Action-angle modeling of tidal streams

Jo Bovy (IAS; Bahcall fellow)

Modeling tidal streams in action-angle coordinates: what are action-angle coordinates?

- In position-velocity space, dynamics follows from Hamilton's equations:

$$\dot{\mathbf{x}} = \mathbf{v};\ \ \dot{\mathbf{v}} = -\mathrm{d}\Phi/\mathrm{d}\mathbf{x}$$

- However, we can express dynamics in any other set of canonical coordinates, using a generating function S(**x**,**J**):

$$\theta = \frac{\partial S}{\partial \mathbf{J}},\ \ \mathbf{v} = \frac{\partial S}{\partial \mathbf{x}}$$

- Then $H \equiv H\left(\mathbf{x}, \frac{\partial S}{\partial \mathbf{x}}(\mathbf{x},\mathbf{J})\right)$ and we can solve the Hamilton-Jacobi equation for S

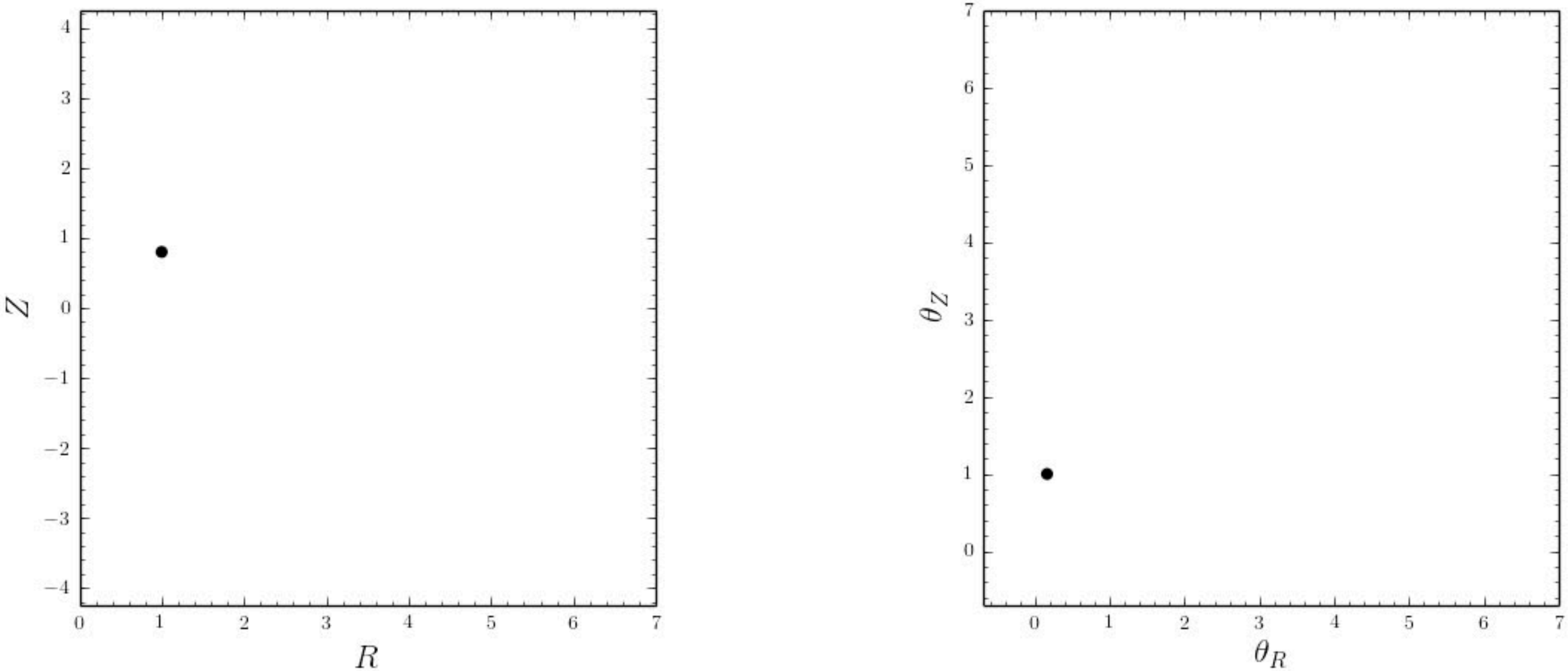
$$H\left(\mathbf{x}, \frac{\partial S}{\partial \mathbf{x}}(\mathbf{x},\mathbf{J})\right) = E$$

