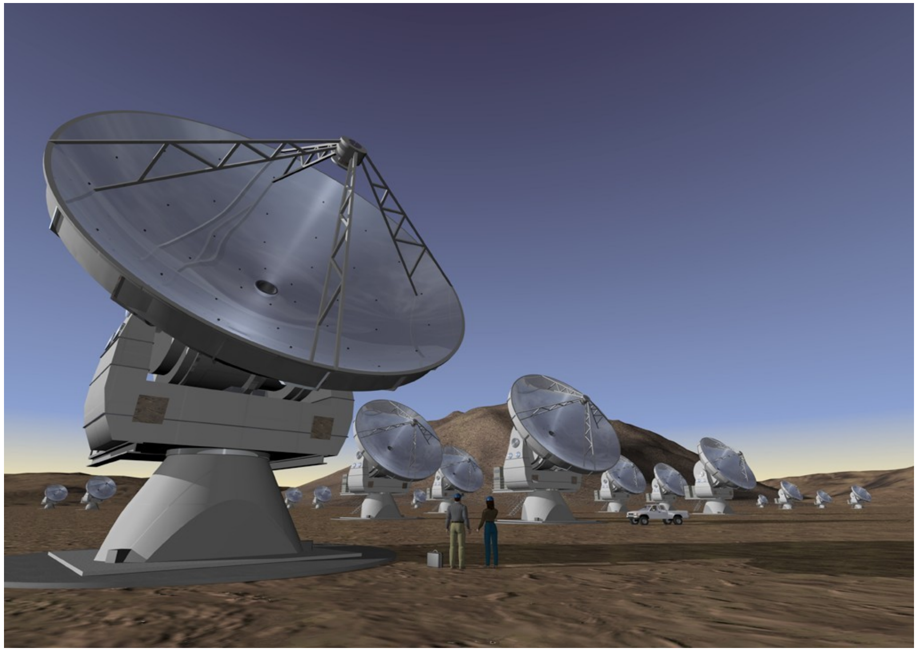
Low Frequency Array (NL)

- uses cheap low-frequency hardware
- huge information-processing problem
- resolution of a few arcseconds

www.lofar.org

Large number of antennas
gives very high sensitivity
up to 240MHz

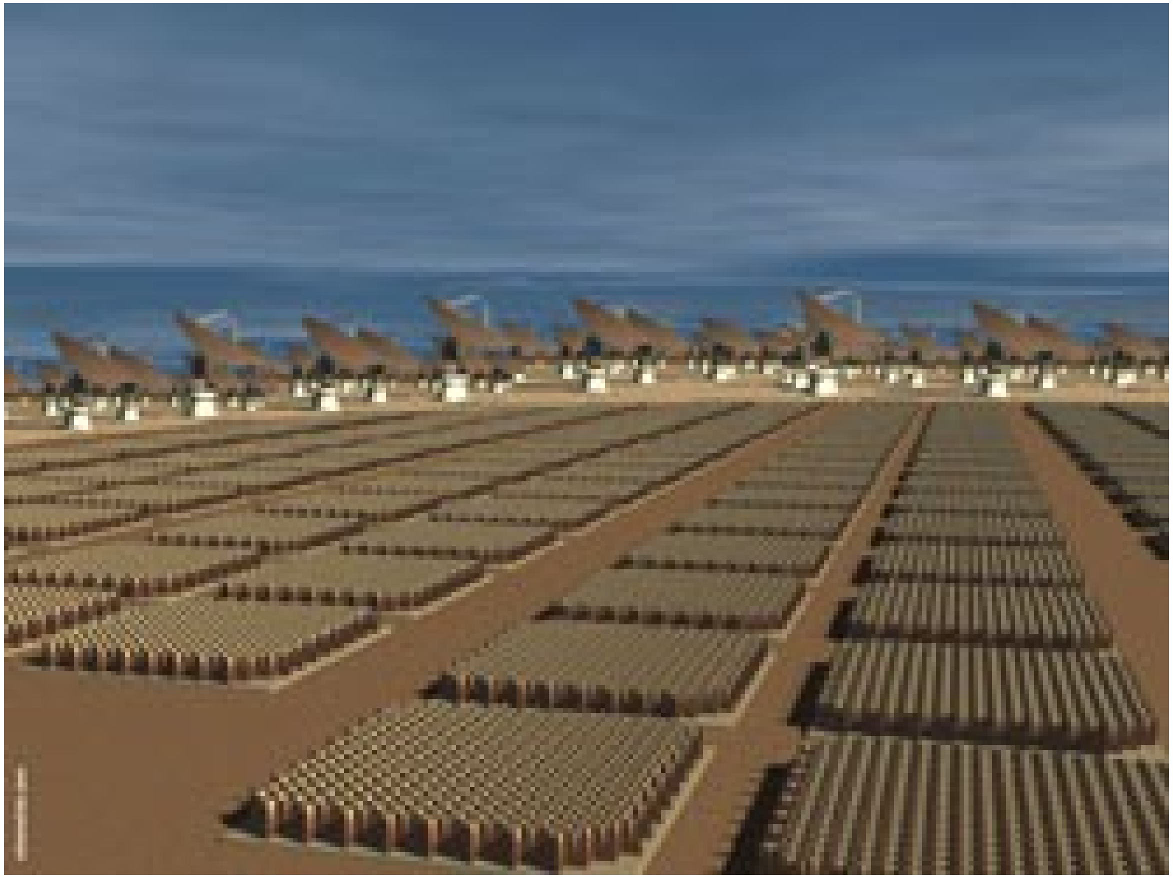
Atacama Large Millimetre Array



Chajnantor, Chile

- 30-950 GHz
- max. baseline ~20km
- molecules in galaxies at cosmological z
- gas in Galactic star-forming regions

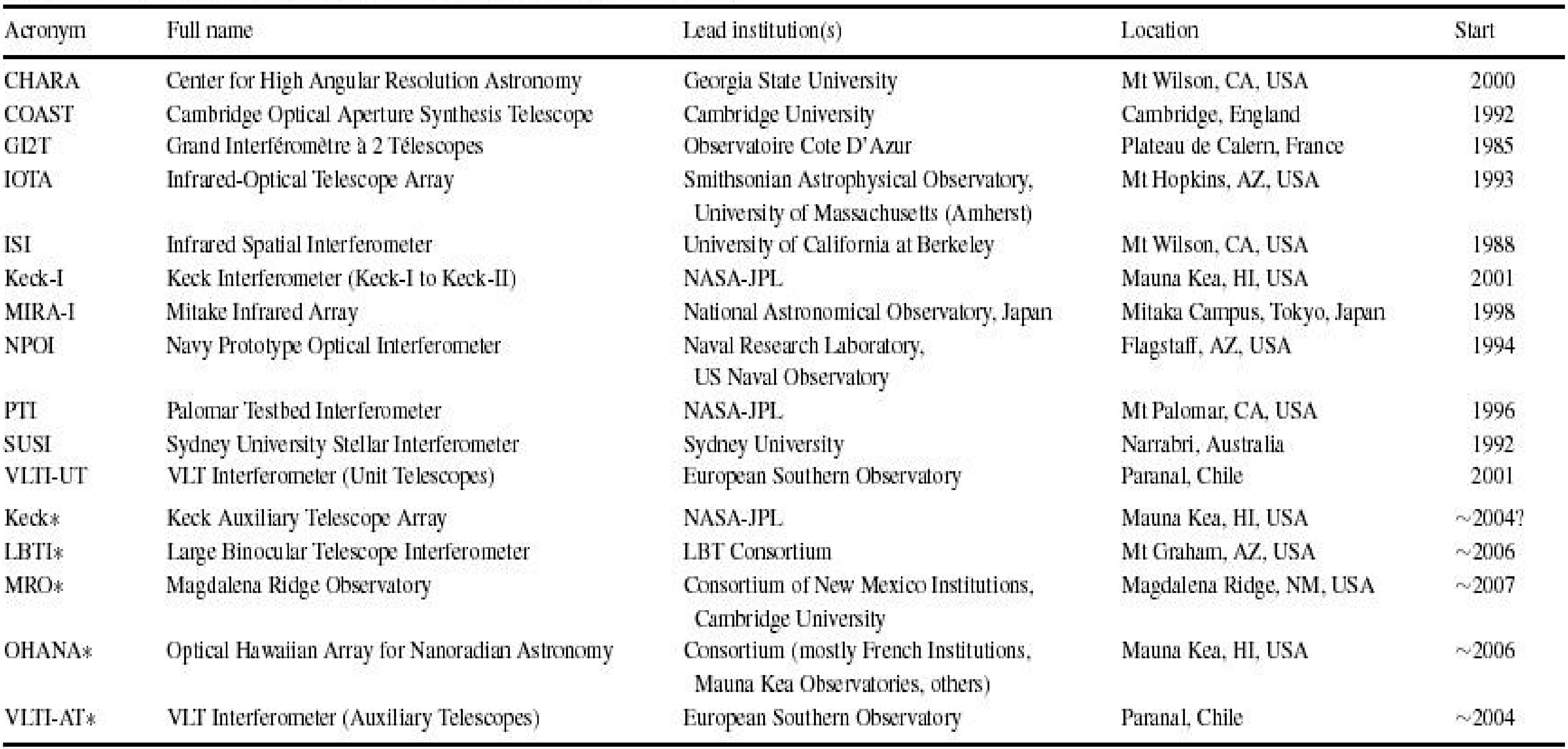
Square Kilometre Array (2017?)



* 1 sq.km collecting area
* HI at cosmological z
* Subarcsec resolution

Vast international project: Europe (UK,NL,IT,FR,ES,PO), US consortium,
Australia, Argentina, Canada, China, India, South Africa

Optical/IR interferometers



Acronym	Full name	Lead institution(s)	Location	Start
CHARA	Center for High Angular Resolution Astronomy	Georgia State University	Mt Wilson, CA, USA	2000
COAST	Cambridge Optical Aperture Synthesis Telescope	Cambridge University	Cambridge, England	1992
GI2T	Grand Interféromètre à 2 Télescopes	Observatoire Cote D'Azur	Plateau de Calern, France	1985
IOTA	Infrared-Optical Telescope Array	Smithsonian Astrophysical Observatory, University of Massachusetts (Amherst)	Mt Hopkins, AZ, USA	1993
ISI	Infrared Spatial Interferometer	University of California at Berkeley	Mt Wilson, CA, USA	1988
Keck-I	Keck Interferometer (Keck-I to Keck-II)	NASA-JPL	Mauna Kea, HI, USA	2001
MIRA-I	Mitake Infrared Array	National Astronomical Observatory, Japan	Mitaka Campus, Tokyo, Japan	1998
NPOI	Navy Prototype Optical Interferometer	Naval Research Laboratory, US Naval Observatory	Flagstaff, AZ, USA	1994
PTI	Palomar Testbed Interferometer	NASA-JPL	Mt Palomar, CA, USA	1996
SUSI	Sydney University Stellar Interferometer	Sydney University	Narrabri, Australia	1992
VLTI-UT	VLT Interferometer (Unit Telescopes)	European Southern Observatory	Paranal, Chile	2001
Keck*	Keck Auxiliary Telescope Array	NASA-JPL	Mauna Kea, HI, USA	~2004?
LBTI*	Large Binocular Telescope Interferometer	LBT Consortium	Mt Graham, AZ, USA	~2006
MRO*	Magdalena Ridge Observatory	Consortium of New Mexico Institutions, Cambridge University	Magdalena Ridge, NM, USA	~2007
OHANA*	Optical Hawaiian Array for Nanoradian Astronomy	Consortium (mostly French Institutions, Mauna Kea Observatories, others)	Mauna Kea, HI, USA	~2006
VLTI-AT*	VLT Interferometer (Auxiliary Telescopes)	European Southern Observatory	Paranal, Chile	~2004

from Monnier 2003

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Imaging

or, How difficult can it be to do a Fourier transform?

Neal Jackson
(with thanks to Tom Muxlow for many images)

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1. Deconvolution basics:
   CLEAN, MEM, multiscale cleaning, details
2. Problems associated with wide fields
3. Problems associated with high dynamic range

Deconvolution

Recall the basic operation of an interferometer baseline:

$$R(x, y) = \int\int I(u, v) e^{2\pi i(ux+vy)} du dv$$

In principle just measure I(u,v) for all u,v.....
But instead we have the "dirty image"

$$R_D(x, y) = \int\int I(u, v) S(u, v) e^{2\pi i(ux+vy)} du dv$$

where the sampling function S is 1 in the parts of the uv plane we've sampled and 0 where we haven't.

Deconvolution (ctd)

We can use the convolution theorem to write

$$R_D(x, y) = R(x, y) * B$$

where B is known as the dirty beam

$$B(x, y) = \int\int S(u, v) e^{2\pi i(ux+vy)} du dv$$

and is the FT of the sampling function.

Problem is then one of deconvolution.

