Advanced radio interferometric imaging

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- Output of an interferometer after calibration:

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i \left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dl dm$$

- (u,v,w) : interferometer's geometrical vector
- (l,m) : position on the sky
- I : sky brightness (“image”)

Imaging : Calculating I(l,m) from V(u,v,w)

Visibility function

- Full visibility function:

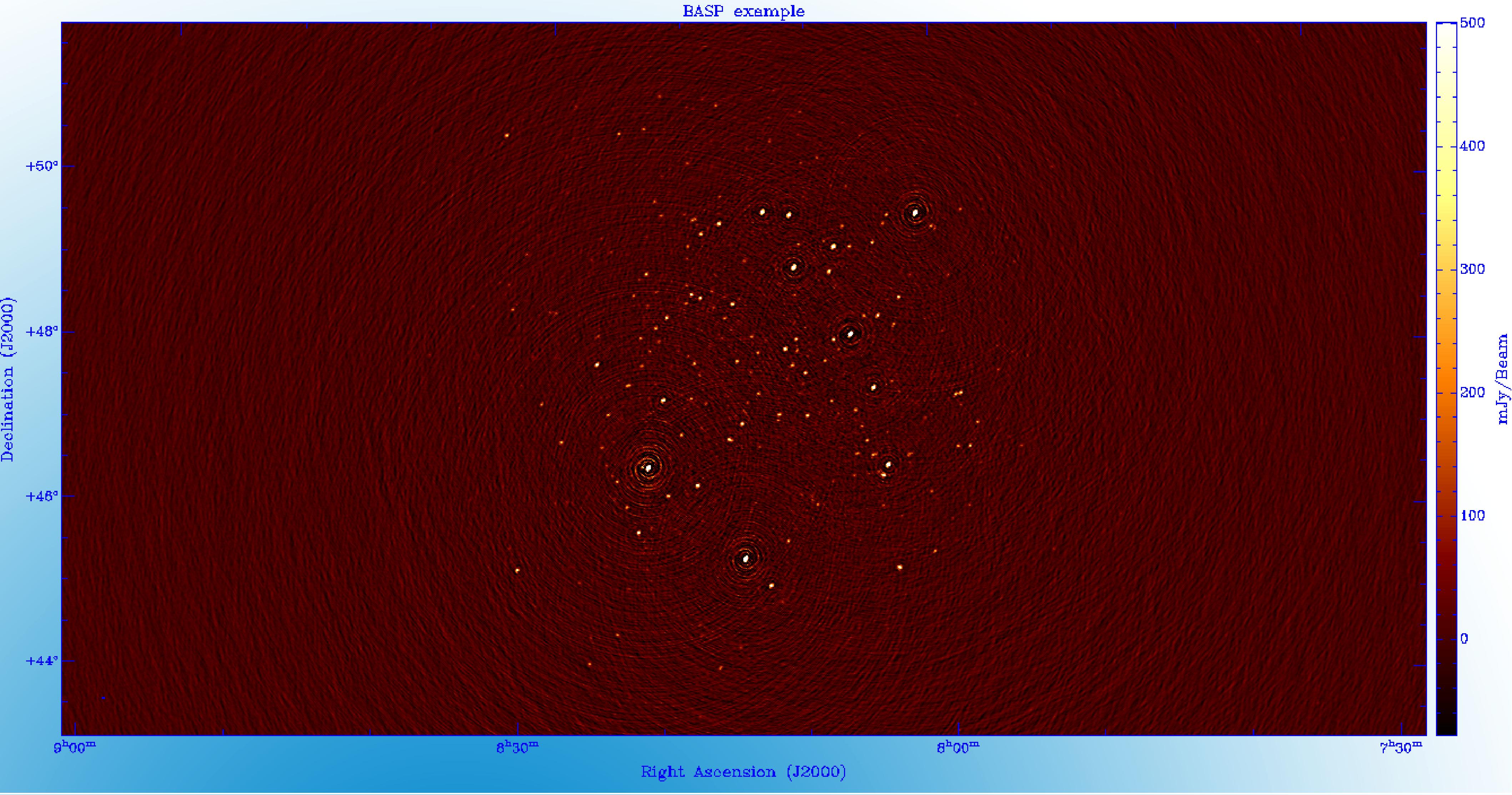
$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i\left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dl dm$$

- For small field of view (l~0, m~0) or w~0 :

$$V(u,v,\not{w}) \approx \iint I(l,m) e^{-2\pi i(ul+vm)} dl dm$$

- (u,v,w) : interferometer's geometrical vector
- (l,m) : position on the sky
- $\boldsymbol{I}$: sky brightness (“image”)

Fourier relation

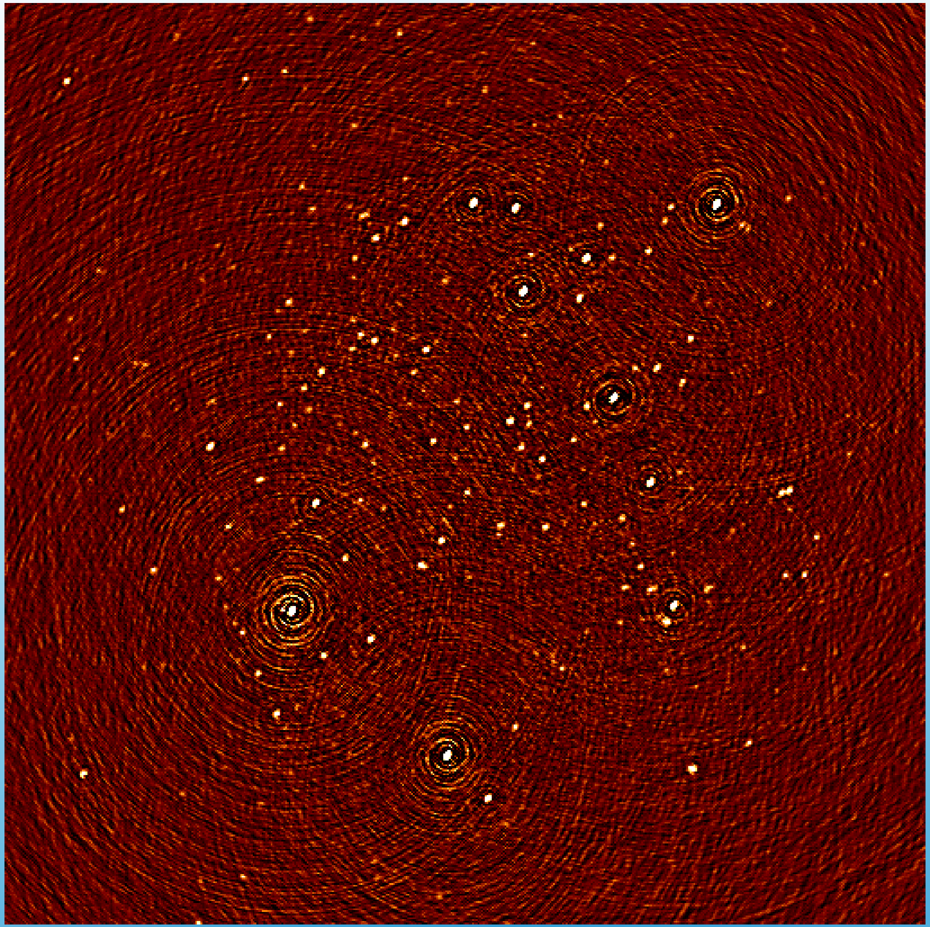


LOFAR dirty image (3c196)

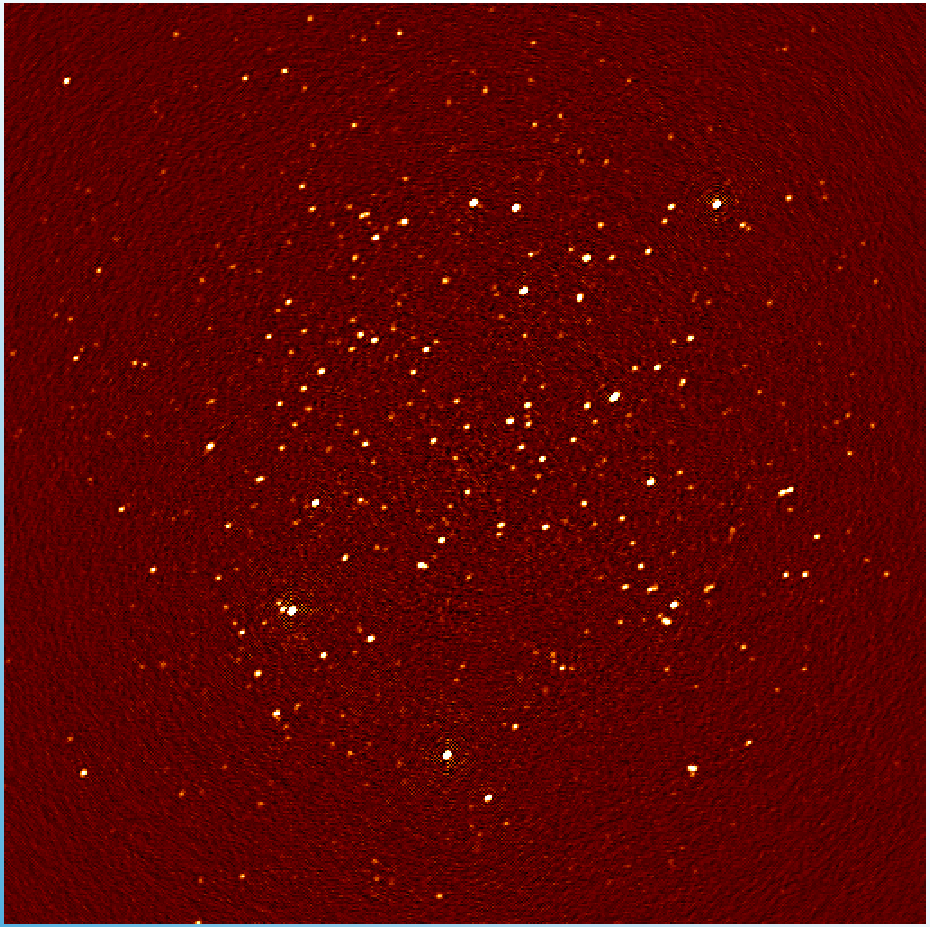
The dirty image

Högbom CLEAN

- Högbom CLEAN algorithm (1974):
  - Find largest peak in image
  - Scale PSF to fraction of peak and subtract
  - Repeat until peak < threshold or nIter > limit
  - Finally: restore subtracted components