- As a PDE this is hard to solve and explicit solutions are rare

- Hamilton's equations for action-angle coordinates:

$$\dot{\mathbf{J}} = -\frac{\partial H}{\partial \theta} = 0;\ \ \dot{\theta} = \frac{\partial H}{\partial \mathbf{J}} = \mathbf{\Omega}(\mathbf{J}) = \mathrm{constant}$$

- Dynamics is extremely simple:
  - Actions are conserved along orbit
  - Angles increase linearly in time



4
3
2
1
0
−1
−2
−3
−4
Z
0 1 2 3 4 5 6 7
R
7
6
5
4
3
2
1
0
θ_Z
0 1 2 3 4 5 6 7
θ_R

Action-angle coordinates: a general solution to a centuries-old problem

- Can calculate 'wrong' action-angle coordinates in an auxiliary isochrone potential (θ^A,J^A)
- Define generating function $S(\boldsymbol{\theta}^A, \mathbf{J}) = \boldsymbol{\theta}^A \cdot \mathbf{J} + 2\sum_{\mathbf{n}>0} S_{\mathbf{n}}(\mathbf{J})\sin(\mathbf{n}\cdot\boldsymbol{\theta}^A)\,,$
- leads to canonical transformation

$$\mathbf{J}^A = \frac{\partial S(\boldsymbol{\theta}^A, \mathbf{J})}{\partial \boldsymbol{\theta}^A} = \mathbf{J} + 2\sum_{\mathbf{n}>0} \mathbf{n}\, S_{\mathbf{n}}(\mathbf{J})\cos(\mathbf{n}\cdot\boldsymbol{\theta}^A)\,,$$

$$\boldsymbol{\theta} = \frac{\partial S(\boldsymbol{\theta}^A, \mathbf{J})}{\partial \mathbf{J}} = \boldsymbol{\theta}^A + 2\sum_{\mathbf{n}>0} \frac{\partial S_{\mathbf{n}}(\mathbf{J})}{\partial \mathbf{J}}\sin(\mathbf{n}\cdot\boldsymbol{\theta}^A)\,.$$

- We can average over the first equation along an orbit

$$\int \mathrm{d}\theta_i^A J_i^A = \int \mathrm{d}\theta_i^A J_i + 2\sum_{\mathbf{n}>0} \mathbf{n}\, S_{\mathbf{n}}(J_i) \int \mathrm{d}\theta_i^A \cos(\mathbf{n}\cdot\boldsymbol{\theta}^A)\,,$$