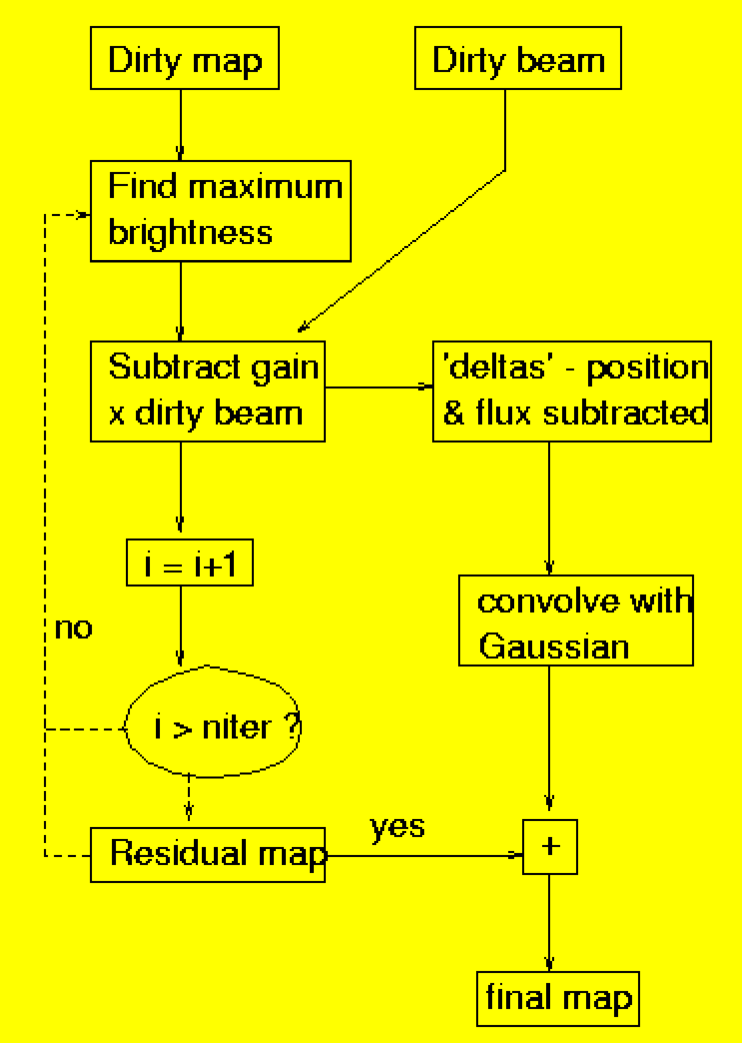
Important comment

Infinite number of images are consistent with the data
(including the dirty map itself)

Extra information/assumptions must be supplied

Simplest (but not only) scheme: sky is mostly empty, and consists of a finite number of point sources

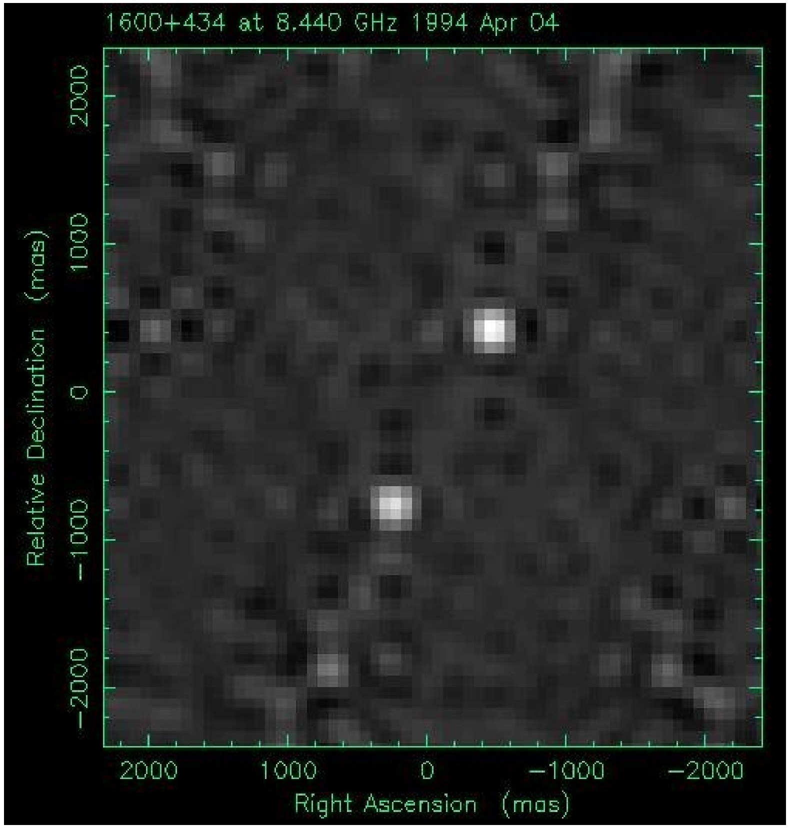
Hogbom CLEAN deconvolution



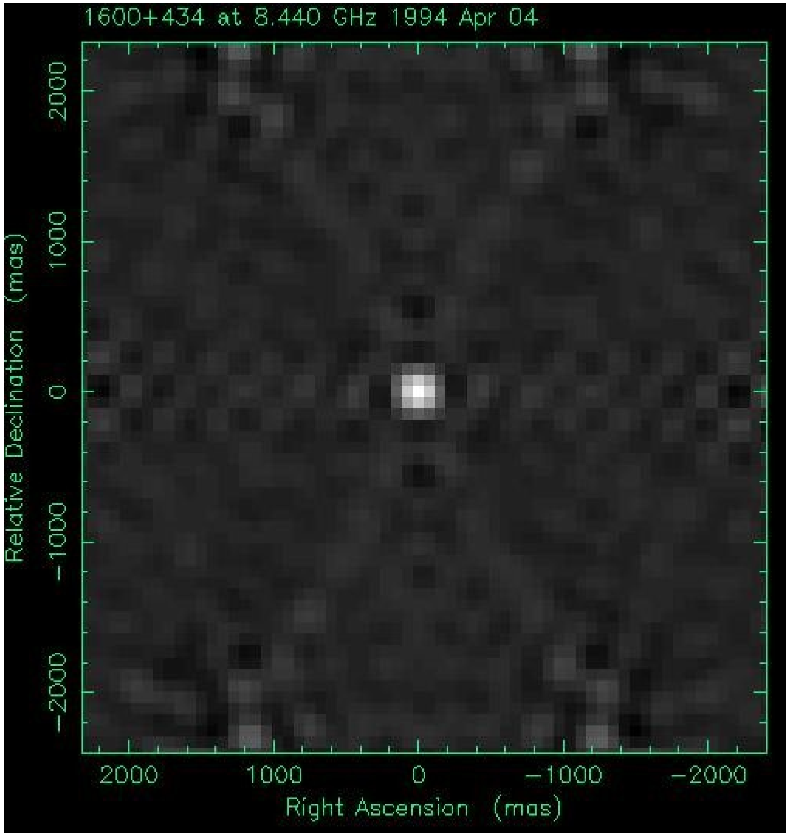
Brute-force iterative deconvolution using the dirty beam

Effectively reconstructs information in unsampled parts of the u-v plane by assuming sky is sum of point sources