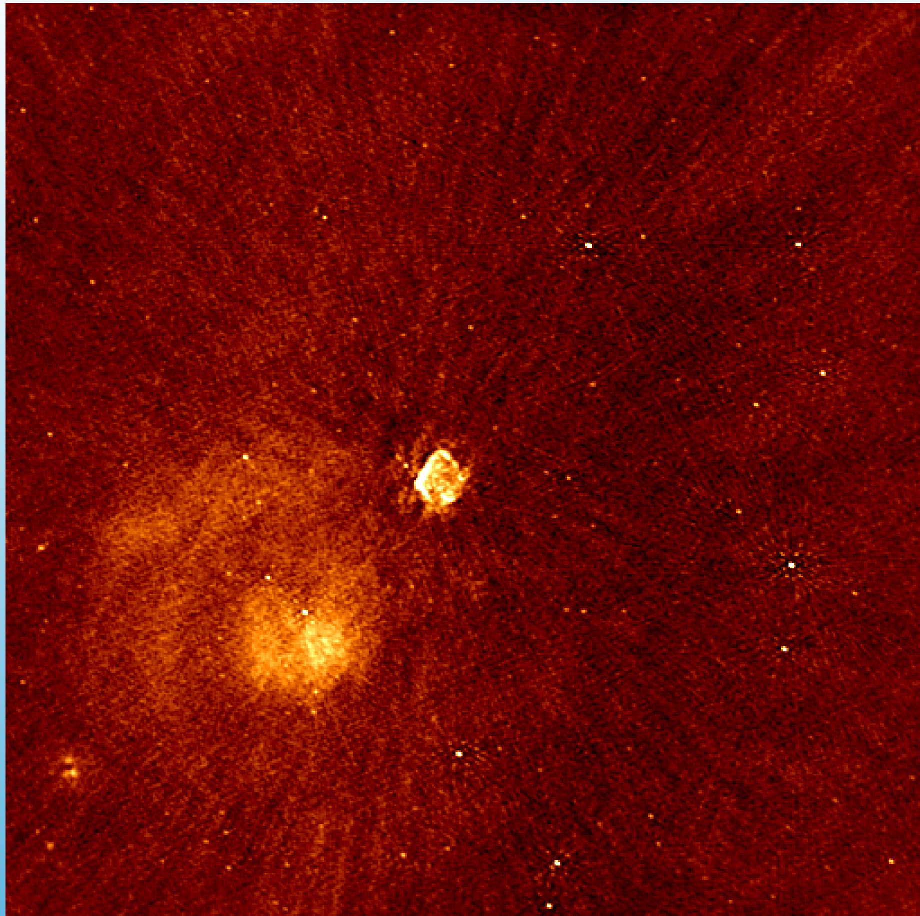


LOFAR undeconvolved (“dirty”) image

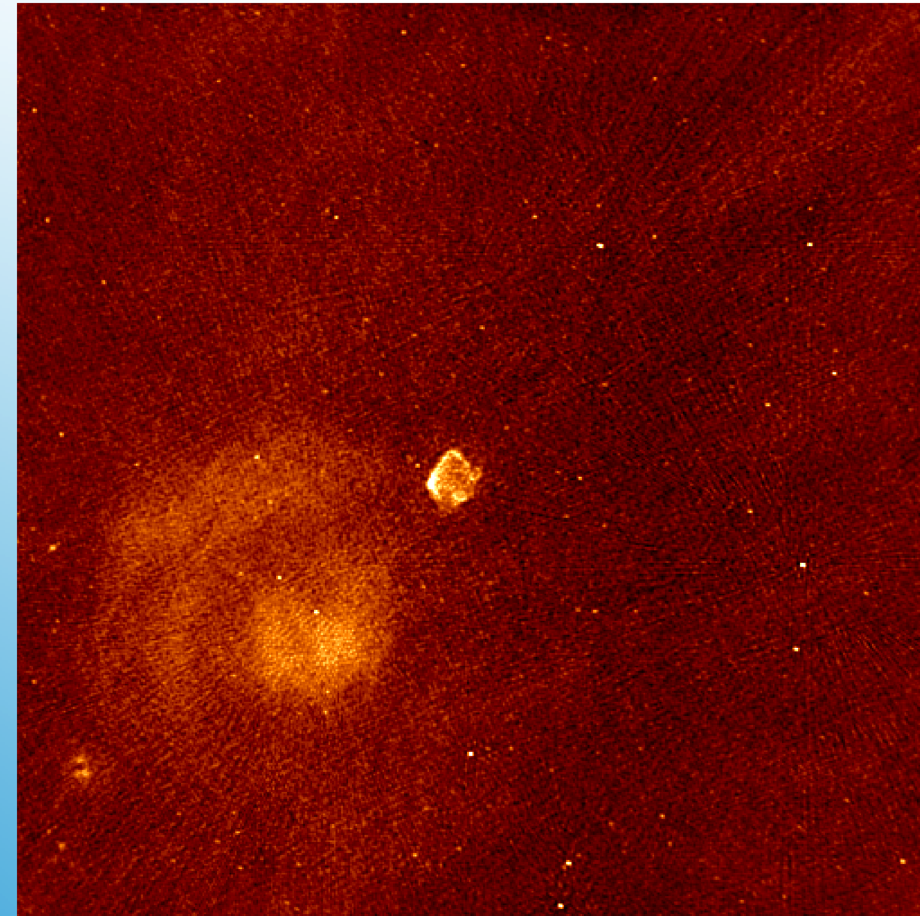


Deconvolved with Högbom CLEAN

Högbom CLEAN

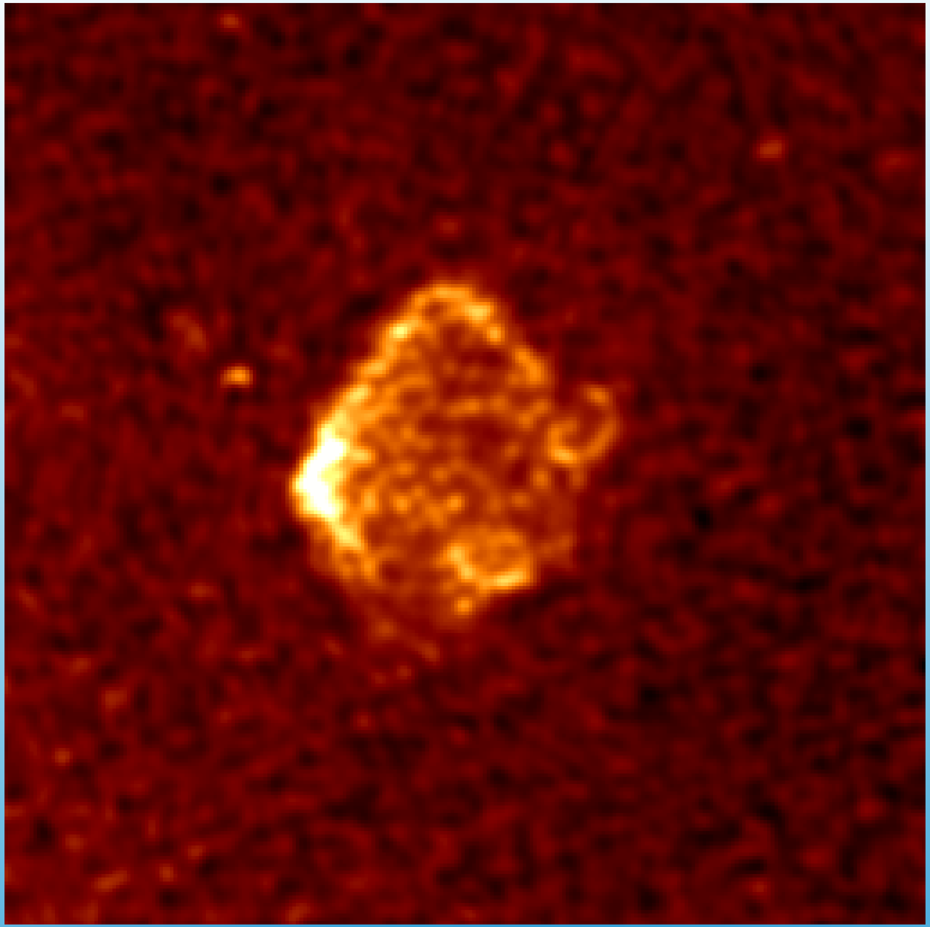


Undeconvolved “dirty” image

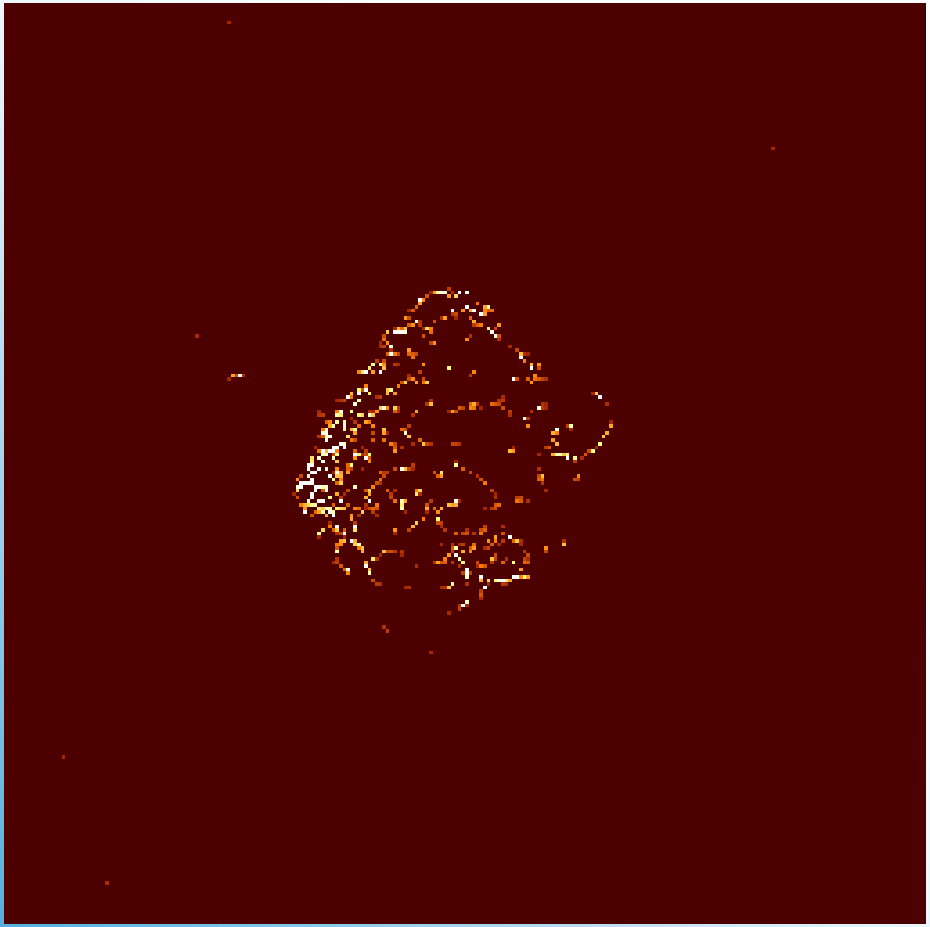


Deconvolved image with Högbom CLEAN

Deconvolving diffuse structures



Deconvolved image (Högbom CLEAN)