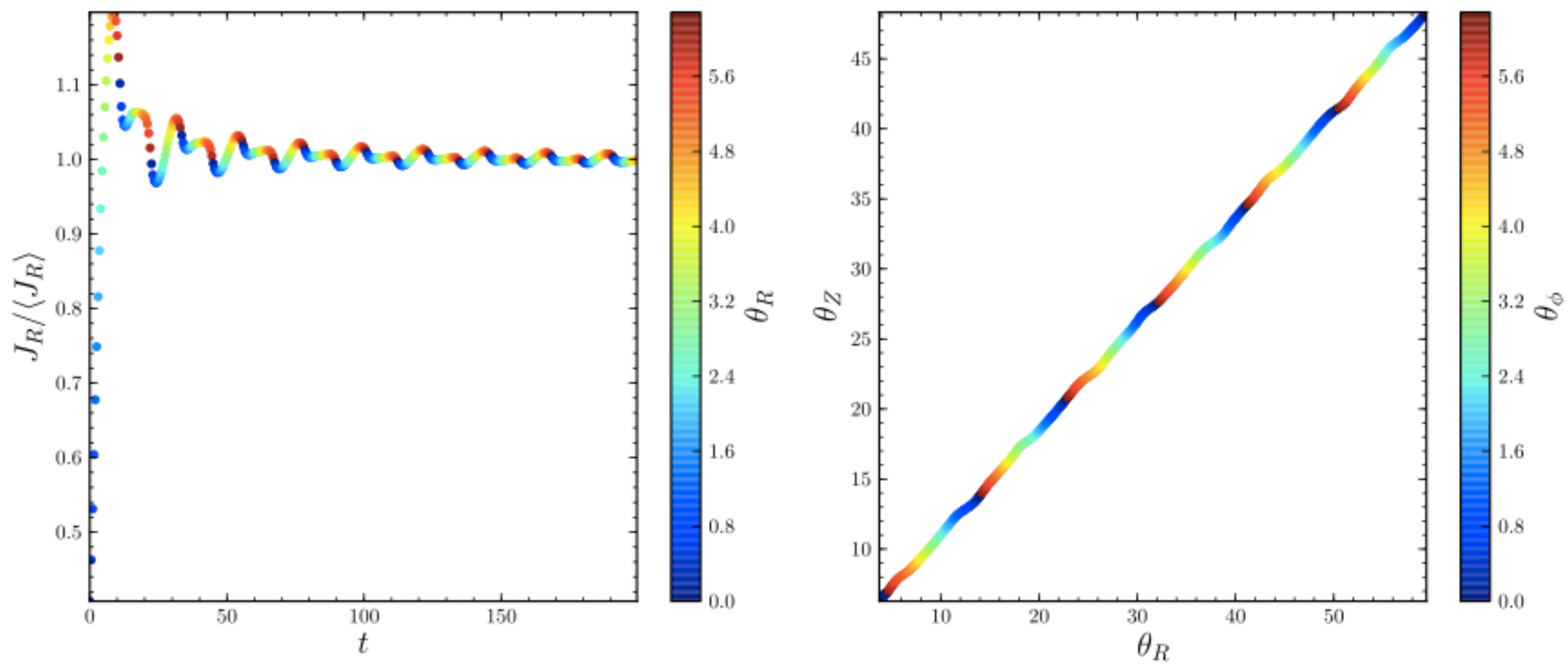
- and simplify to

$$J_i = \frac{\int \mathrm{d}\theta_i^A J_i^A}{\int \mathrm{d}\theta_i^A}\,,$$

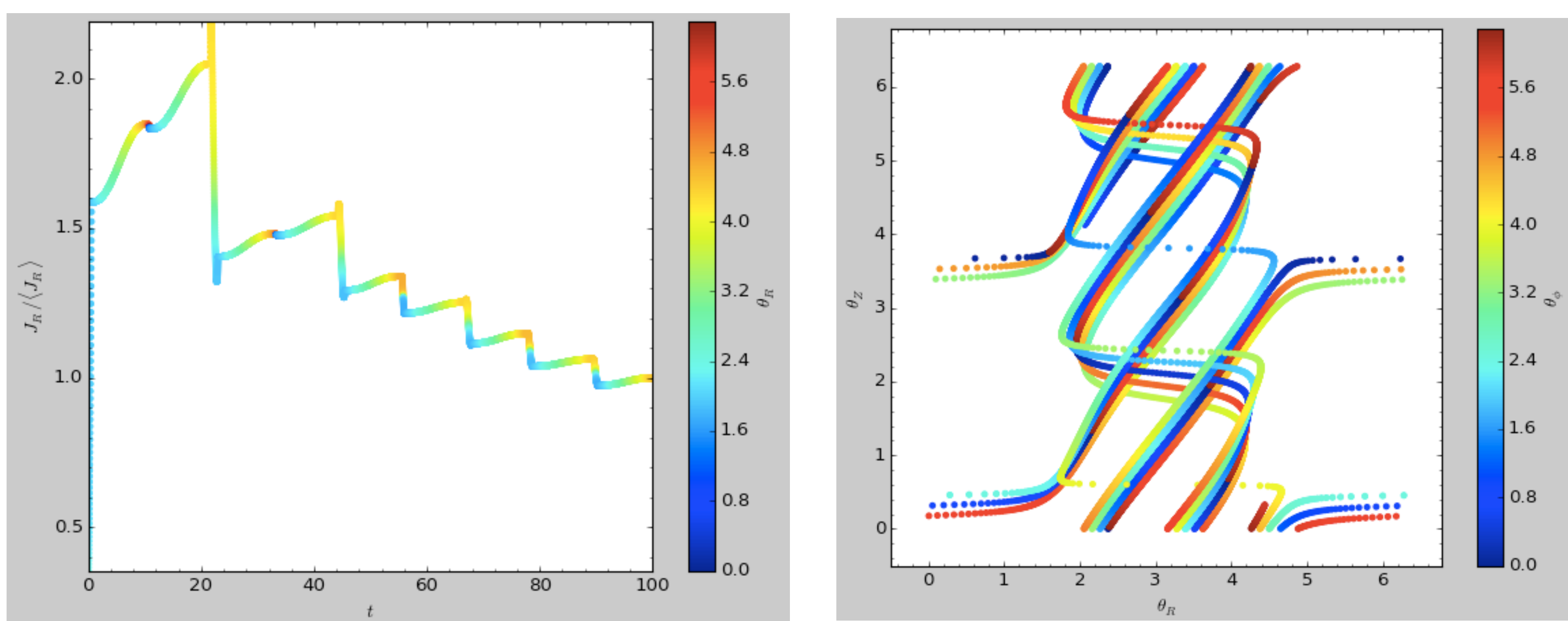
Bovy (2014), see also Sanders & Binney (2014)