(Quasi-)Hogbom CLEAN in action

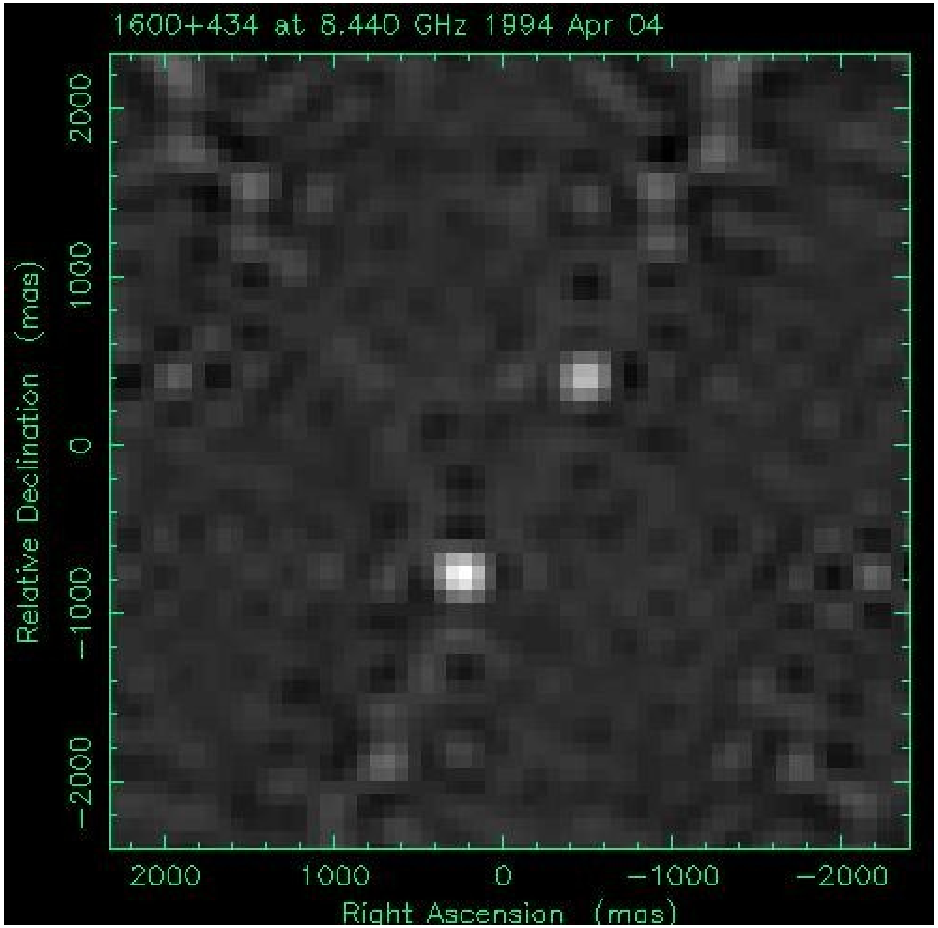


Dirty map



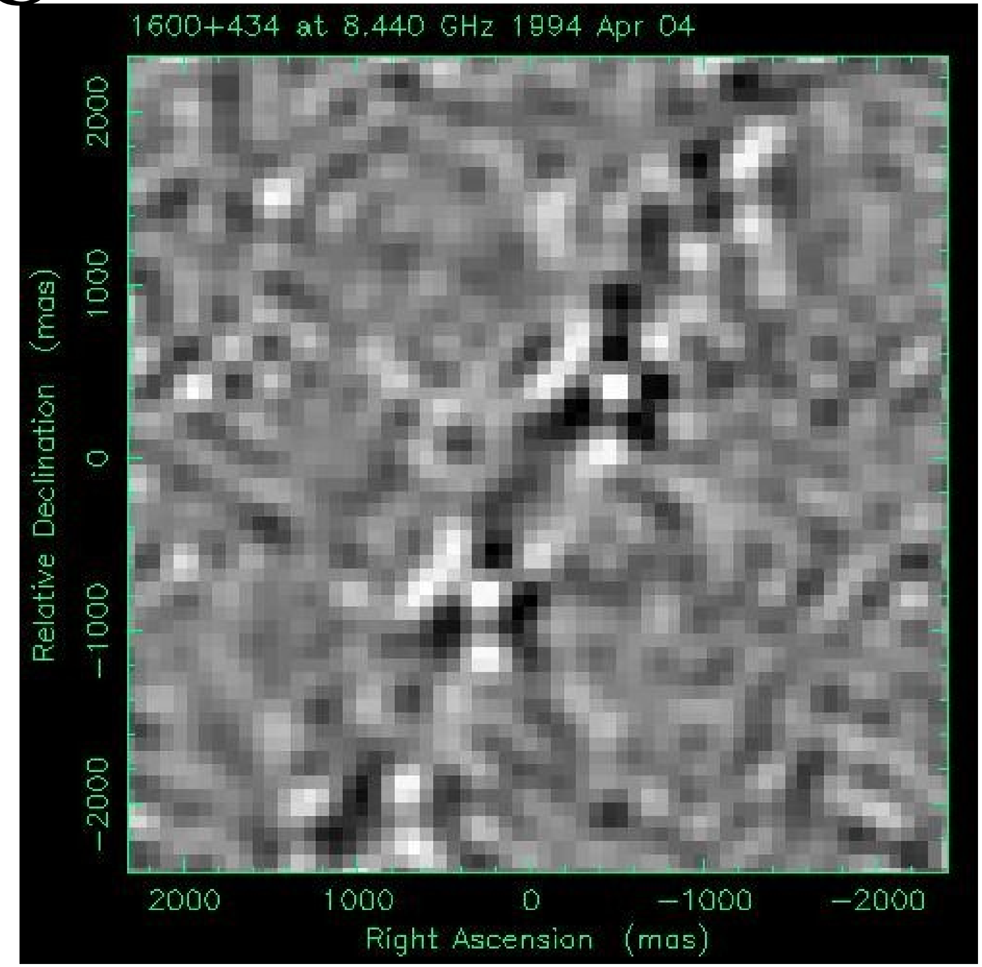
Dirty beam

Hogbom CLEAN in action



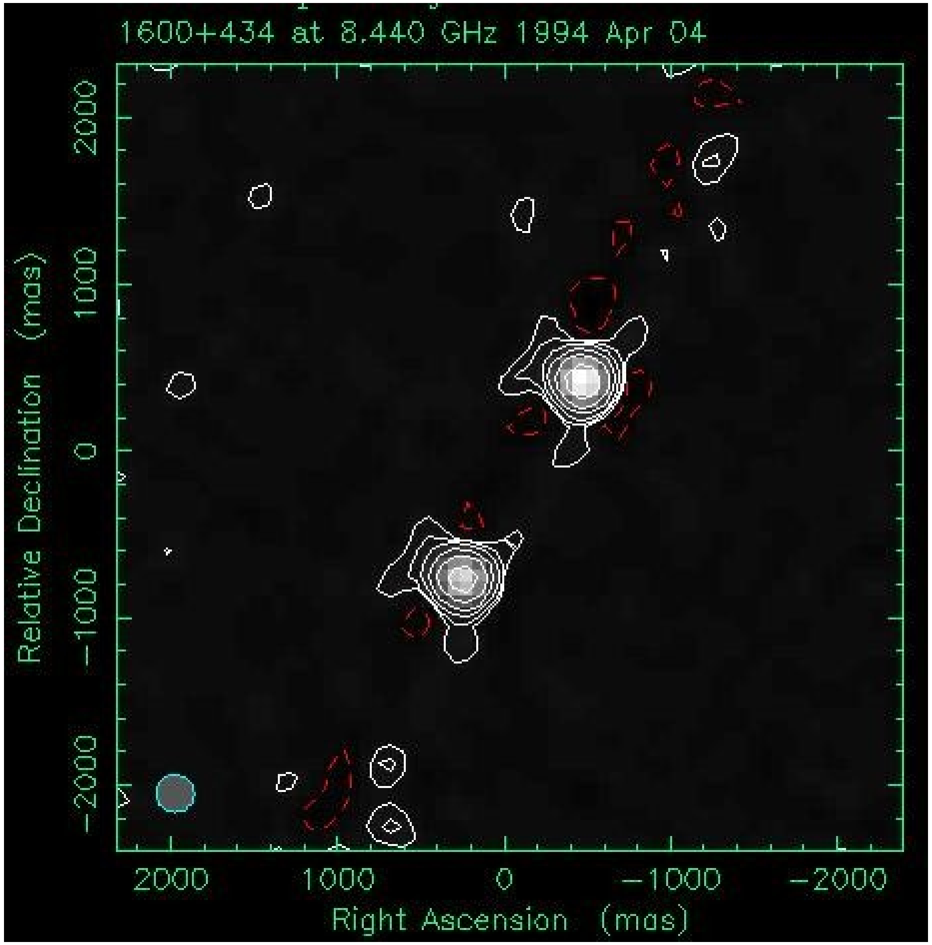
Residual after 1 CLEAN (gain 0.5)

Hogbom CLEAN in action



Residual after 100 CLEANs (gain 0.1)

Hogbom CLEAN in action



CLEAN map (residual+CCs) after 100 CLEANs (gain 0.1)
Remaining artefacts due to phase corruption (see later lectures)

CLEAN in practice (i) details of algorithm:

“minor cycles” - subtract subimages of dirty beam

“ major cycle” - FT residual map and subtract

CLEAN in practice: (ii) how to help the algorithm

"minor cycles" - subtract subimages of dirty beam

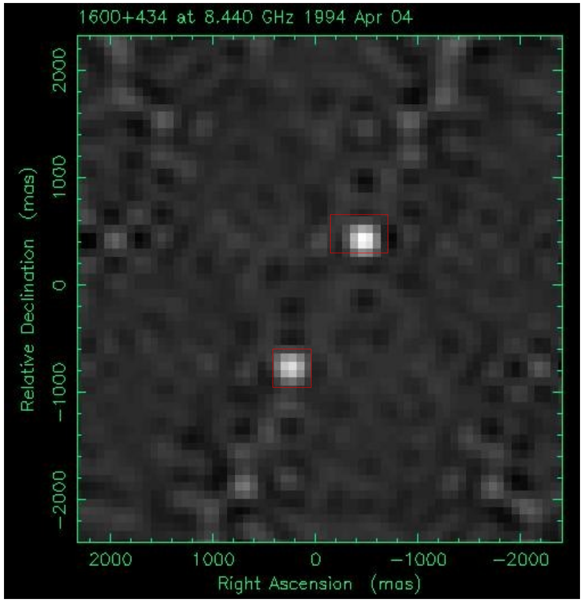
" major cycle" - FT residual map and subtract

CLEAN can be helped by "windows" (areas in which you tell the algorithm flux lies and within which it is allowed to subtract)

CLEAN in practice: (ii) how to help the algorithm

“minor cycles” - subtract subimages of dirty beam

“ major cycle” - FT residual map and subtract



CLEAN can be helped by “windows” (areas in which you tell the algorithm flux lies and within which it is allowed to subtract)

CLEAN in practice: (iii) multifrequency synthesis

- large fractional bandwidth
- require a spectral solution at each point of the image
- known as MFS (multi-frequency synthesis)

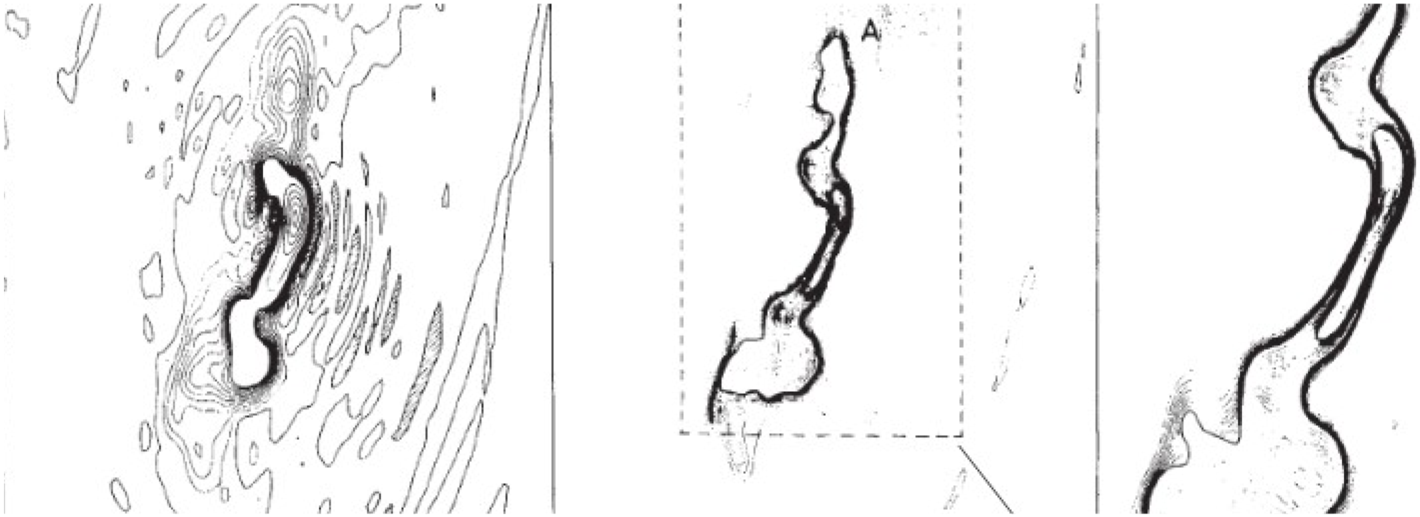
Maximum entropy method

All deconvolution methods supply missing information.
CLEAN does this by saying that sky is Σpoint sources.