Actual model

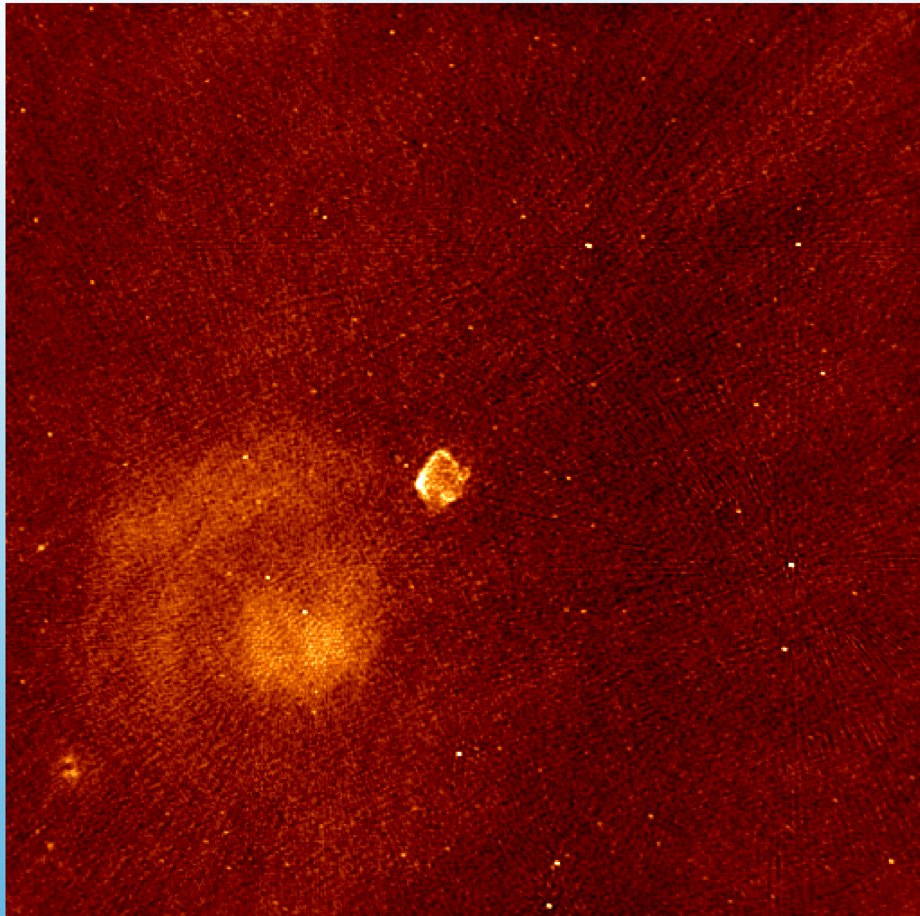
Deconvolving diffuse structures

Improved algorithm by Cornwell (2008) :

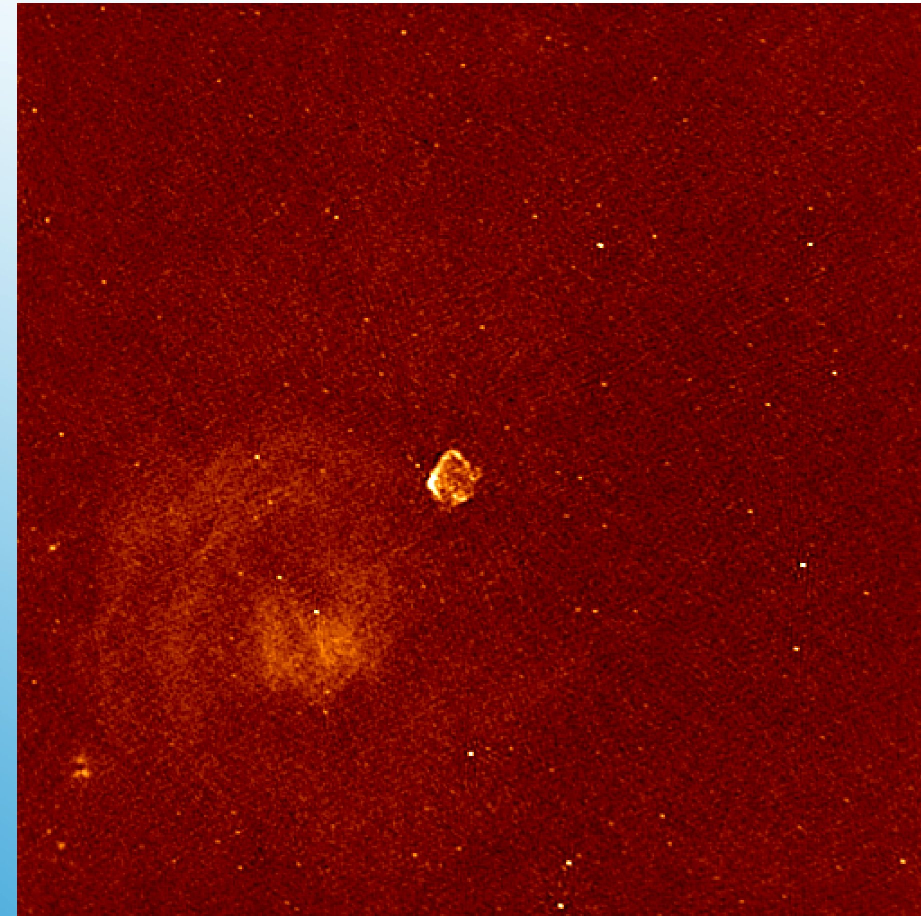
- “Multi-scale clean”

- Fits small smooth kernels (and delta functions) during a Högbom CLEAN iteration

Multi-scale CLEAN



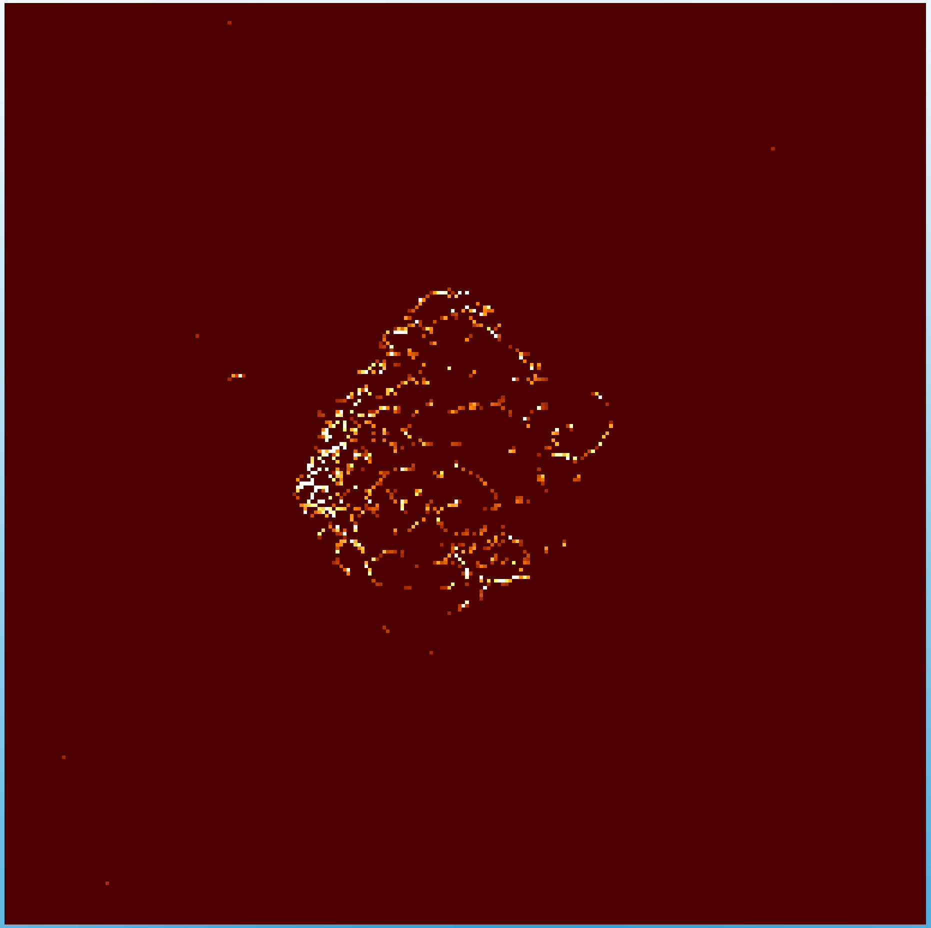
Normal Högbom CLEAN



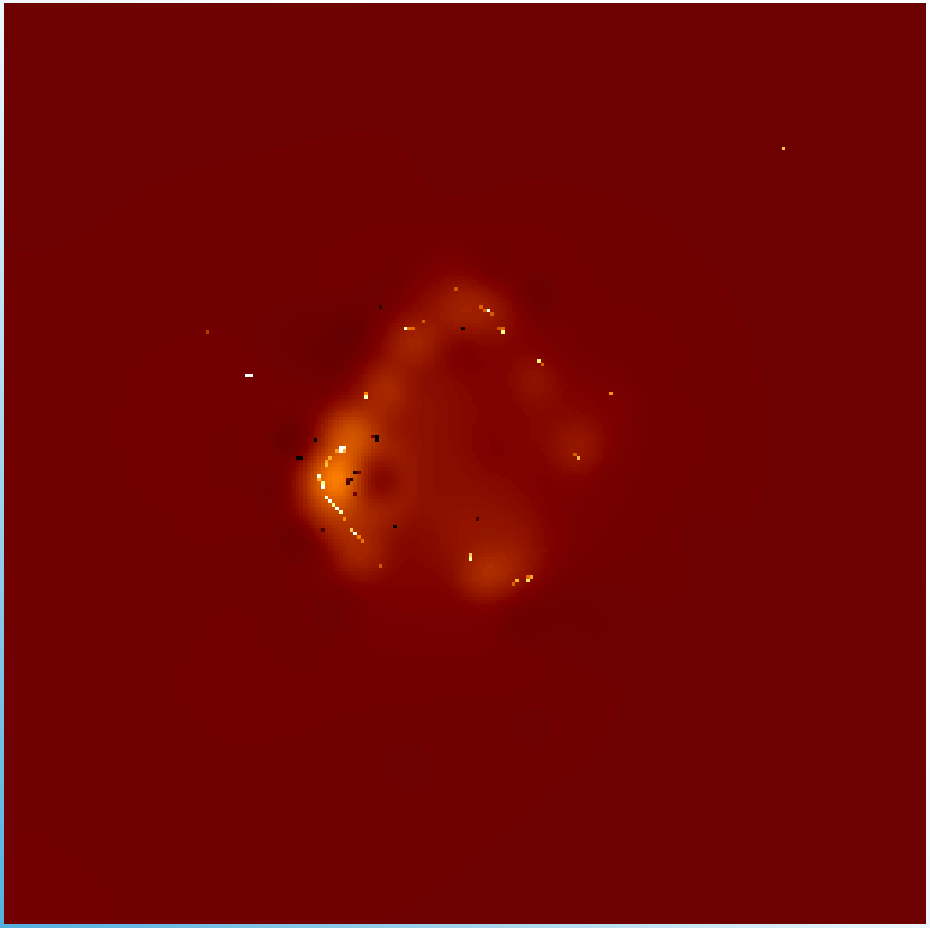
Multi-scale CLEAN
(implementation in WSClean)