- For actions and frequencies write $\boldsymbol{\theta} = \boldsymbol{\theta}(t=0) + \boldsymbol{\Omega}(\mathbf{J})\,t = \boldsymbol{\theta}^A + 2\sum_{\mathbf{n}>0} \frac{\partial S_{\mathbf{n}}(\mathbf{J})}{\partial \mathbf{J}}\sin(\mathbf{n}\cdot\boldsymbol{\theta}^A)$
- and then fit for $\theta(t=0)$, Ω, and all of the d $\mathbf{Sn(J)}$/d $\mathbf{J}$ (linear fit)

Action-angle coordinates: a general solution

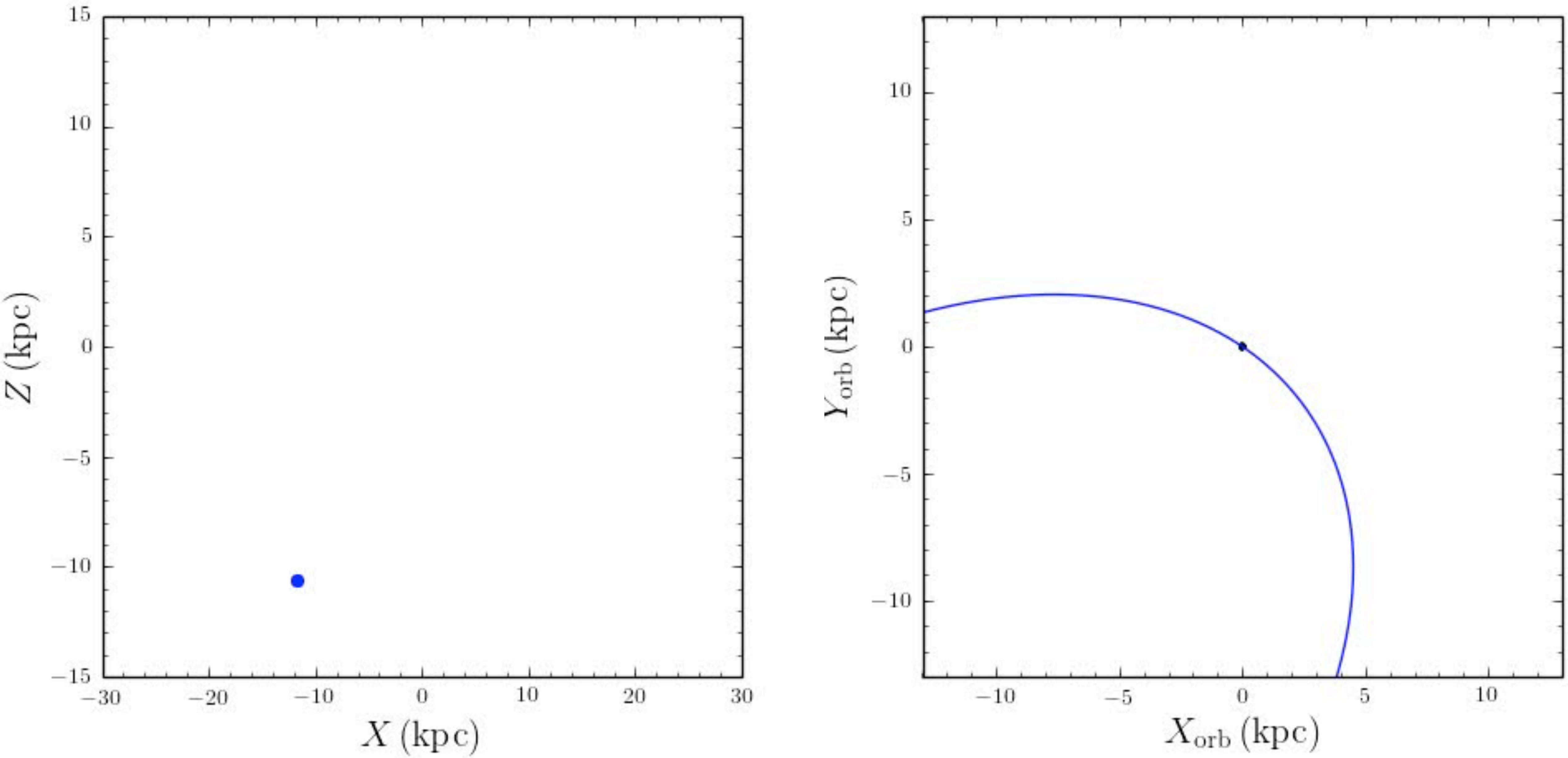


"bad" auxiliary potential

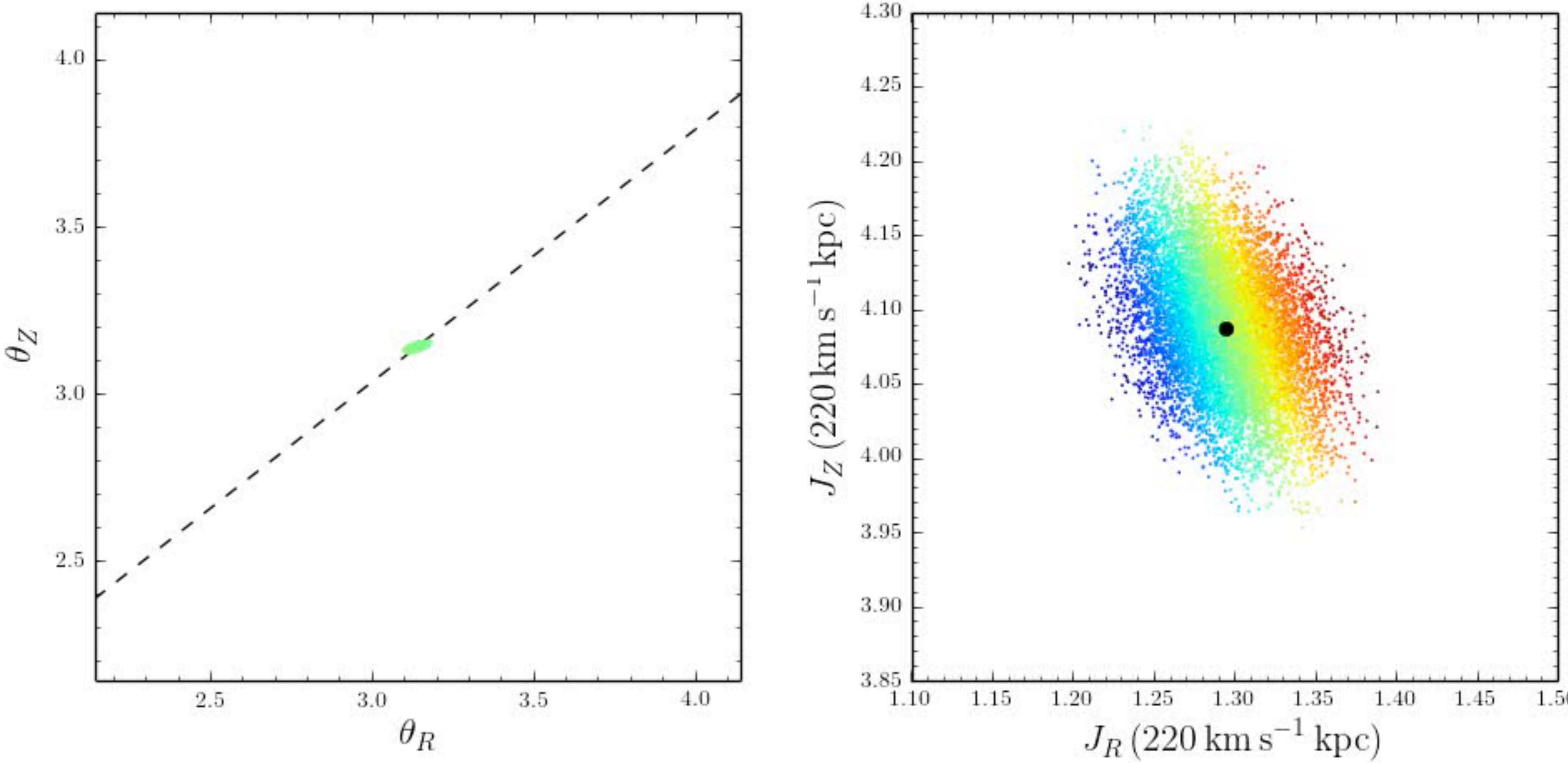


Bovy (2014)