MEM demands that the smoothest map consistent with the data is the most likely

using an "entropy" estimator e.g. $\lambda p_i \ln p_i$

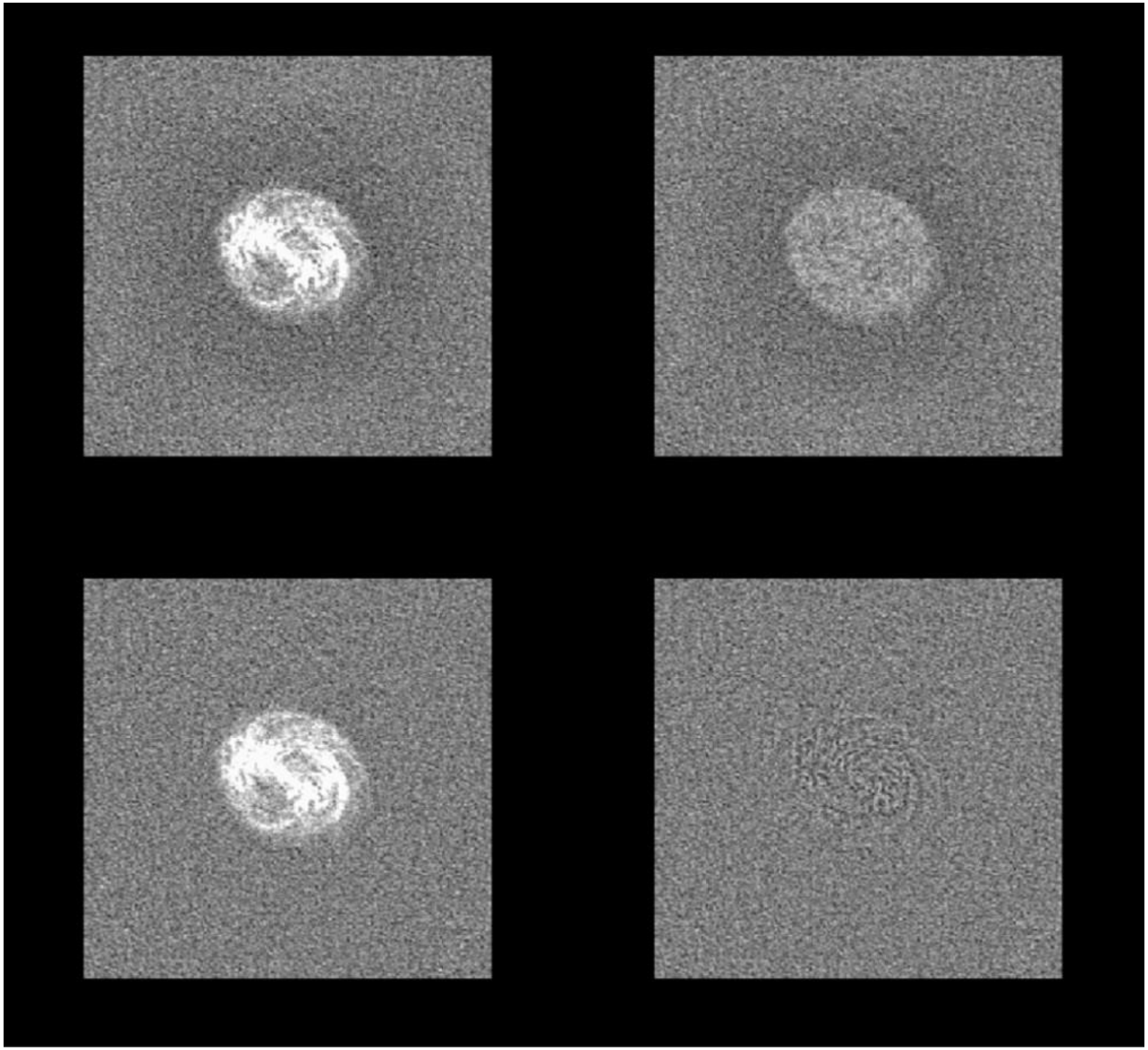
Maximum entropy method



Gull & Daniell 1978

Multi-scale clean – better images of complex data

smooth RM/beam; subtract from scale with maximum residual at each iteration



Conventional

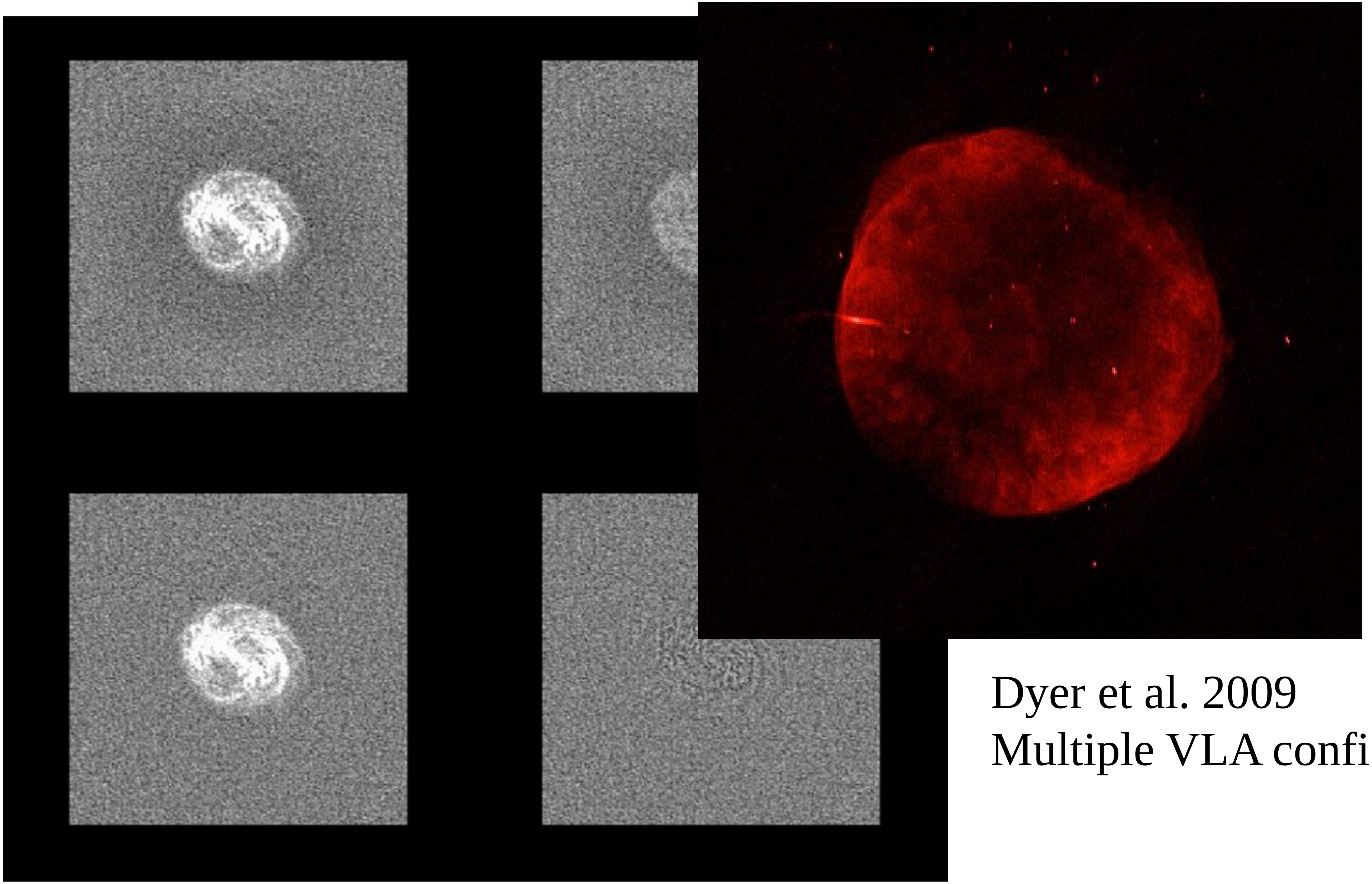
Multi-scale

Map

Residual

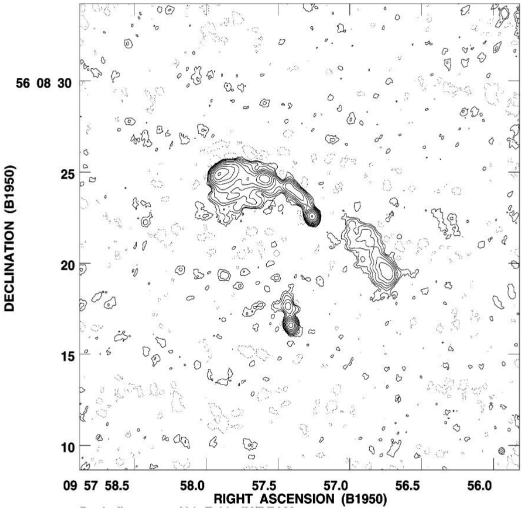
Multi-scale clean – better images of complex data

smooth RM/beam; subtract from scale with maximum residual at each iteration

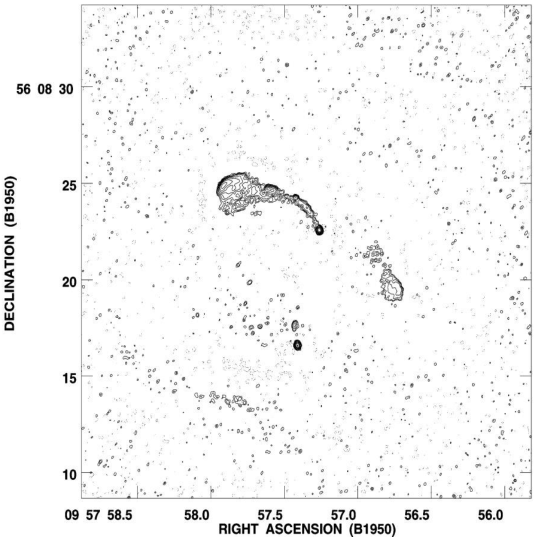


Dyer et al. 2009
Multiple VLA configs

Data weighting in the u-v plane



natural



uniform

Generally more u-v tracks on inner part

Can choose to

* weight all data equally (natural)
  - gives best S:N, less good beam

* weight all u-v grid points equally (uniform)
  - gives good resolution, less S:N

* Compromises possible
  - Briggs “robust” parameter -5 → 5

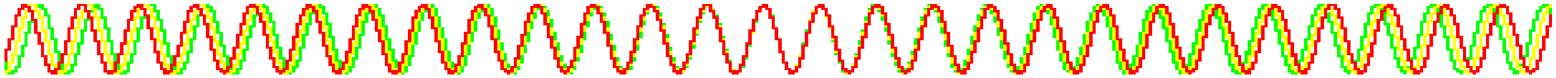
Data weighting by telescope

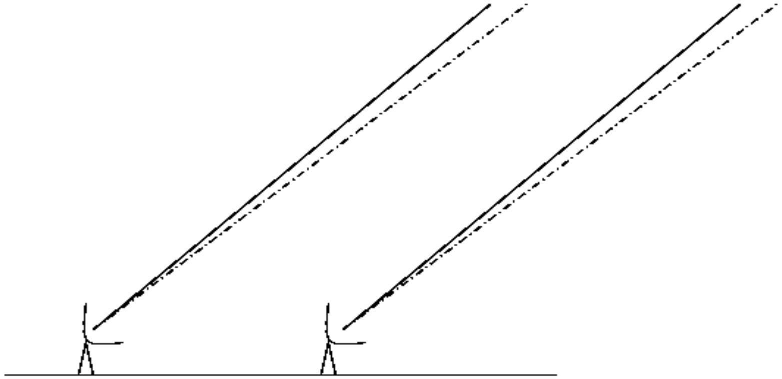
Many arrays have unequal size telescopes – international LOFAR, eMERLIN, VLBI

For best S:N, adjust weights so larger telescopes contribute more

Wide-field imaging: limits due to bandwidth smearing

Finite range of wavelengths





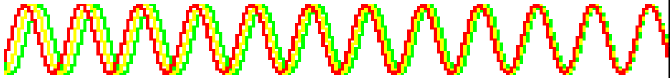
Fringe pattern OK at field centre
but different colours out of phase
at higher relative delay

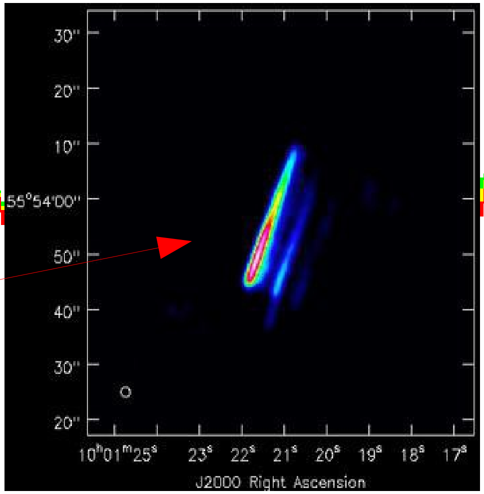
Bigger range – smaller field of view (FT again)

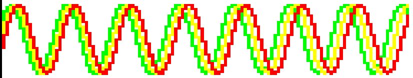
FOV = $(\lambda/\Delta\lambda) \times (\lambda/L)$ i.e. $\lambda/\Delta\lambda$ resolution elements

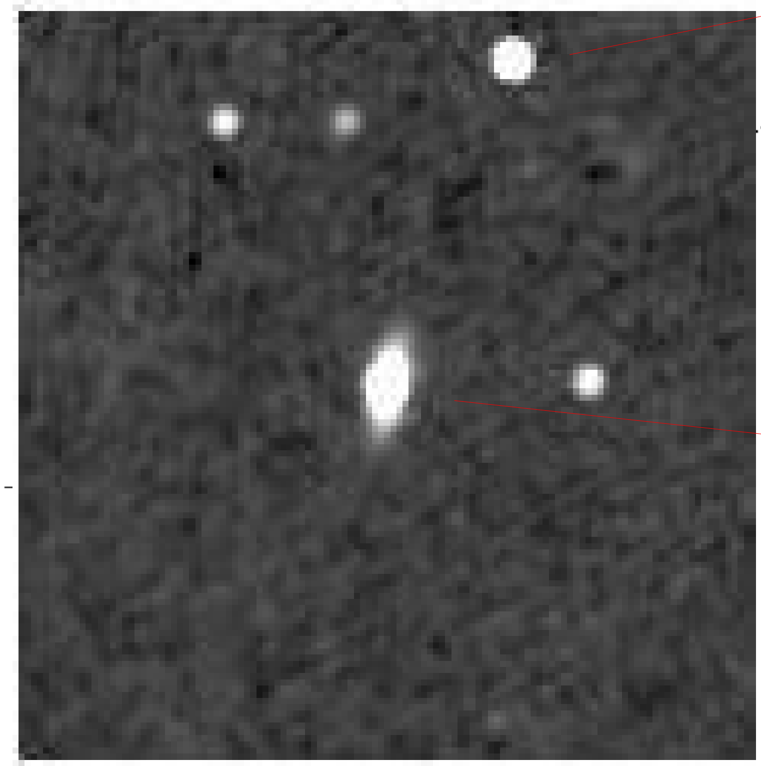
Wide-field imaging: limits due to bandwidth smearing

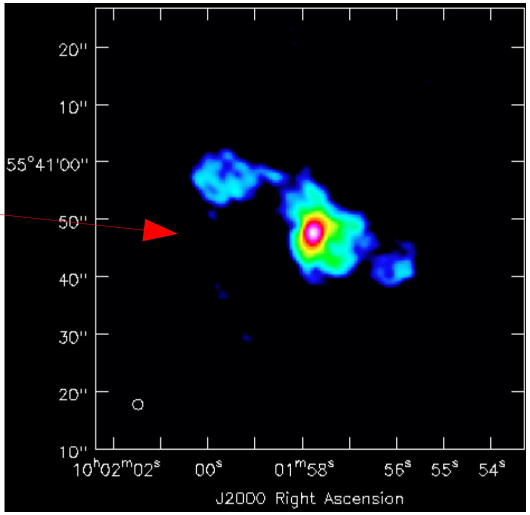
Finite range of wavelengths











Effect is a radial smearing, corresponding to radial extent of measurements in u-v plane

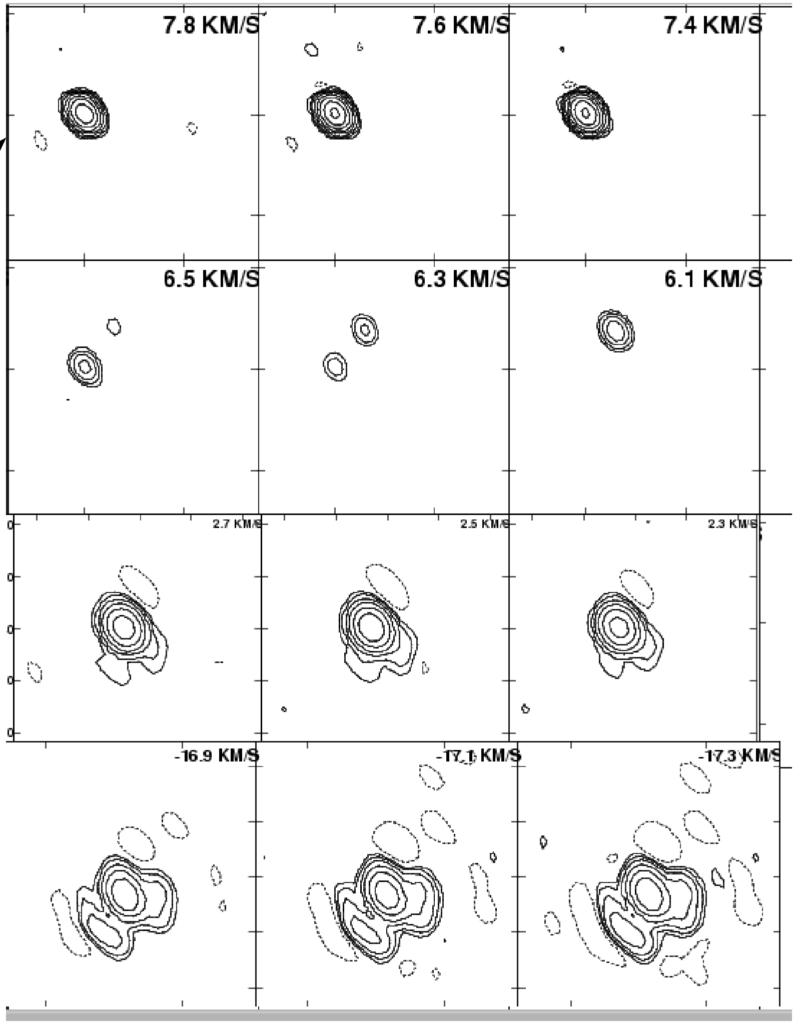
Wide-field imaging: limits due to time-smearing