Multi-scale CLEAN

Multi-scale CLEAN

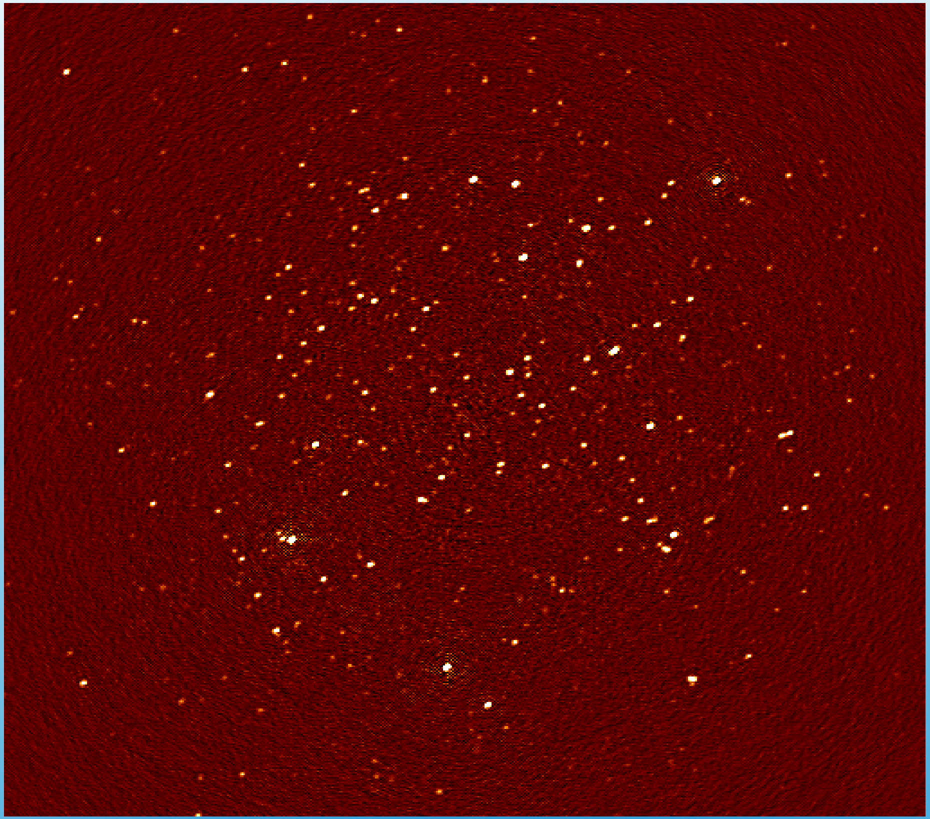


Normal Högbom CLEAN
Output model

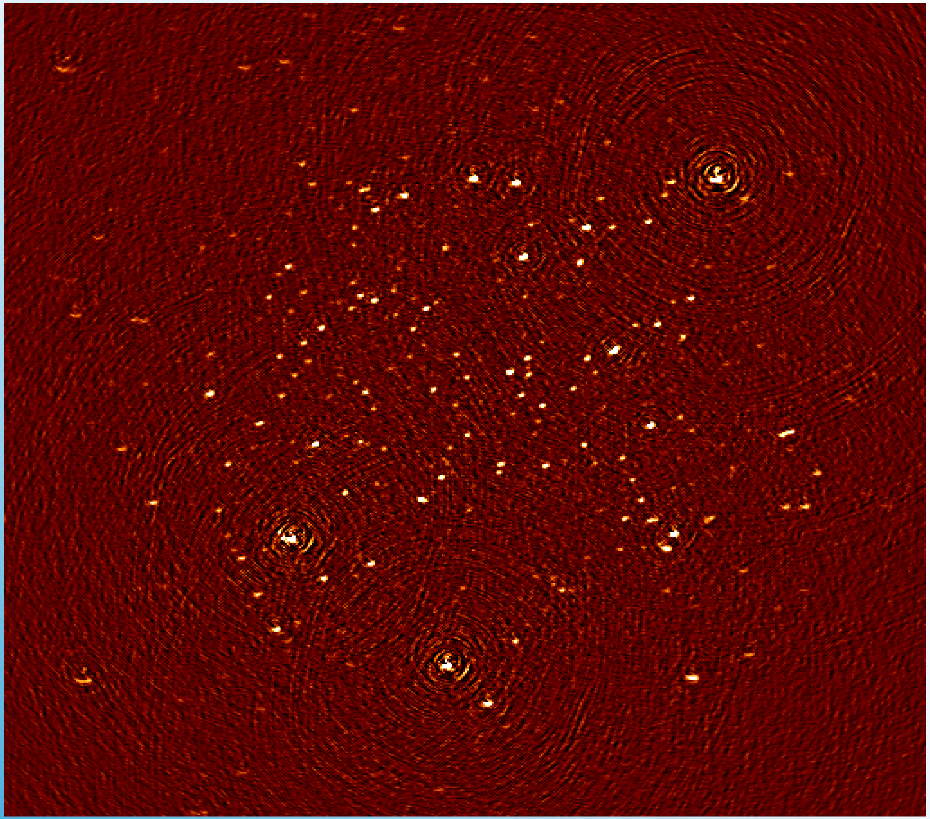


Multi-scale CLEAN
(as implementation in WSClean)

- 2D FT does not hold for new arrays: l,m,w >> 0



Correcting w-terms



Without correcting w-terms

The w-term

- 2D FT relationship does not hold for new arrays: l,m,w >> 0
- Have to use full function:

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i \left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dl dm$$

- Easy solution: facetting
    - But: slow, stitching artefacts
- Better & most used solution: 'w-projection'

The w-term

- Visibility function:

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i \left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dl dm$$

- W-projection: *(Cornwell et al, 2008)*

$$V(u,v,w) * \mathcal{F}(e^{-2\pi i w(\sqrt{1-l^2-m^2}-1)}) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i (ul+vm)} dl dm$$

This convolution turns out to have a "limited" support

- Performance very dependent on zenith angle, coplanarity of array, field of view and resolution.

w-projection

- Another problem; convolution theorem no longer works when w-terms present in

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i \left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dl dm$$

- Högbom CLEAN assumes constant PSF
- But PSF changes (slightly) over the image
- Solved with Cotton-Schwab algorithm (schwab 1984)
- Normal CASA imaging mode will automatically use CS

w-projection

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$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i \left(ul+vm+w(\sqrt{1-l^2-m^2}-1)\right)} dldm$$

- Högbom CLEAN assumes constant PSF
- But PSF changes (slightly) over the image
- Solved with Cotton-Schwab algorithm (schwab 1984)
- Normal CASA imaging mode will automatically use CS (i.e., **C**otton-**S**chwab, *not* **C**ompressed **S**ensing)

w-projection

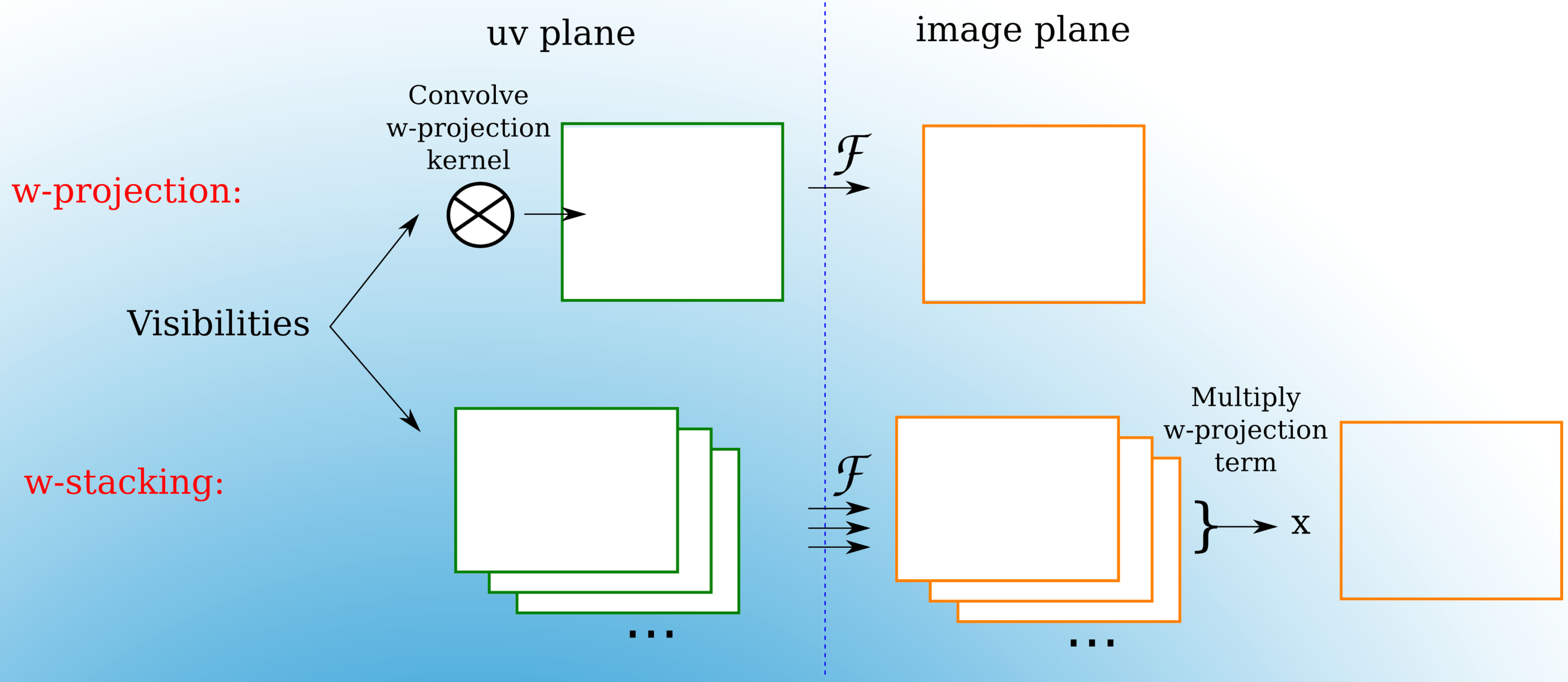
- The Cotton-Schwab + w-projection algorithm:
  - Make initial dirty image & central PSF
  - Perform minor iterations:
    - Find peak
    - Subtract scaled PSF at peak with small gain
    - Repeat until highest peak ~ 80-90% decreased
  - Major iteration: “Correct” residual
    - Predict visibility for current model
    - Subtract predicted contribution and re-image

w-projection

- W-projection is the standard way to solve w-terms in radio astronomy
- W-term convolution can be *s..l..o..w...*
  - Imaging 2 minutes of data of the MWA telescope (30 degree FOV) costs hours
- New imager with new algorithm implemented: WSClean[1] ("w-stacking clean").
  - Offringa et al, 2014