stream modeling

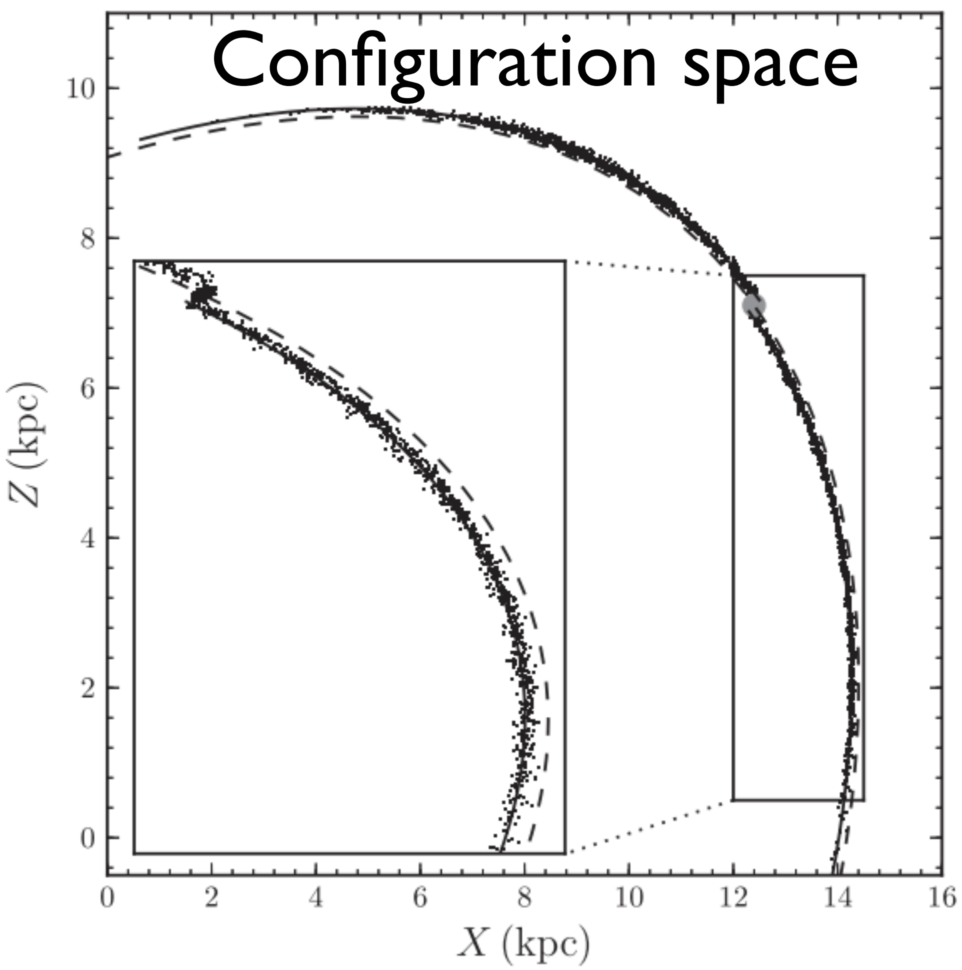


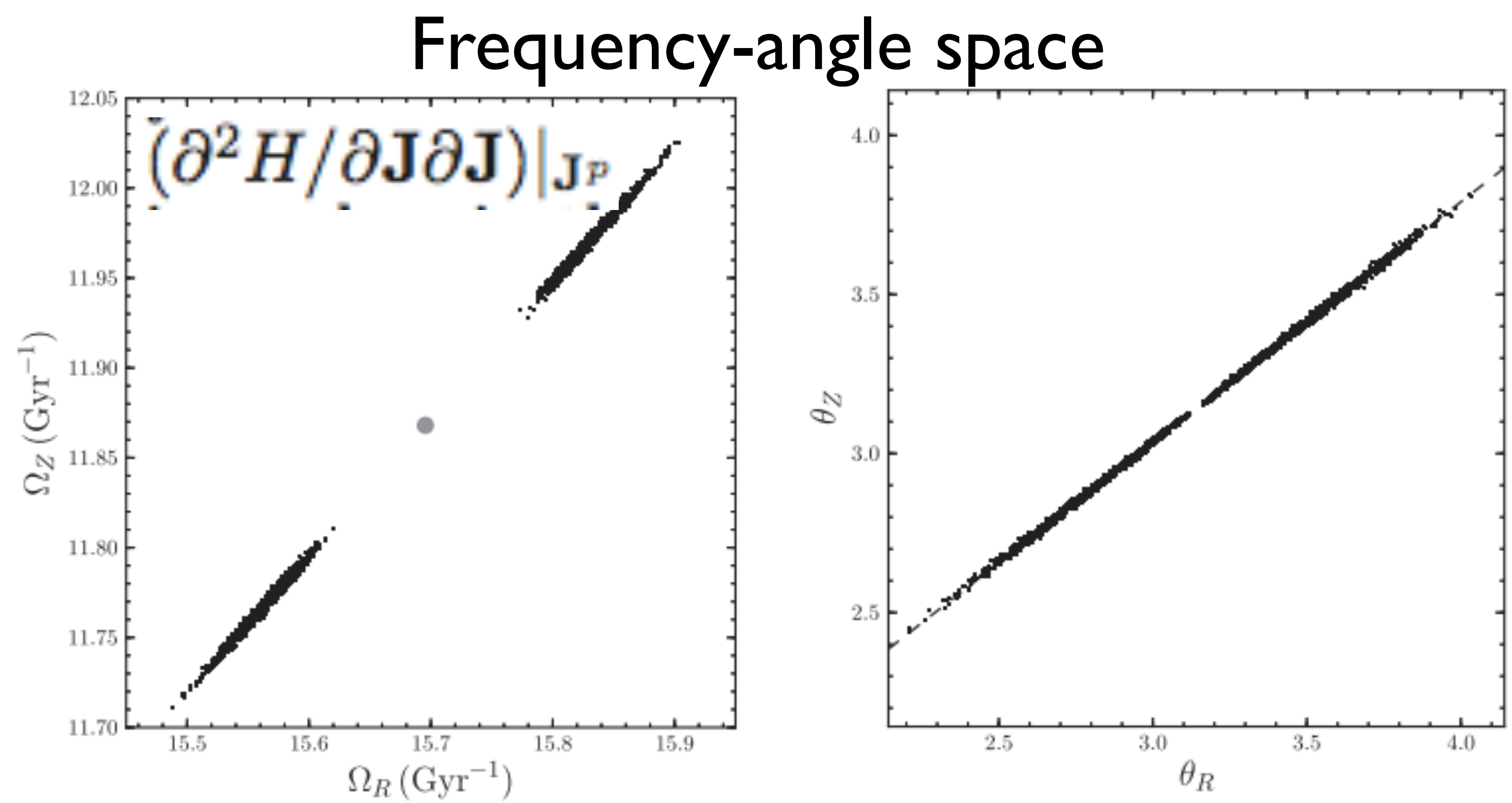
Z (kpc)
X (kpc)
15
10
5
0
−5
−10
−15
−30
−20
−10
0
10
20
30
Y_orb (kpc)
X_orb (kpc)
10
5
0
−5
−10
−10
−5
0
5
10