Tangential smearing, corresponding to smearing over u-v tracks

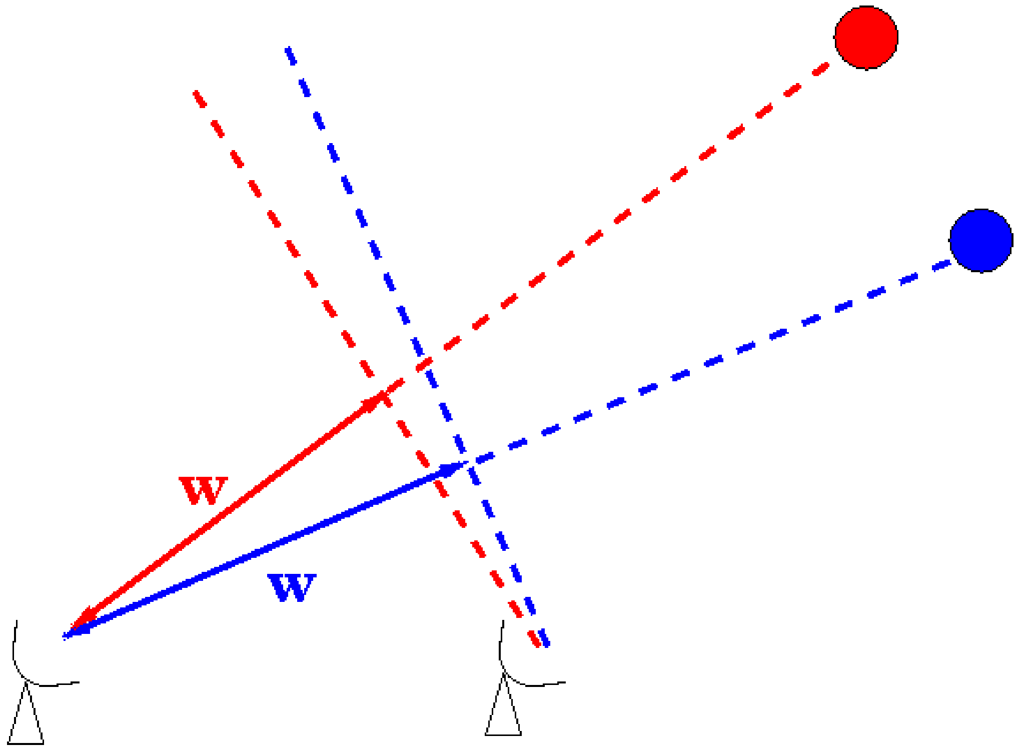
(limit is ~14000/T times the resolution)

Unsmeared

Smeared

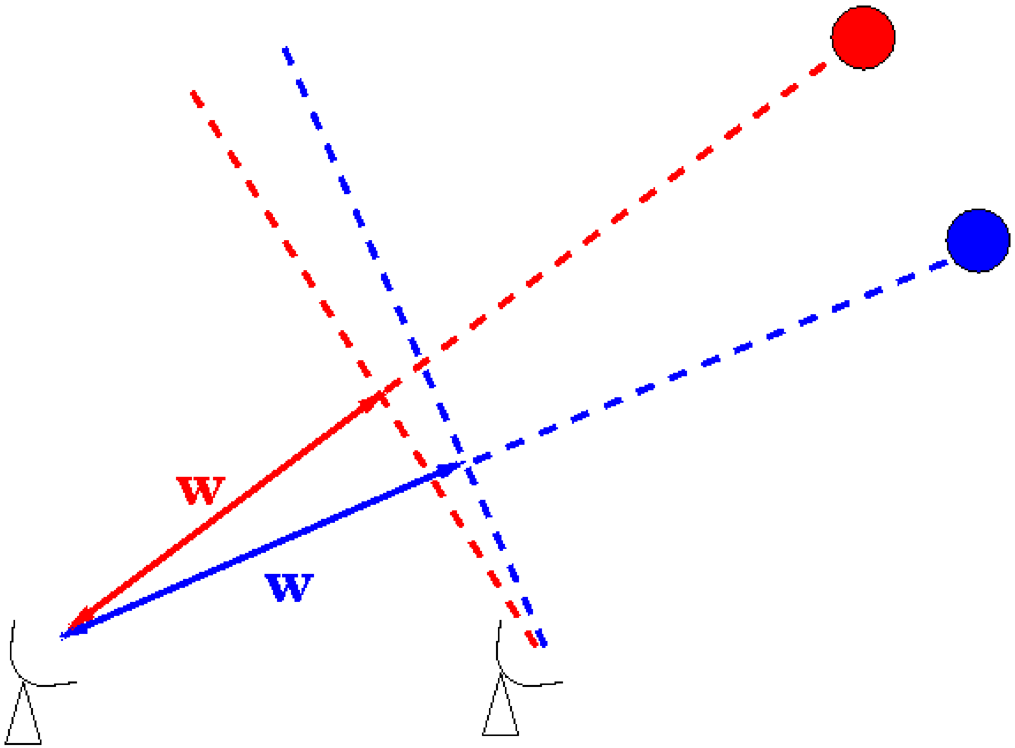


Wide-field imaging: limits due to non-coplanar baselines ("w-term")



- Component of baseline in source direction
- Not a problem for small fields of view; take out a phase term and it goes away across the whole field
- Quadratically worse with distance from centre of field

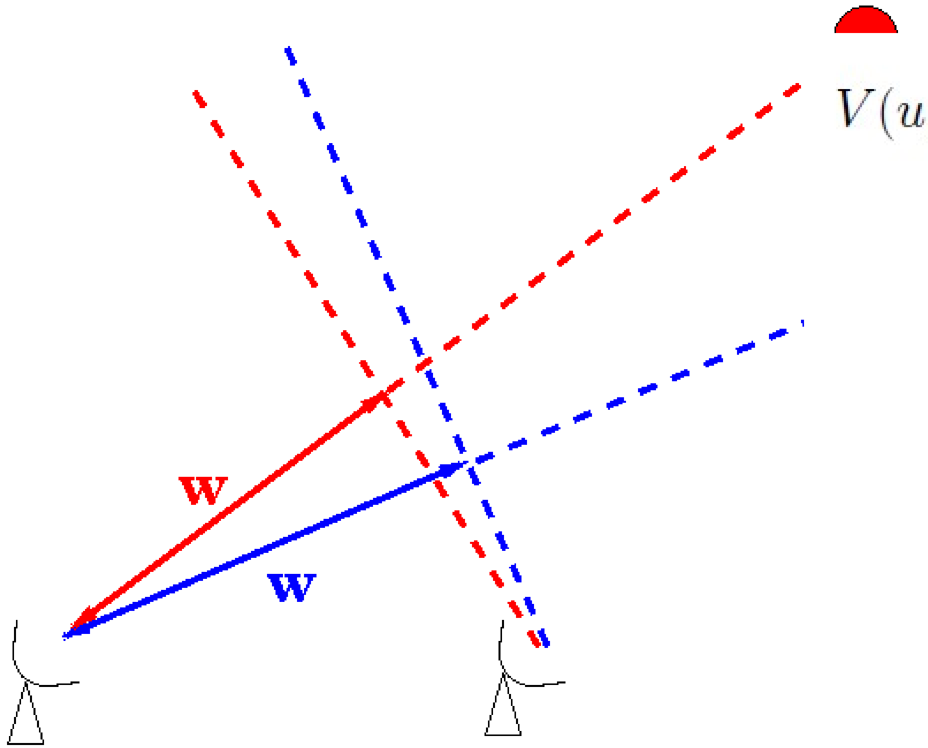
Wide-field imaging: limits due to non-coplanar baselines (“w-term”)



- Component of baseline in source direction
- Not a problem for small fields of view; take out a phase term and it goes away across the whole field
- Quadratically worse with distance from centre of field

Solutions:
- image in “facets” - small regions of sky, then stitch together
- “W-projection”

Wide-field imaging: limits due to non-coplanar baselines (“w-term”)



$$V(u,v,w) = \int\int I(x,y) e^{-2\pi i(ux+vy+\sqrt{1-x^2+y^2})} dxdy$$

$$= \int\int I(x,y) W(x,y) e^{-2\pi i(ux+vy)} dxdy$$

$$V(u,v,w) = V(u,v,0) * \tilde{W}(u,v)$$

Solutions:

- “W-projection”
- project back into V(u,v,0) for all points together with particular (u,v)

Wide-field imaging limits: the primary beam

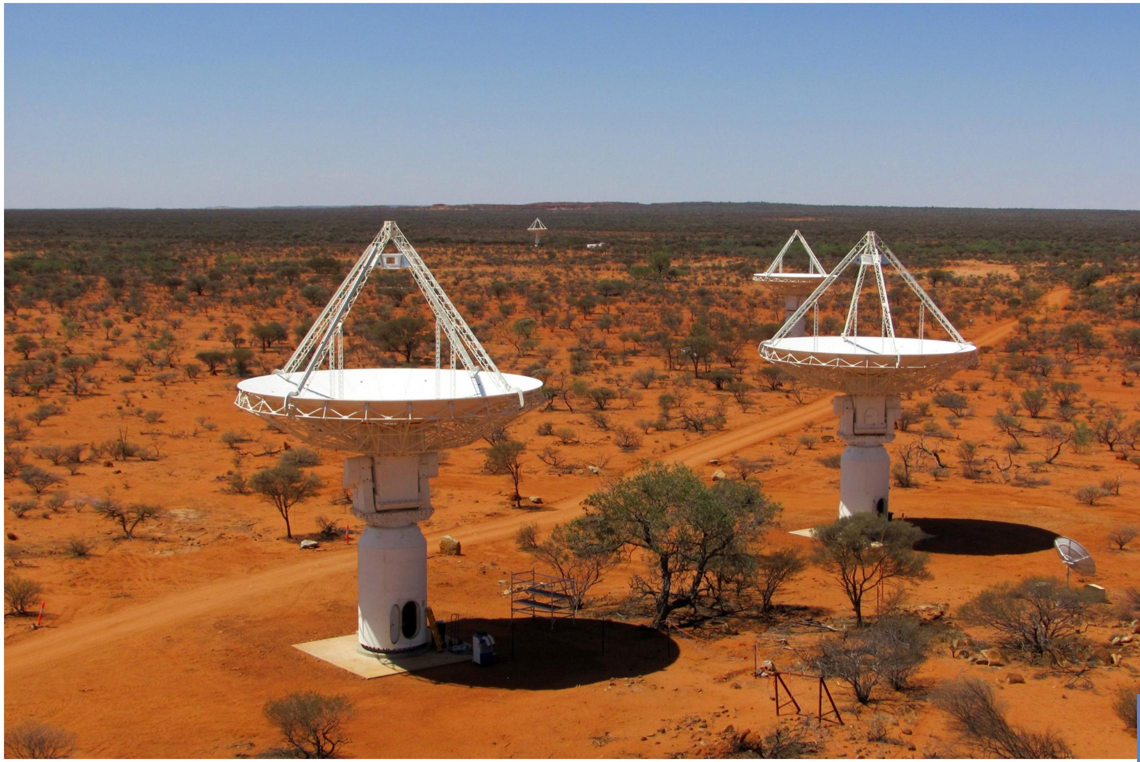
Similar principle to 2 wide slits in Young's slits experiment:

Pattern of 2 WS = Pattern of 2 NS * Pattern of 1 WS

Convolution with sinc function → visibilities drop at large angles to field centre

This happens at λ/w, where w = width of individual telescopes

Wide-field imaging limits: the primary beam



its experiment:

f 1 WS

drop at large angles to field centre

lividual telescopes

Phased-array feeds (e.g. ASKAP) – allow telescope beams to point in multiple directions – here 30 at once

(Also retrofit of WSRT with PAFs: “APERTIF”)