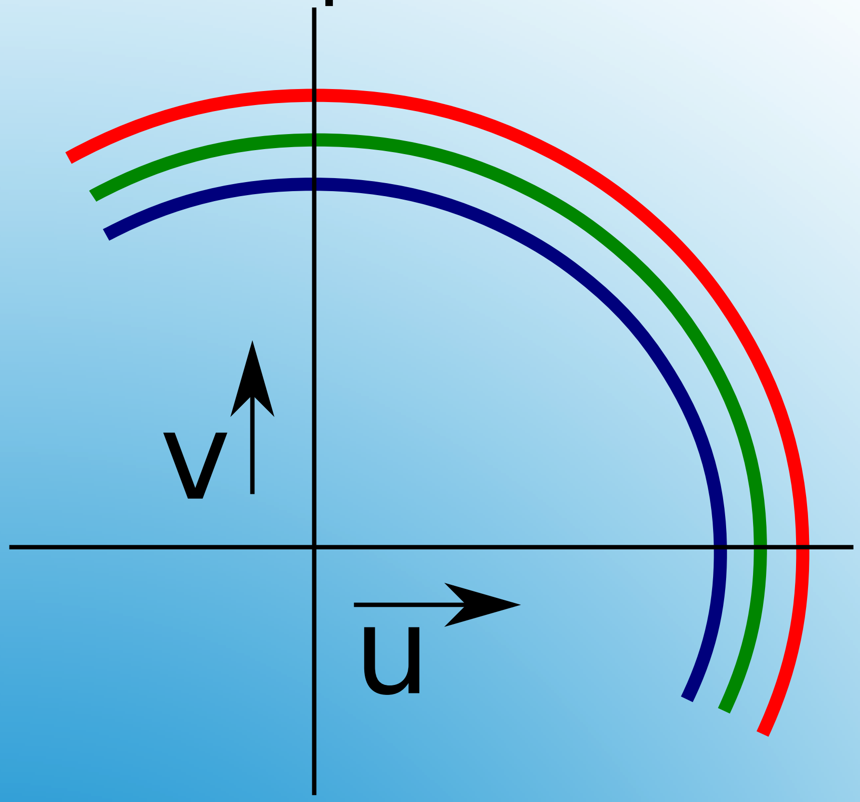
[1]see http://wsclean.sourceforge.net/

w-projection


uv plane
image plane
Convolve
w-projection
kernel
w-projection:
Visibilities
w-stacking:
Multiply
w-projection
term
x

w-stacking

- Multi-frequency synthesis (MFS) means gridding different frequencies on the same uv grid:



- This is the standard for modern telescopes

Multi-frequency synthesis

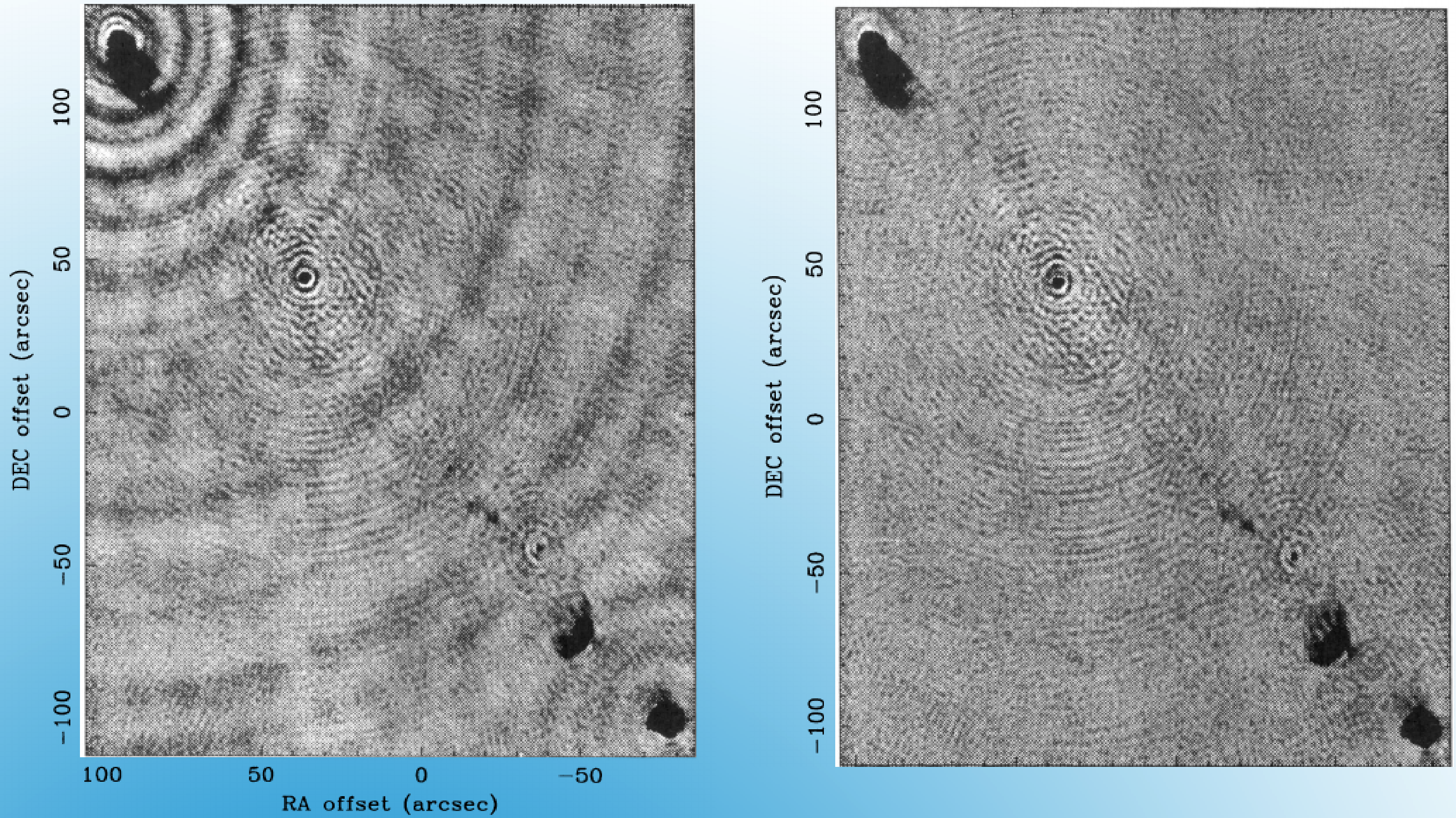
Related, but not the same:

- **Multi-frequency deconvolution** *(see Rau and Cornwell, 2011)* sometimes called **multi-term deconvolution**

  Selected by setting `nterms` *in CASA's clean task*

- Takes spectral variation into account during deconvolution
- Useful for wideband, sensitive imaging

Multi-frequency deconvolution

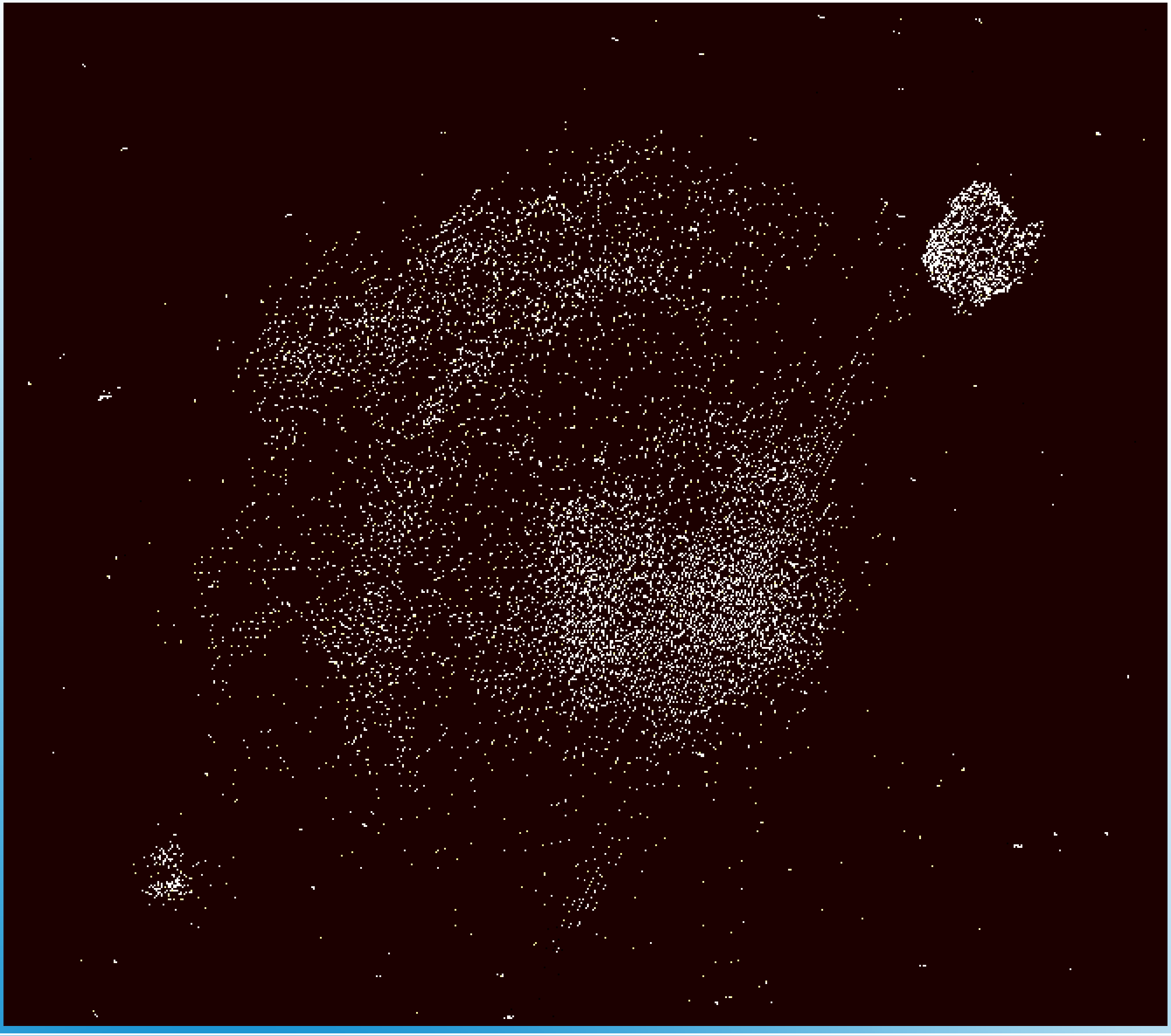


- Right image: fit for flux over frequency to improve deconvolution (Sault & Wieringa, 1994)