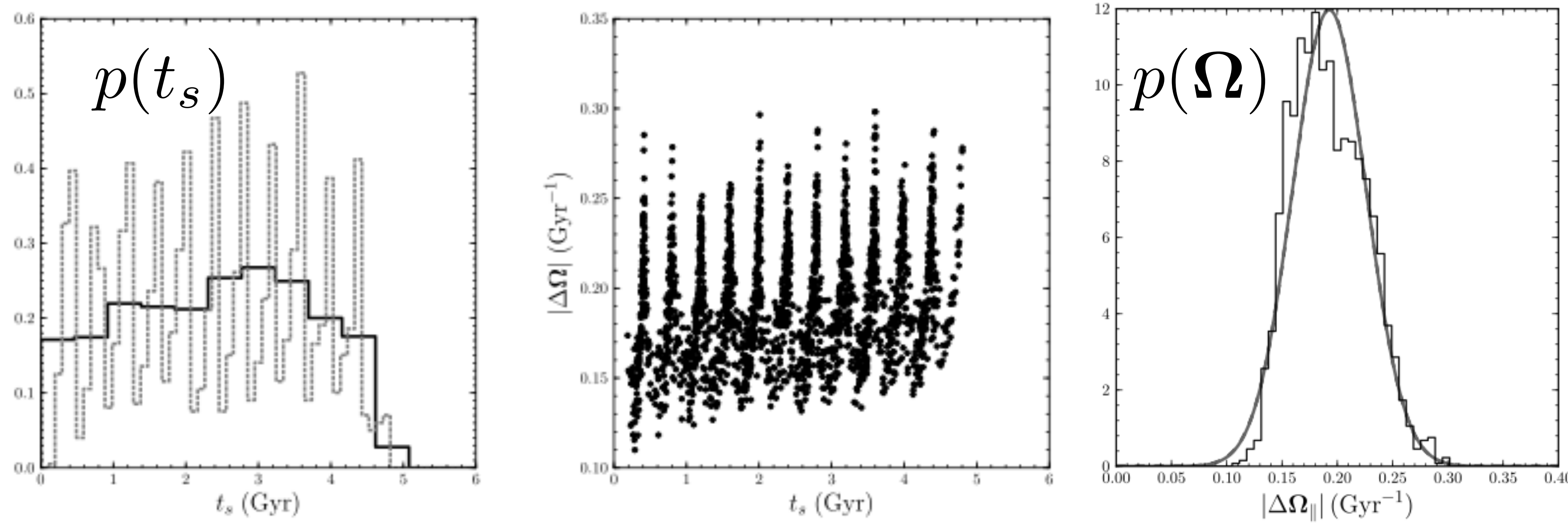
θ_Z
θ_R
2.5
3.0
3.5
4.0
$J_Z\,(220\,\mathrm{km\,s^{-1}\,kpc})$
$J_R\,(220\,\mathrm{km\,s^{-1}\,kpc})$
3.85
3.90
3.95
4.00
4.05
4.10
4.15
4.20
4.25
4.30
1.10
1.15
1.20
1.25
1.30
1.35
1.40
1.45
1.50