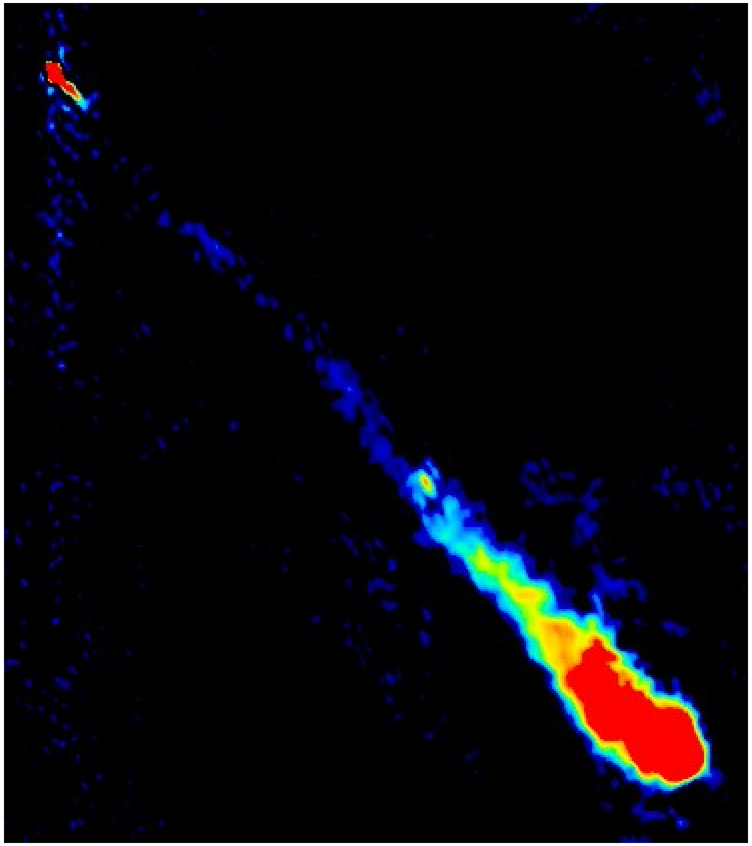
Achieving high dynamic range

- Usually two major problems prevent achievement of very high dynamic range

- Lack of u-v coverage

- Closing errors (factorisable by telescope) removable using self-cal (next lecture)

- Non-closing errors (baseline-based)
  * mismatched bandpasses in correlator
  * calibrate on very bright source



3C273, Davis et al. (MERLIN)
1,000,000:1 peak – RMS

See also de Bruyn & Brentjens work with WSRT

Signal-to-noise

Radio interferometer noise level =

$$\frac{\sqrt{2}k_B T_{\rm sys}}{\sqrt{n_b T \Delta\nu} A\eta}$$

Tsys = system temperature, nb = number of baselines, T=integration time, Δν=bandwidth in Hz, A=area of apertures, η=aperture efficiency

NB linearly as 1/A not $(1/A)^{1/2}$

In practice you rarely get to this!

Optical: need photons in coherence vol $r_0^2 c t_0$

taking throughput into account

END

5. Dealing with the atmosphere

What the atmosphere does

Corrupts phase and amplitude of incoming signals

Corruption is different for different telescopes/apertures

Corruption changes with time (scales ms to mins)

Corruption varies with position (<size of tel. for optical)

Sources: water vapour..., ionosphere (LF radio)

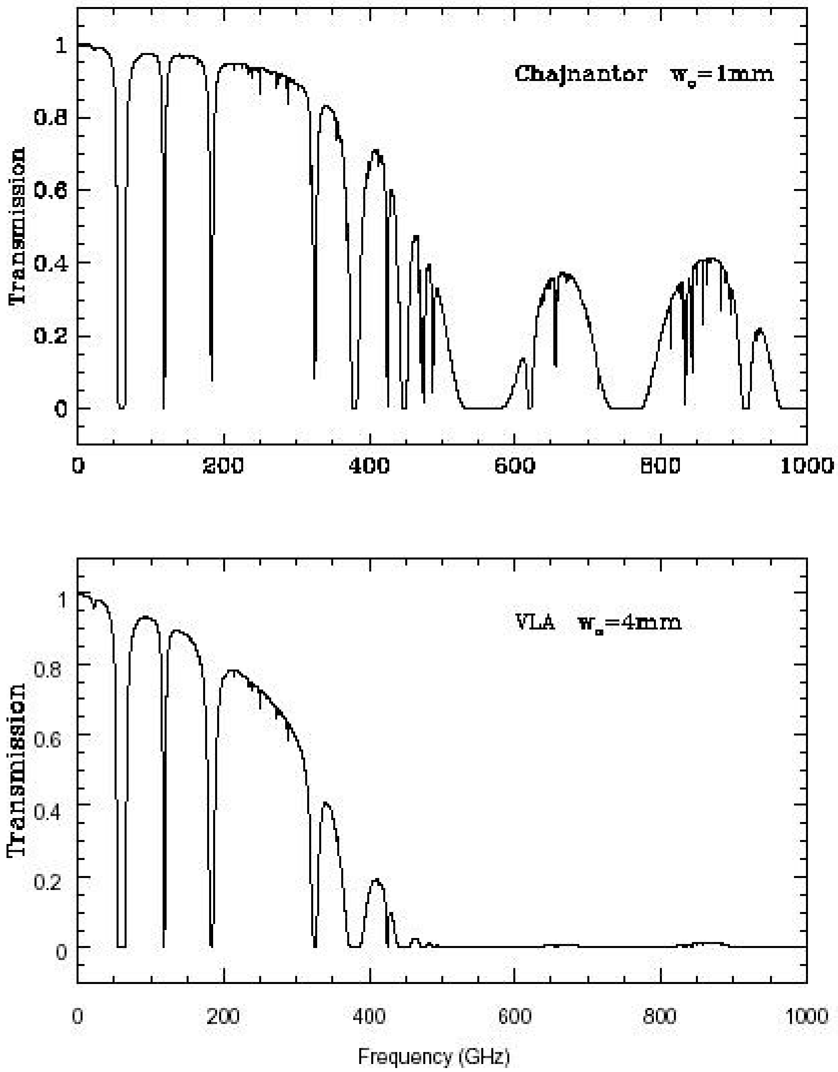
How bad is the problem? I. Phase

Waveband		Problem	Phase variation timescale
Radio	<300MHz	ionosphere	seconds-minutes
	few GHz	water &c	minutes
	>20GHz	water	sec (site dependent)
mm		water	highly site dependent
near-IR		atm cells	~100 milliseconds
optical			1-10 milliseconds

Optical: Fried parameter r_0 – length scale of fluctuations
Timescale = r_0/wind velocity

The shorter the wavelength, the more rapid the phase fluctuation and the harder the problem becomes.

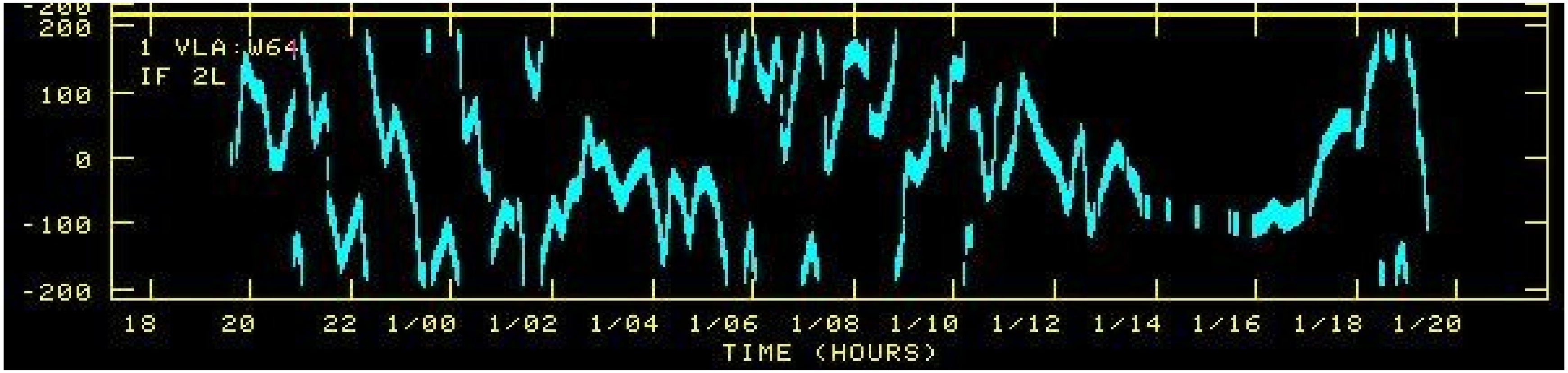
How bad is the problem? II. Amplitude



Carilli & Holdaway 1999

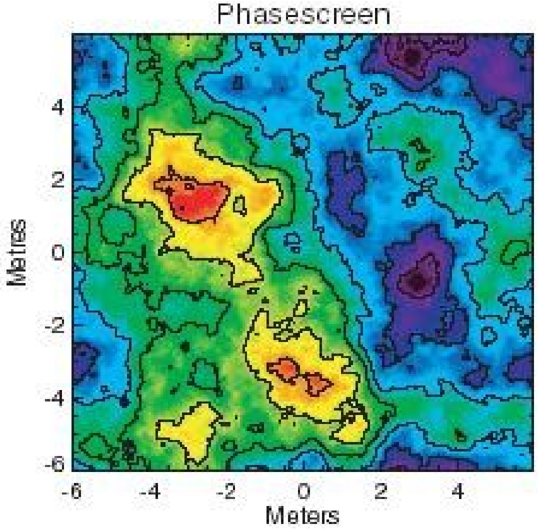
Mm wavelengths: amplitude drops precipitously with atmospheric water vapour column

Typical examples

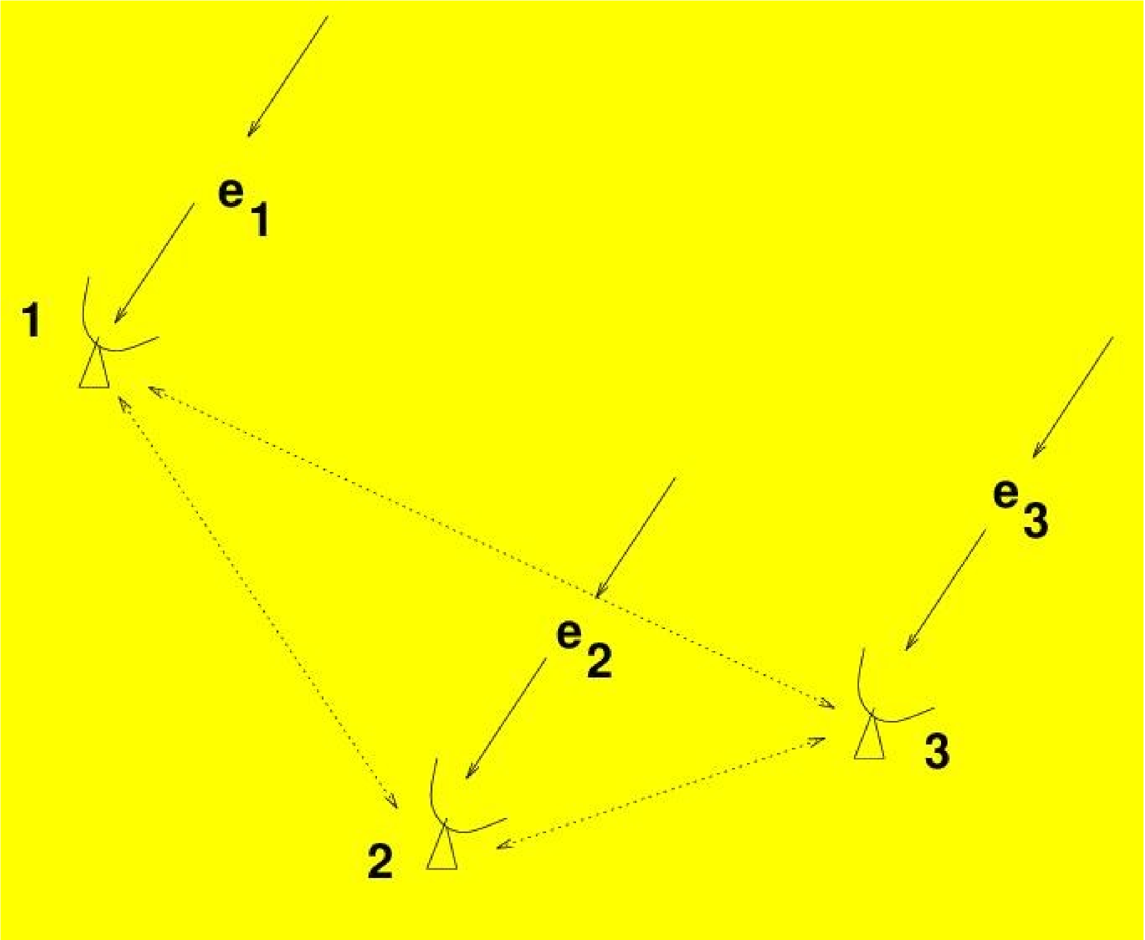


VLA, 8.4GHz

Realization of phase-screen with r_o=50cm (Monnier 2003)



One approach: closure phase



Phase error on baseline a-b is e_a-e_b ($\neq 0$)