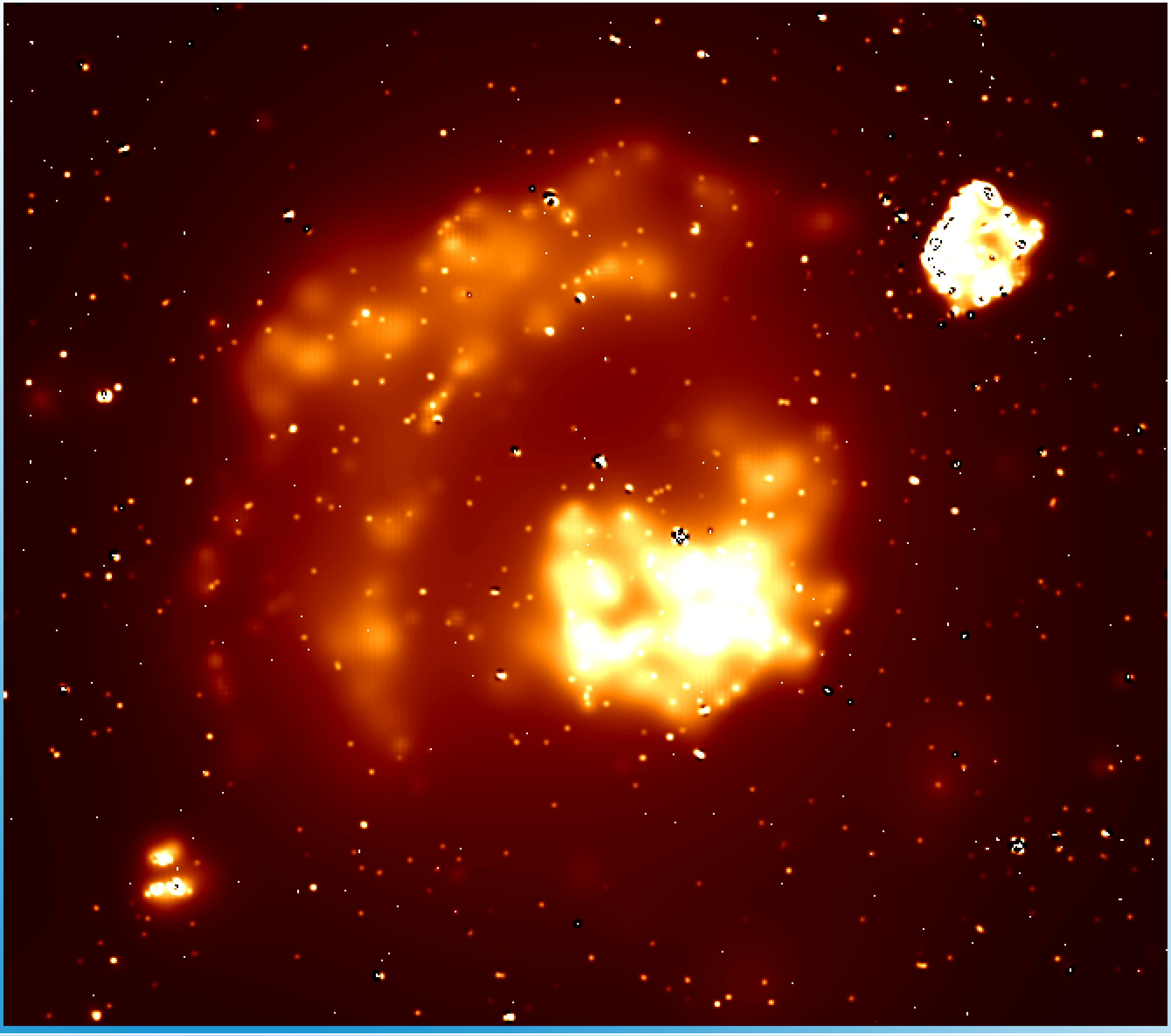
Frequency-dependent deconvolution

- Recent focus on deconvolution using '**c**ompressed **s**ensing' (abbrev. CS – but CS can mean "Cotton-Schwab" too)
- CS methods assume the sky is 'sparse' ("solution matrix is sparse in some basis")
- Minimizes "L1-norm" (= abs sum of CLEAN components)
- Högbom clean is actually (almost) a compressed sensing method called "Matching Pursuit"
- CS considers MP to be non-ideal… but radio data is not the perfect CS case: Calibration errors, w-terms

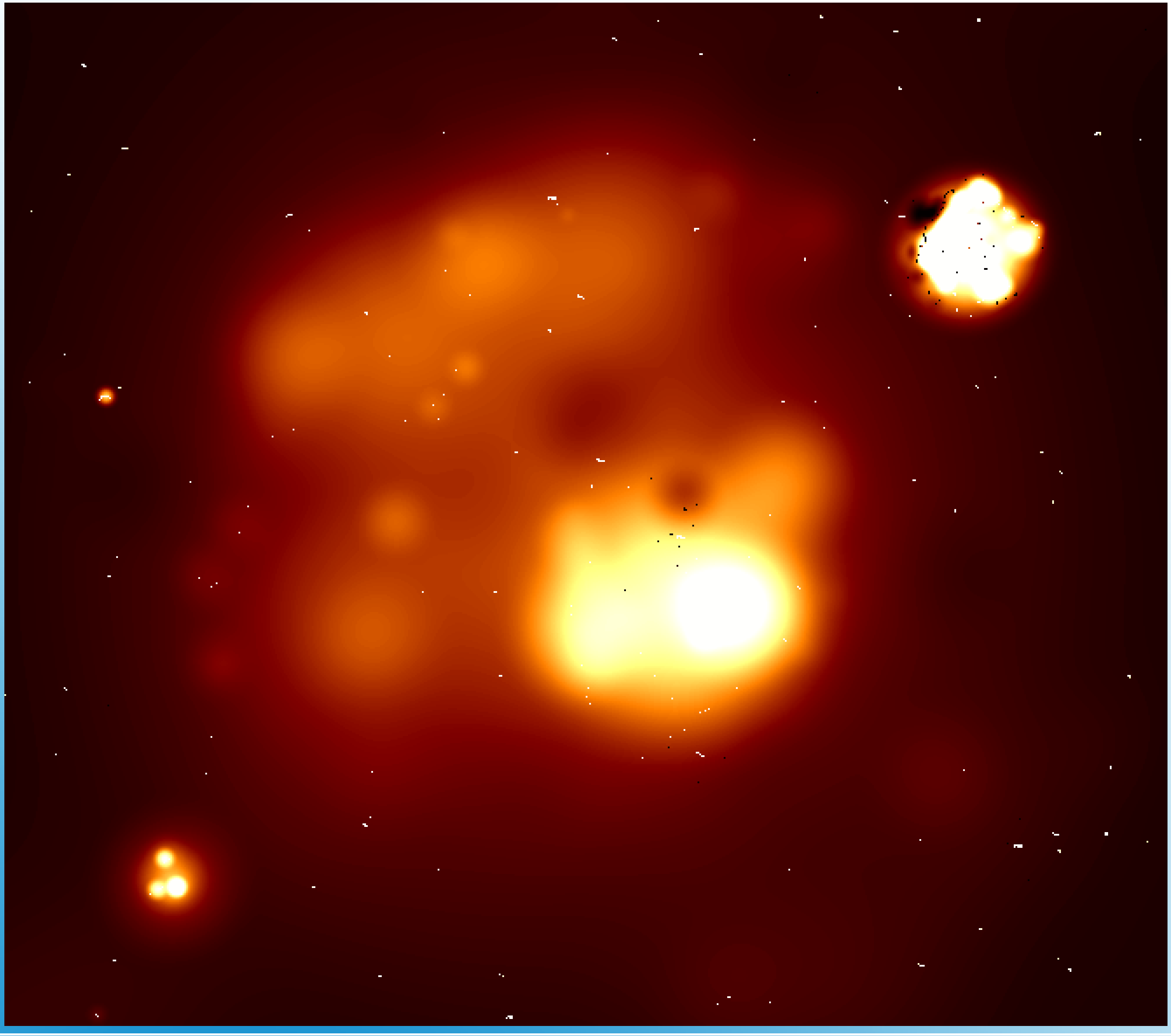
Compressed sensing



Model created by Högbom clean



Model created by a CS method
(“non-linear conjugate gradient using IUWT”)



Model created by multi-scale clean

- Compressed sensing does not work well with calibration artefacts
- Multi-scale is more robust
- On well-calibrated data:
  - CS gives more accurate model
  - But residuals don't improve much

Compressed sensing

- Clean components can be used as calibration model
- Often applied as:

Phase cal
↓
Shallow clean
↓
Phase cal
↓
Deep clean
↓
Phase & amplitude cal
↓
Deep clean

Self-cal & CLEAN

- Clean components can be used as calibration model
- Often applied as:

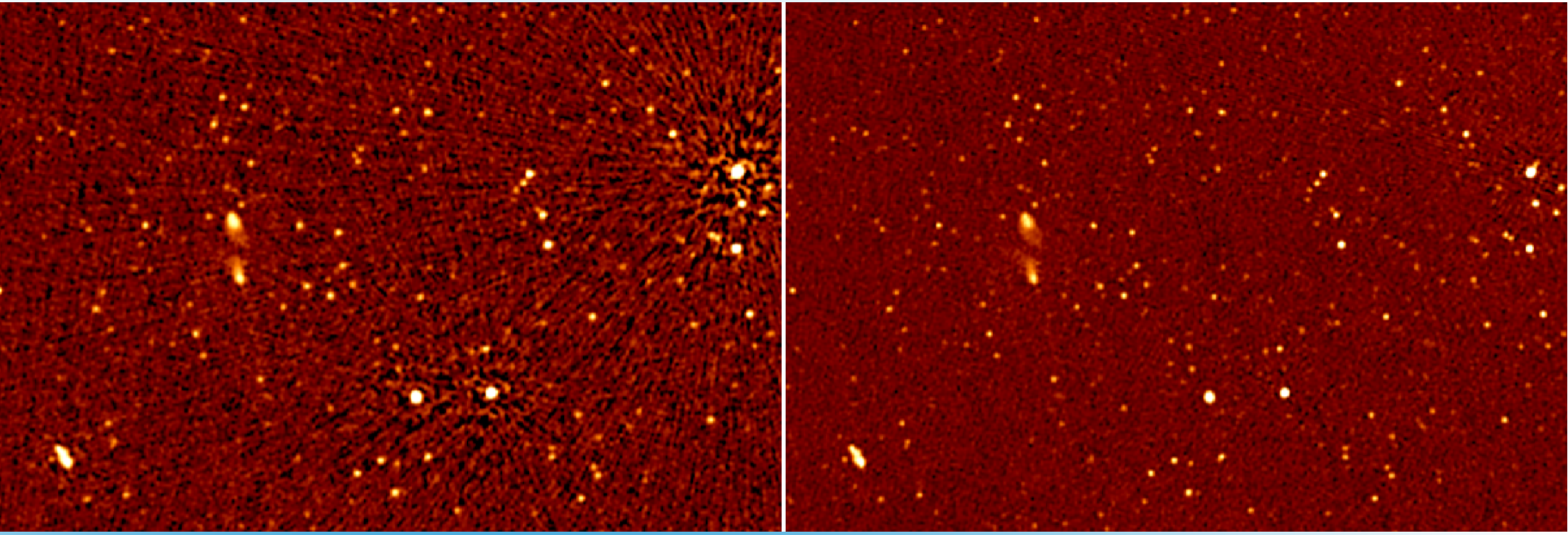
Phase cal
↓
Shallow clean
↓
Phase cal ······ **Why only phase?**
↓
Deep clean
↓
Phase & amplitude cal
↓
Deep clean

Self-cal & CLEAN

Self-cal & CLEAN

- Clean components can be used as calibration model
- Often applied as:

Phase cal
↓
Shallow clean
↓
Phase cal — **Why only phase?**
→ Avoid clean bias
↓
Deep clean
↓
Phase & amplitude cal
↓
Deep clean

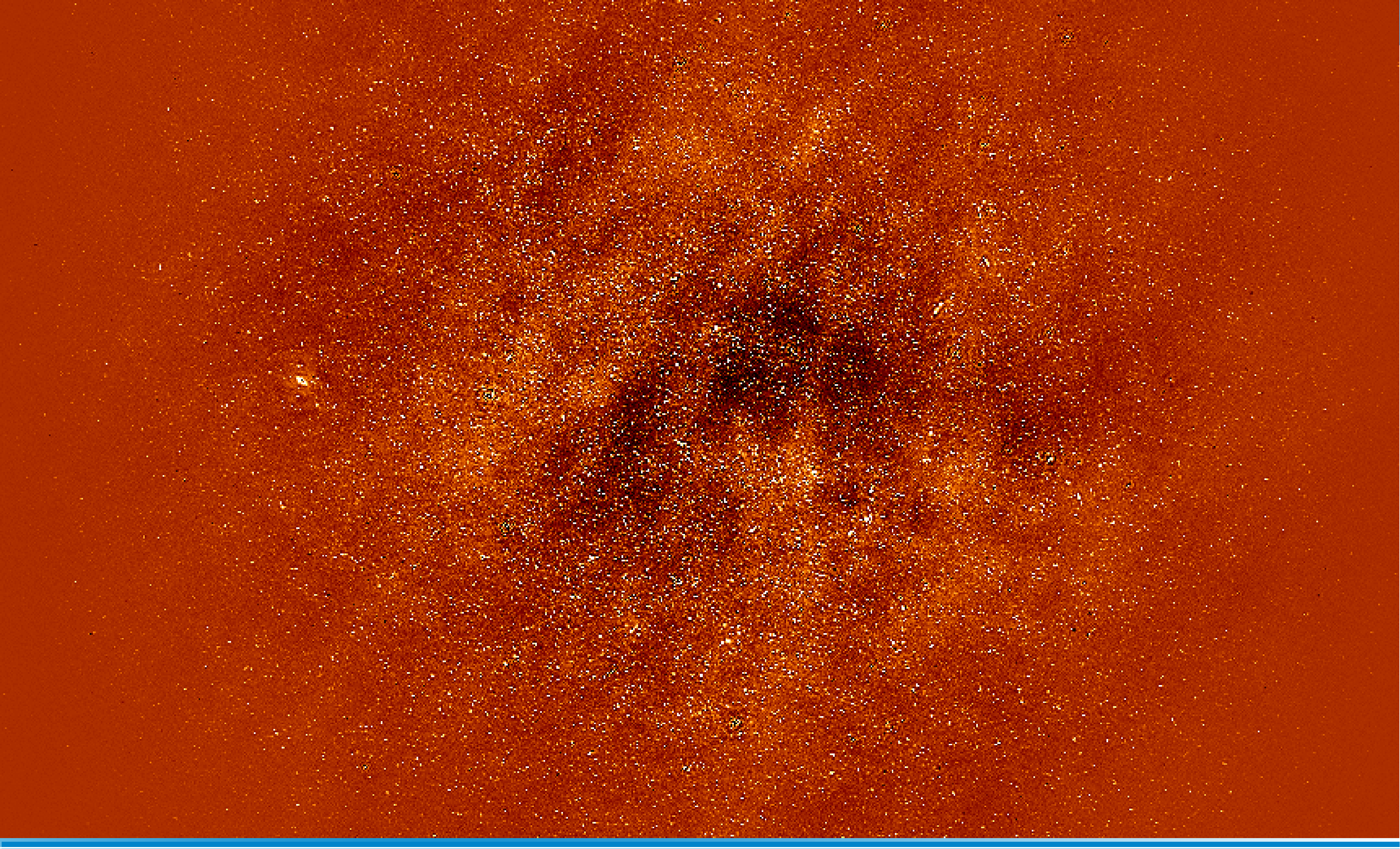


After initial calibration

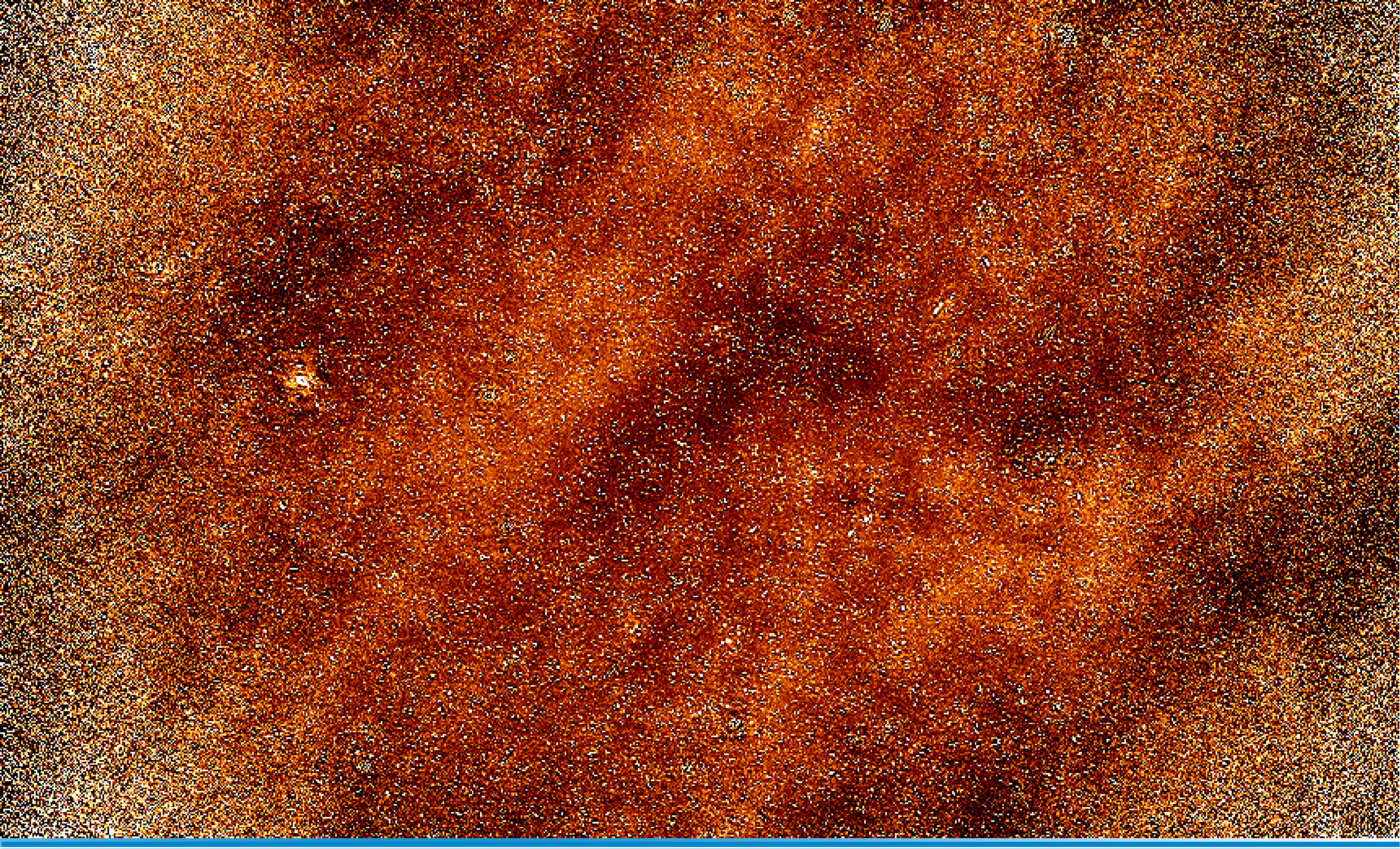
After self-cal on clean components

Image credit: N. Hurley walker (using the MWA)

Self-calibration using CLEAN



- **Result of imaging – is this how the sky looks like?**
  (and I don't mean the orange colour)



- **Result of imaging – is this how the sky looks like?**
  (and I don't mean the orange colour)

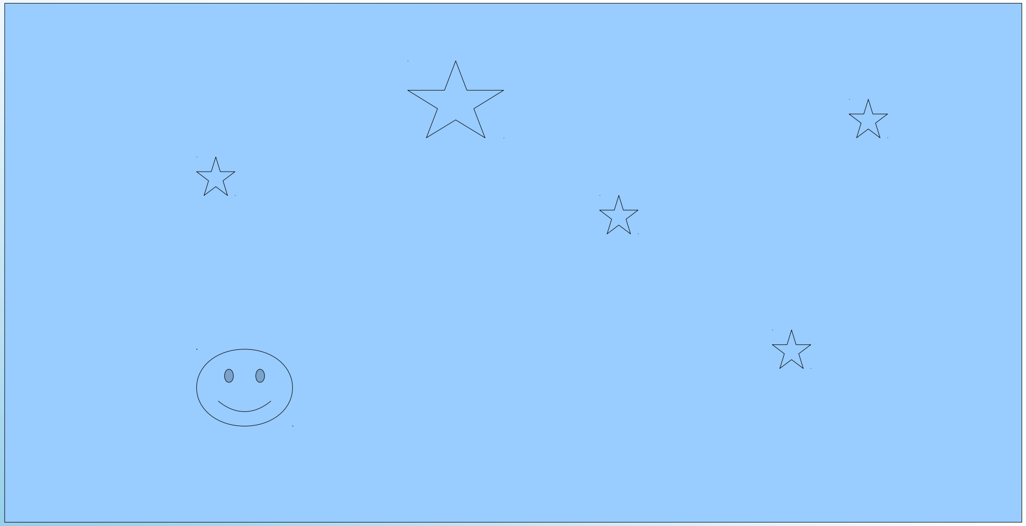
- Correction is required for the antenna response
- This is called "primary beam" correction (as opposed to the synthesized beam / psf)

- For dishes, the primary beam is ~constant
- To correct for: multiply final image with the inverse beam
- **Scalar** for total brightness, **matrix** for polarized

Primary beam correction

What if…

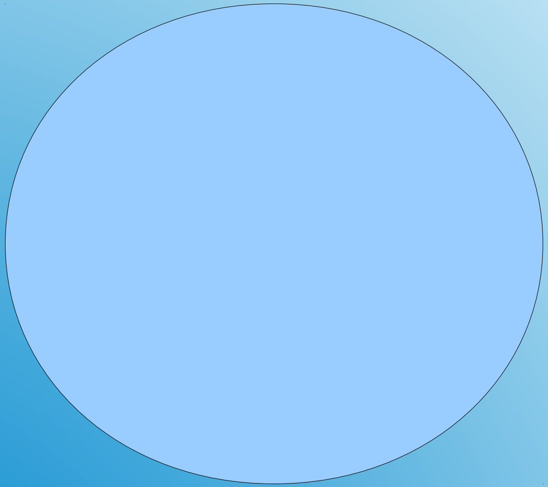
This is our field of interest→



(In practice, actual galaxies look different)