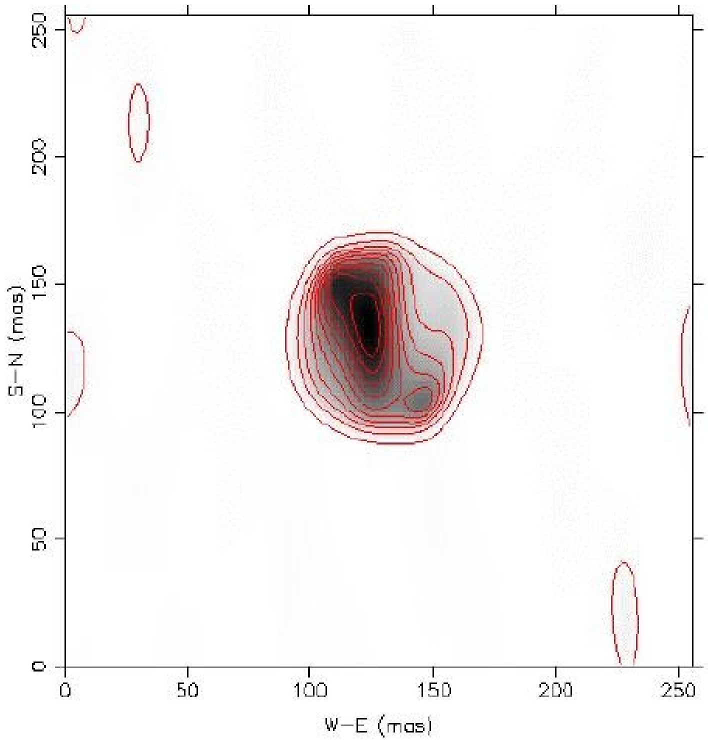
Add phases around triangle:

$CP=\Psi_{12}+\Psi_{23}+\Psi_{31}$

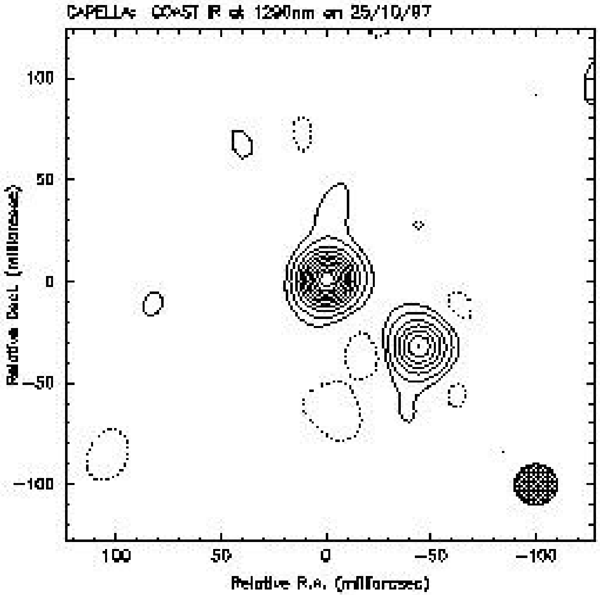
$e_{CP}=0$!!!

BUT: requires bright source (S:N in one coherence time)

Closure-phase mapping: COAST



Betelgeuse (Young et al. 2004, Proc. Nat. Astr. Meeting)



Capella (www.mrao.cam.ac.uk/telescopes/coast)

Self-calibration

Development of closure techniques which implicitly preserves closure phase and amplitude (Cornwell & Wilkinson 1981)

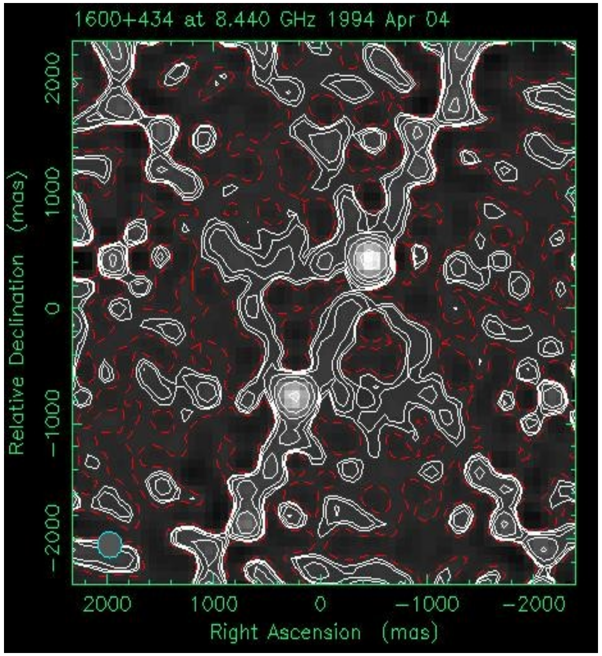
Given $R_{ij}^{\text{obs}} = g_i g_j R_{ij}^{\text{real}}$

use an arbitrary model to determine g_i and g_j for an arbitrary model, and then replace the observed visibilities with

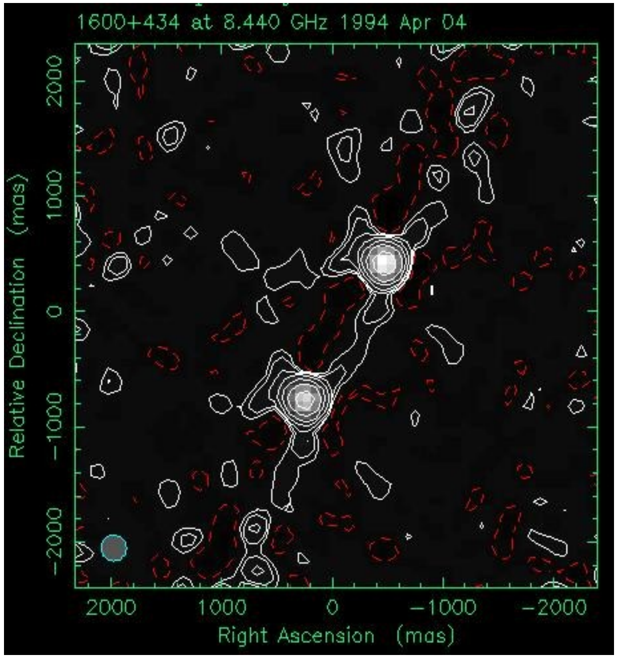
$$R_{ij}^{\text{obs}} / g_i g_j$$

Shocking but it works; relies on errors separating by telescope.

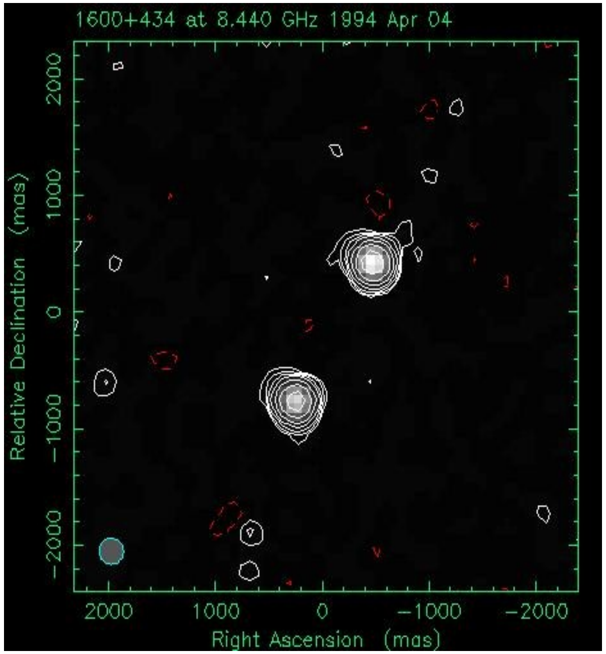
Self-calibration in action



Dirty map

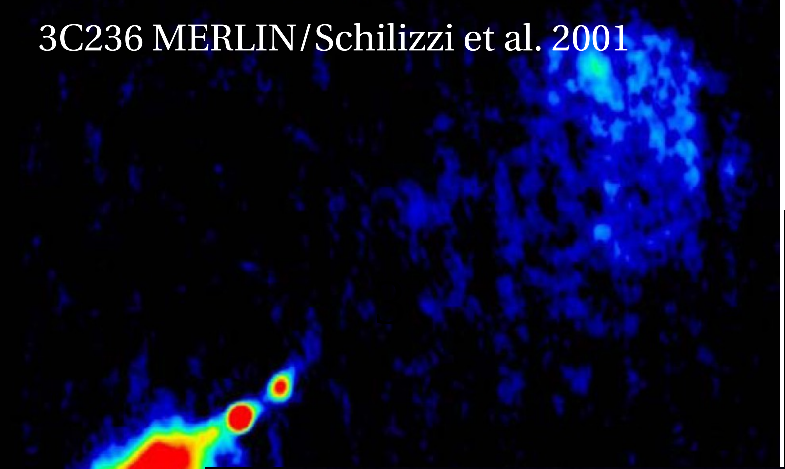


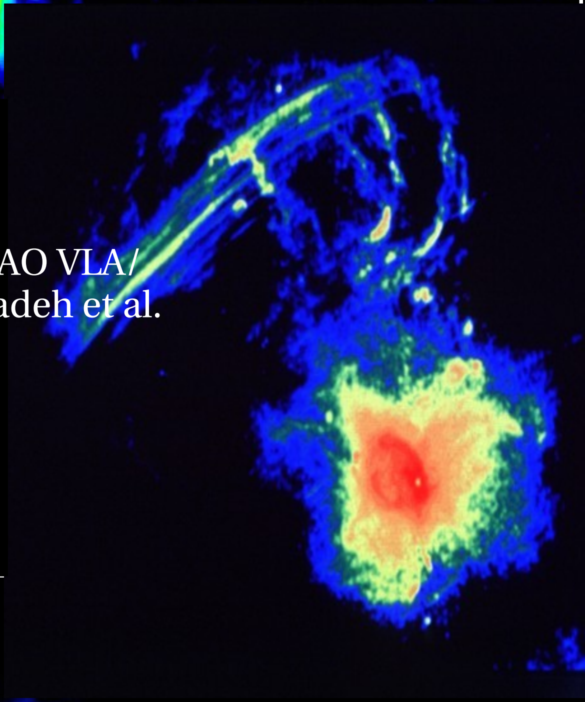
CLEANed map

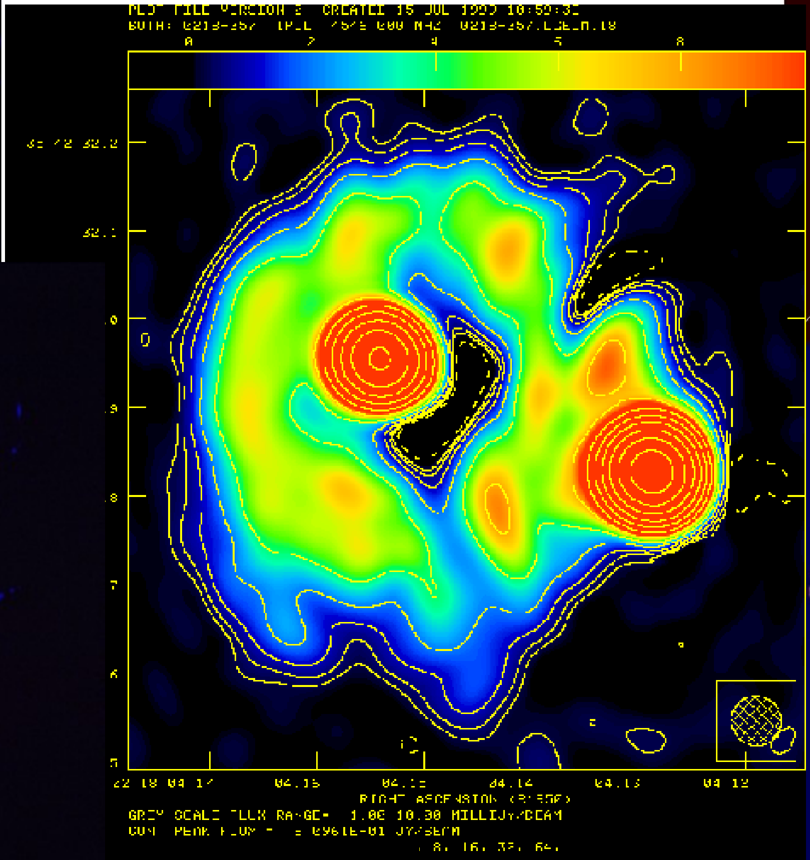


CLEANed map with phase selfcalibration

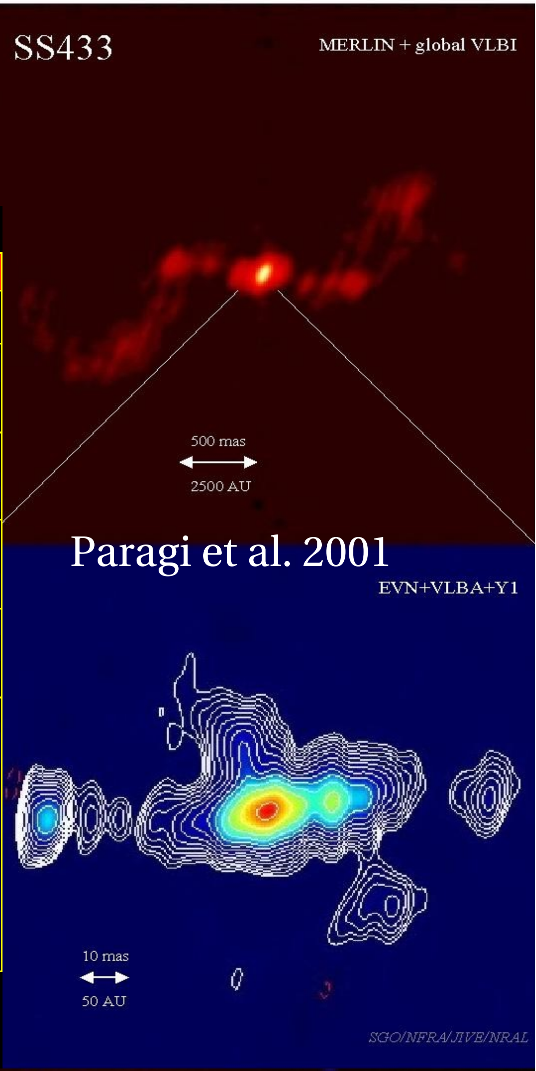
Some radio images
from interferometers


3C236 MERLIN/Schilizzi et al. 2001

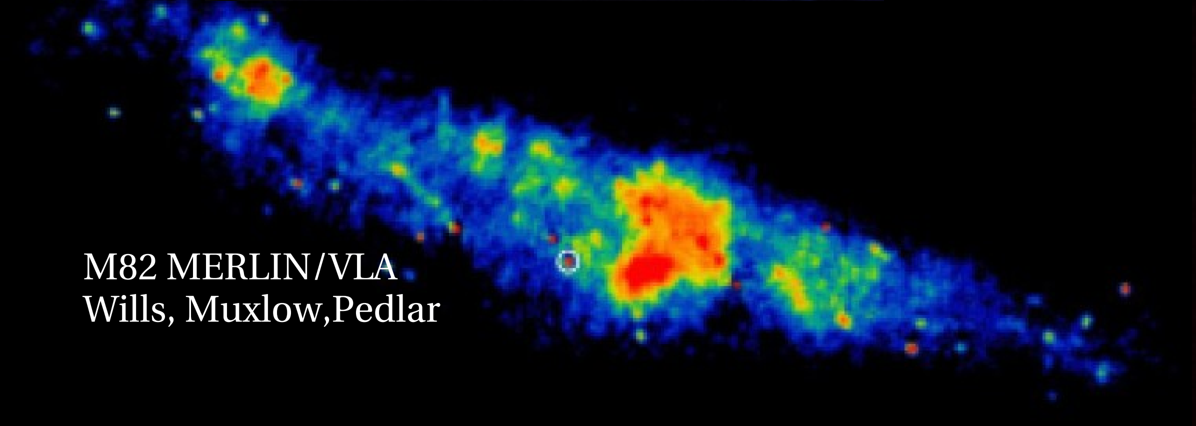

SgrA NRAO VLA/
Yusuf-Zadeh et al.

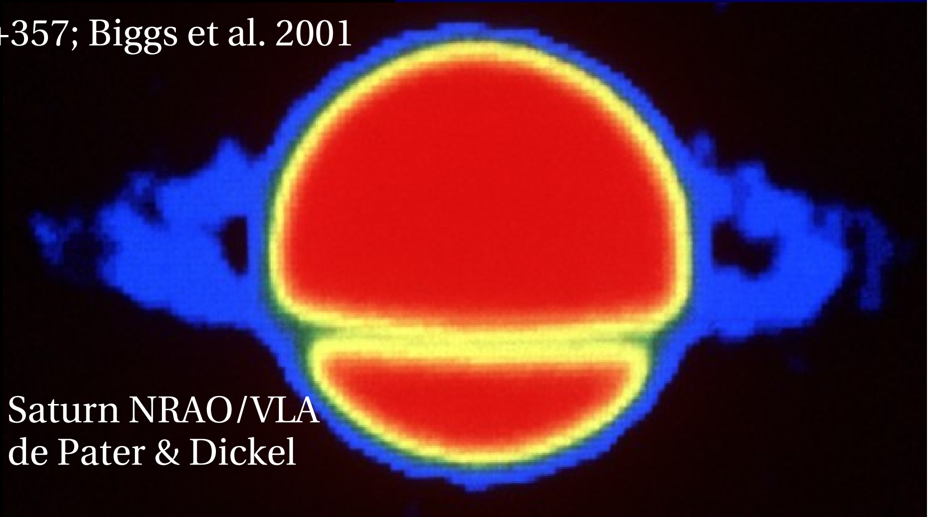


Grav. Lens 0218+357; Biggs et al. 2001
MERLIN/EVN

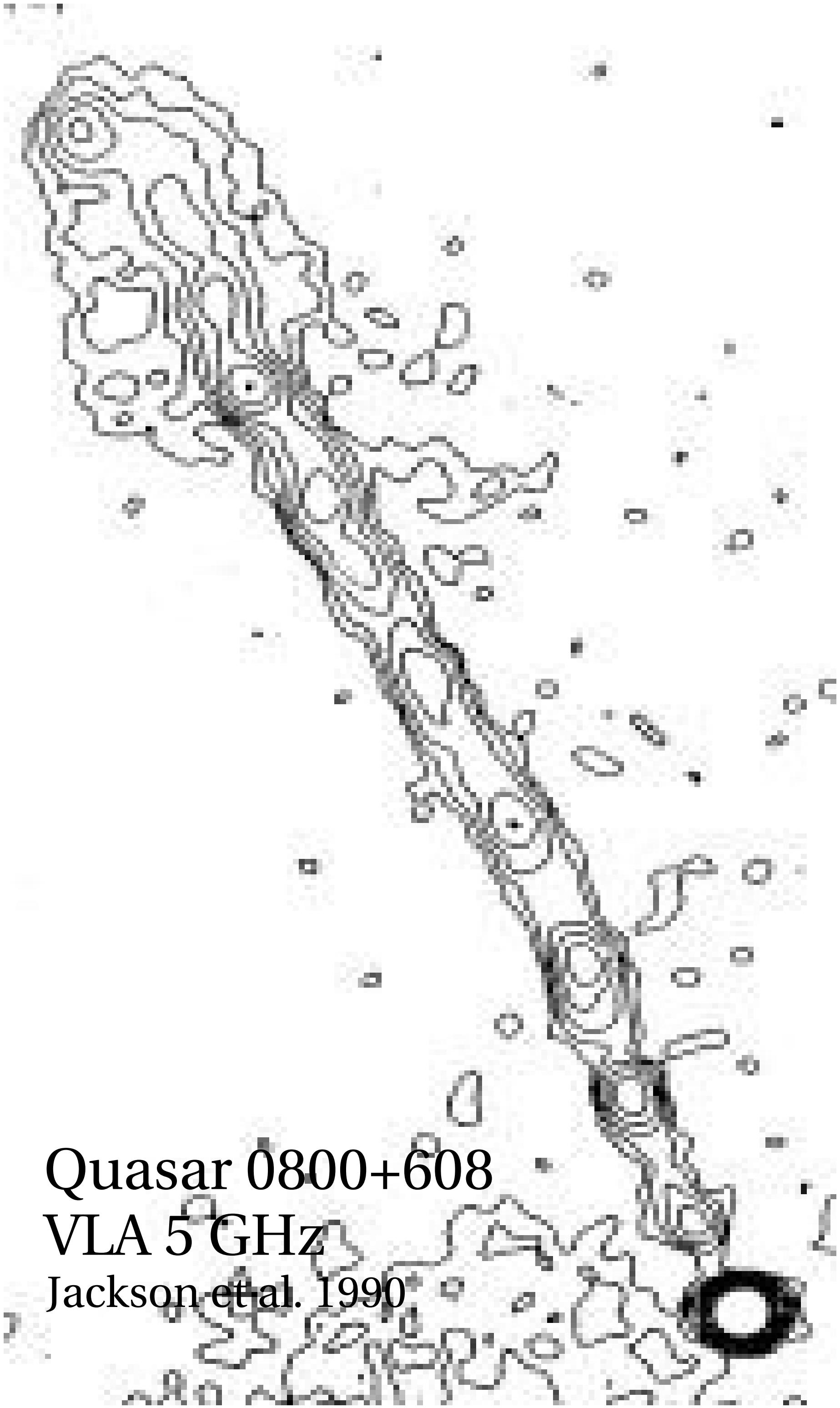


Paragi et al. 2001


M82 MERLIN/VLA
Wills, Muxlow,Pedlar

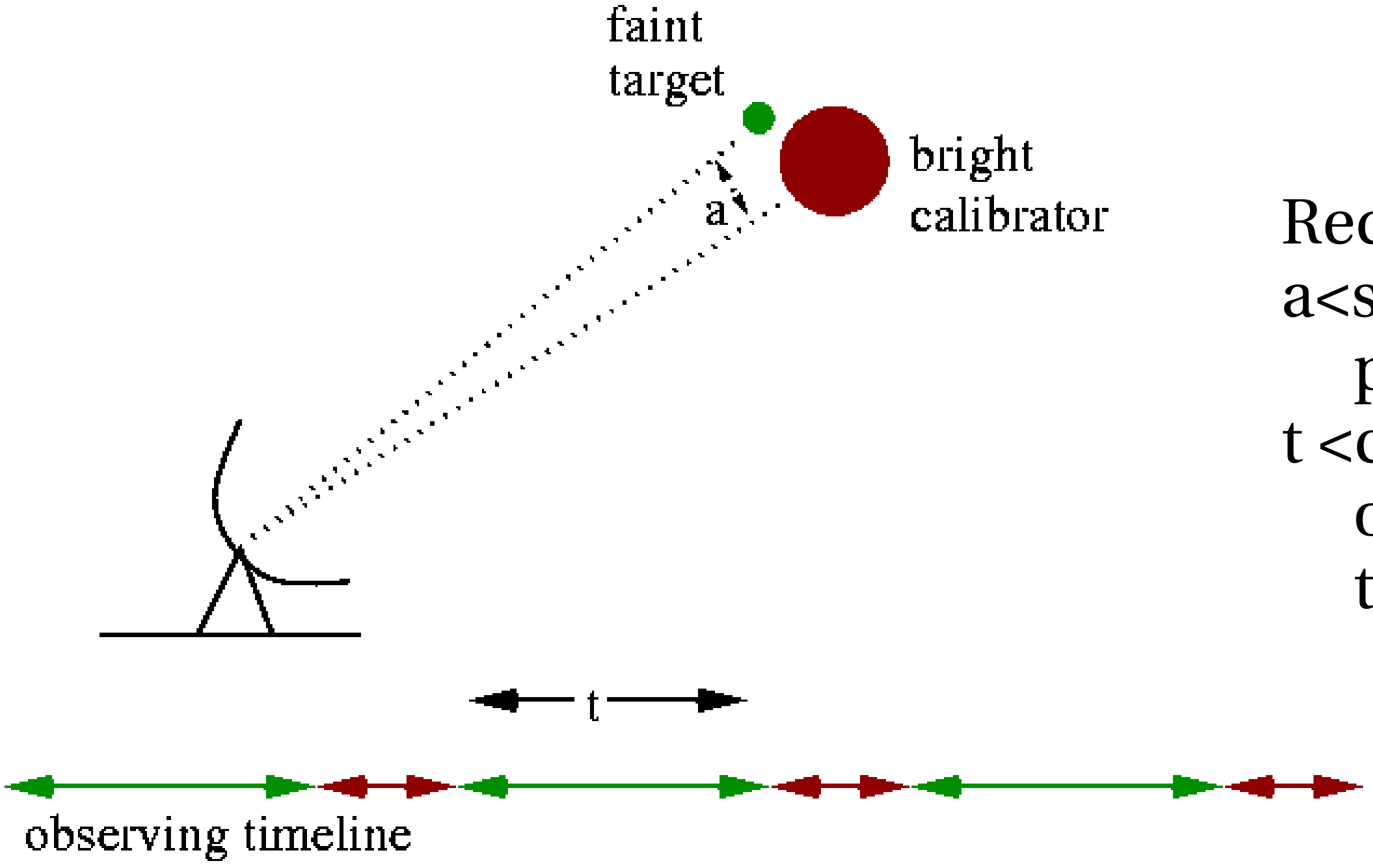

Saturn NRAO/VLA
de Pater & Dickel

...and one more



Quasar 0800+608
VLA 5 GHz
Jackson et al. 1990

Phase calibration



Require
a<size of isoplanactic
    patch
t <coherence time
    of atmosphere at
    this wavelength

Can nod back and forth, or have target and calibrator in same FOV

Phase calibration

Phase calibrator must be
- bright (S:N in reasonable time/atmospheric coh. time)
- close (same corruption)

(cf. adaptive optics on single telescopes)

If isoplanactic patch is small
- calibrator may not exist

Summary

Interferometry is hard because
- it is more technically demanding
- you have to worry about atmospheric effects

Interferometry is worthwhile because
- it is the only way to obtain the high resolution needed to observe a vast range of astrophysical phenomena (including jets!)