… and

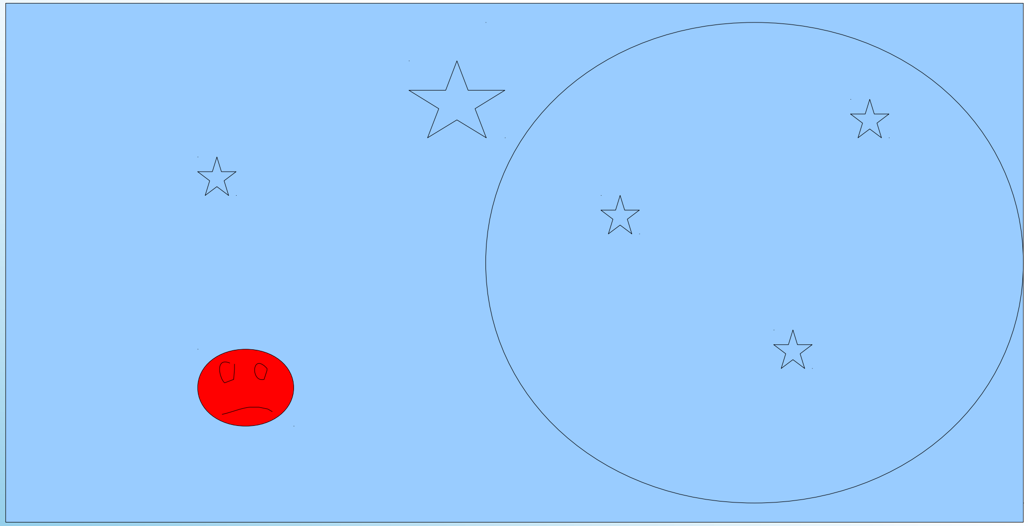
this is our primary beam→



Mosaicing

What if…

This is our field of interest→

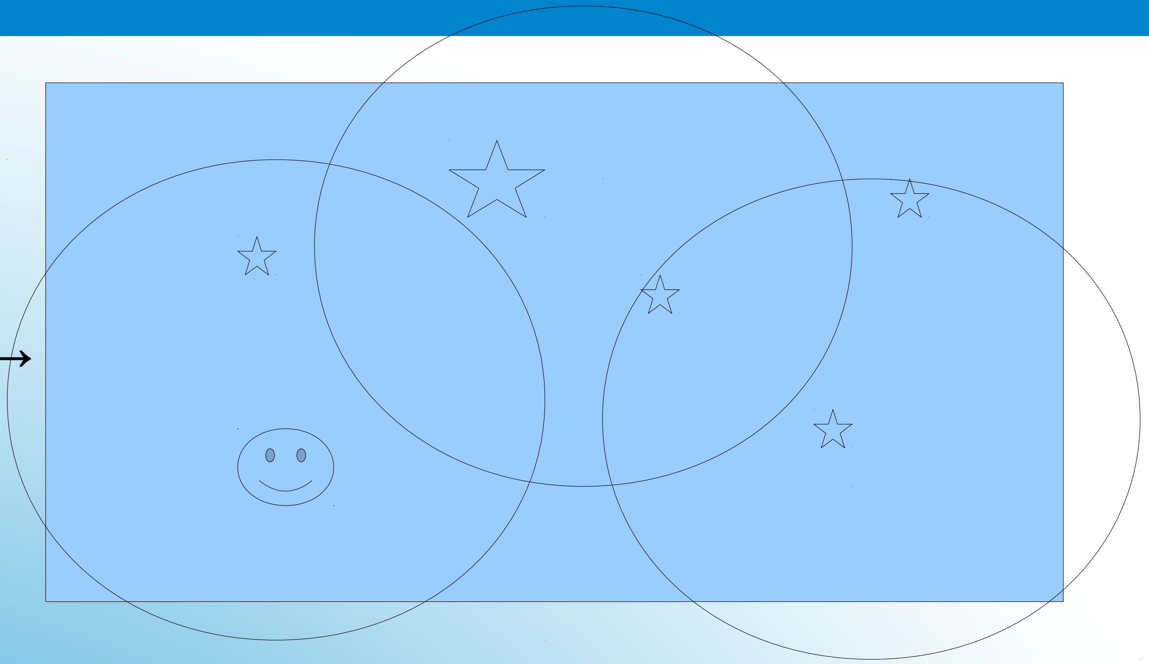


(In practice, actual galaxies look different)

Mosaicing

What if…

This is our field of interest→



- This is called mosaicing
- Should we average the 3 primary-beam-corrected images together?

Inverse-variance weighting

Primary-beam-corrected image

$$M(l,m) = \frac{\sum_i B_i^2(l,m)\left(I_i(l,m)/B_i(l,m)\right)}{\sum_i B_i^2(l,m)} = \frac{\sum_i B_i(l,m) I_i(l,m)}{\sum_i B_i^2(l,m)}$$

- This is called mosaicing
- Should we average the 3 primary-beam-corrected images together?

No → Weight with $1/\sigma^2$ = (primary beam)^2

Mosaicing

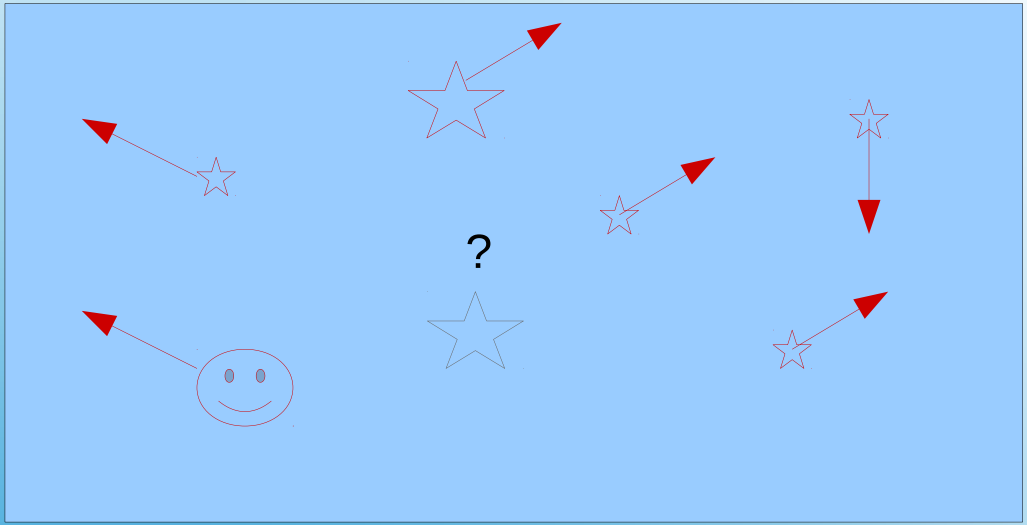
- Primary beam of tiled arrays varies in time, per station

Variable primary beam

- Primary beam of tiled arrays varies in time
  - Or even per station
- Has to be accounted for during cleaning
- Algorithm to do this is “aw-projection”
  - similar to w-projection
  - Specialized software package for LOFAR (“**AWImager**”)
- Homogeneous arrays can also use snapshot imaging

Variable primary beam

- **Direction-dependent effects** might require further correction during imaging:



- Positions of 'calibrators' (red) are known
- Apparent position has moved due to ionosphere

More variable effects...

- **Direction-dependent effects** might be time-variable (e.g. ionosphere)

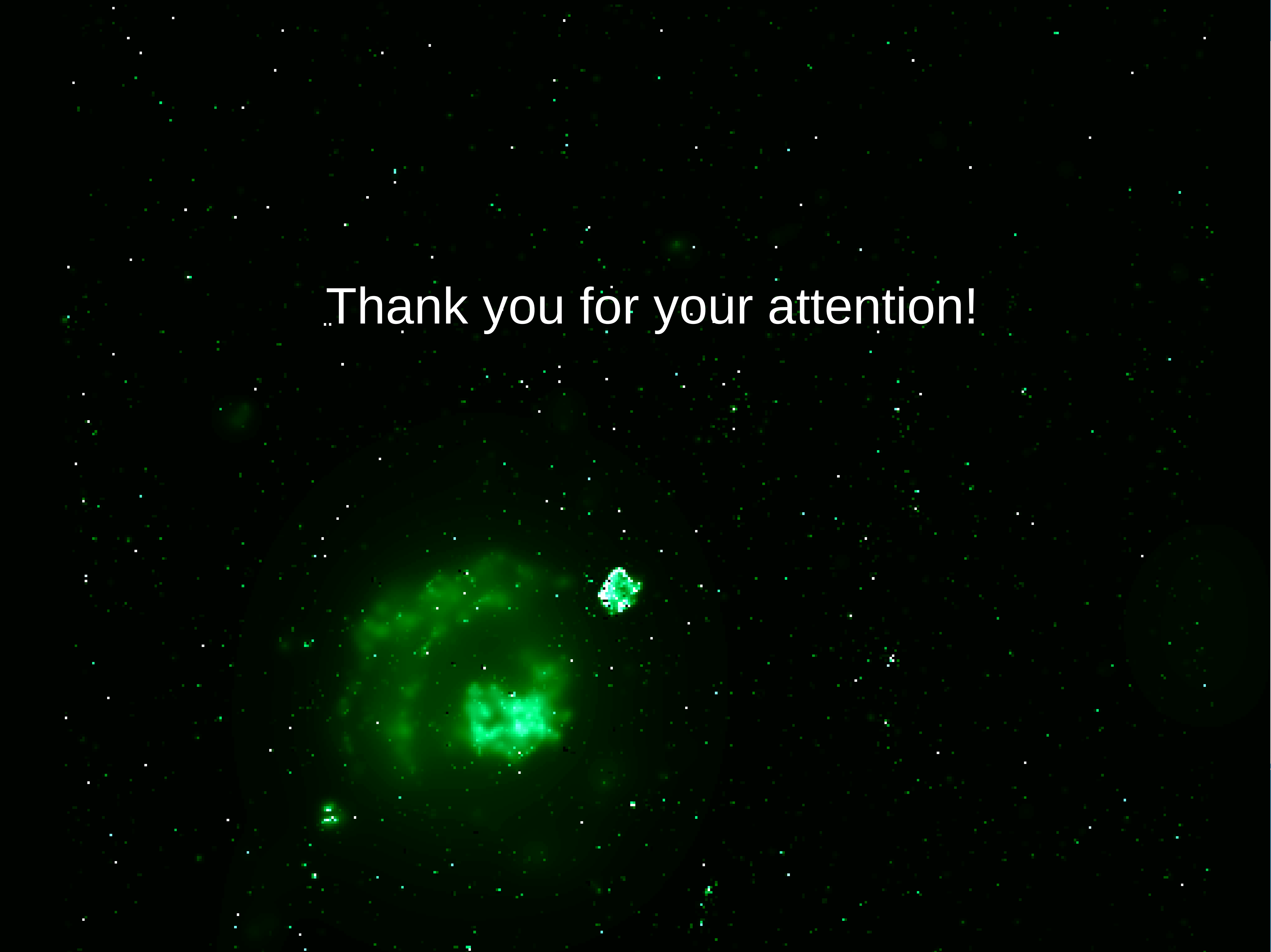
- Besides position, DD effects can also affect polarization angle and brightness
- Not a fully solved problem, but possible solutions:
  - image in small “facets” where DDE's are constant
  - or interpolation – AWImager can do this.
  - Peeling

Direction-dependent effects

Discussed topics:

- CLEAN
- When to use Multi-scale or other deconvolution methods
- The effect of and solution to w-terms
- Multi-term deconvolution
- Self-cal using CLEAN components
- Primary beam correction
- Mosaicing
- Direction-dependent effects during imaging

Summary


Thank you for your attention!