Streams in action-angle coordinates





Simple stream model



$$p(\mathbf{x}, \mathbf{v}, t_s) \left|\frac{\partial \mathbf{J}}{\partial \mathbf{\Omega}}\right| = p(\theta, \mathbf{\Omega}, t_s)$$

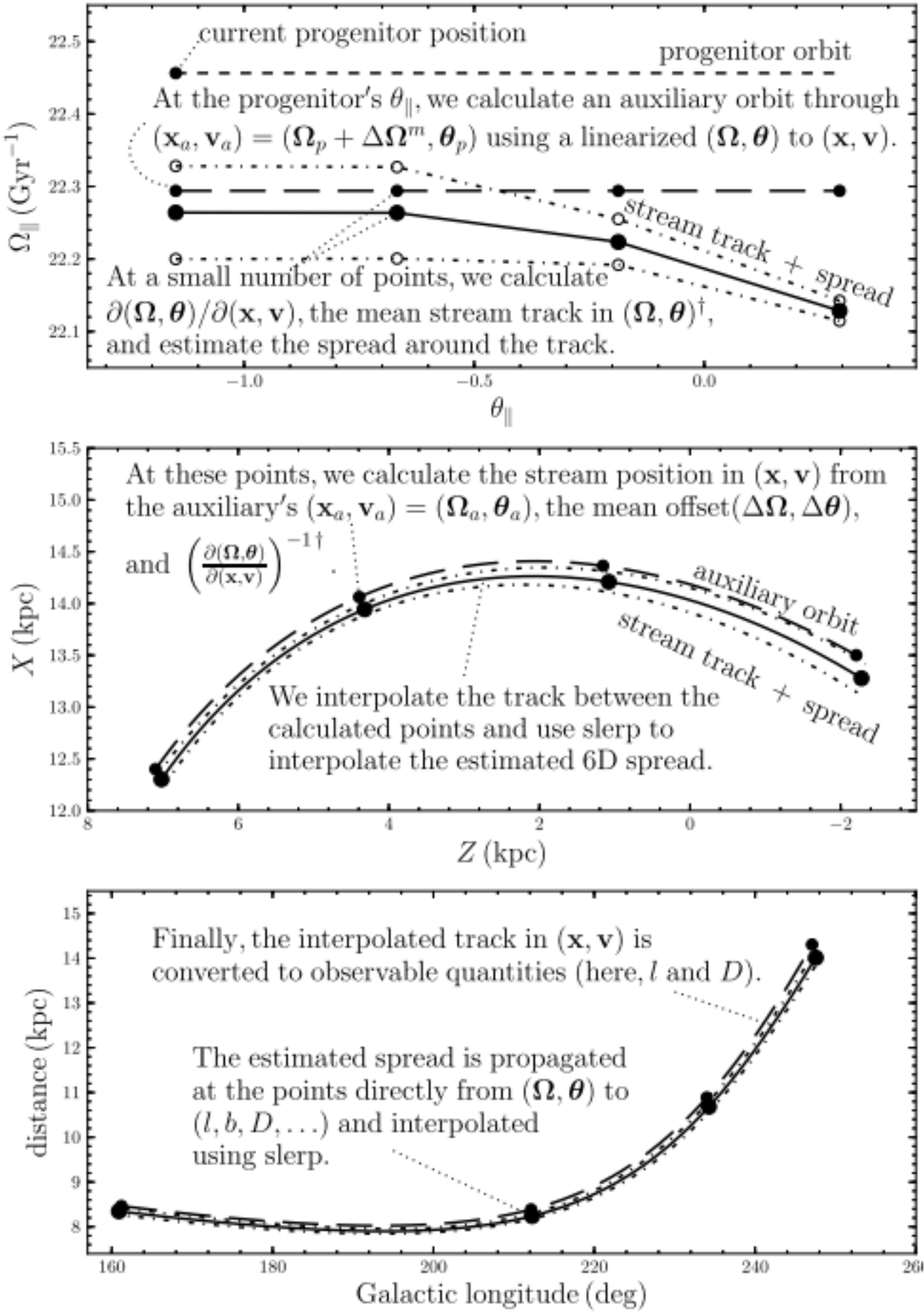
$$= p(\mathbf{\Omega}|\theta, t_s)\, p(\theta|t_s)\, p(t_s)$$

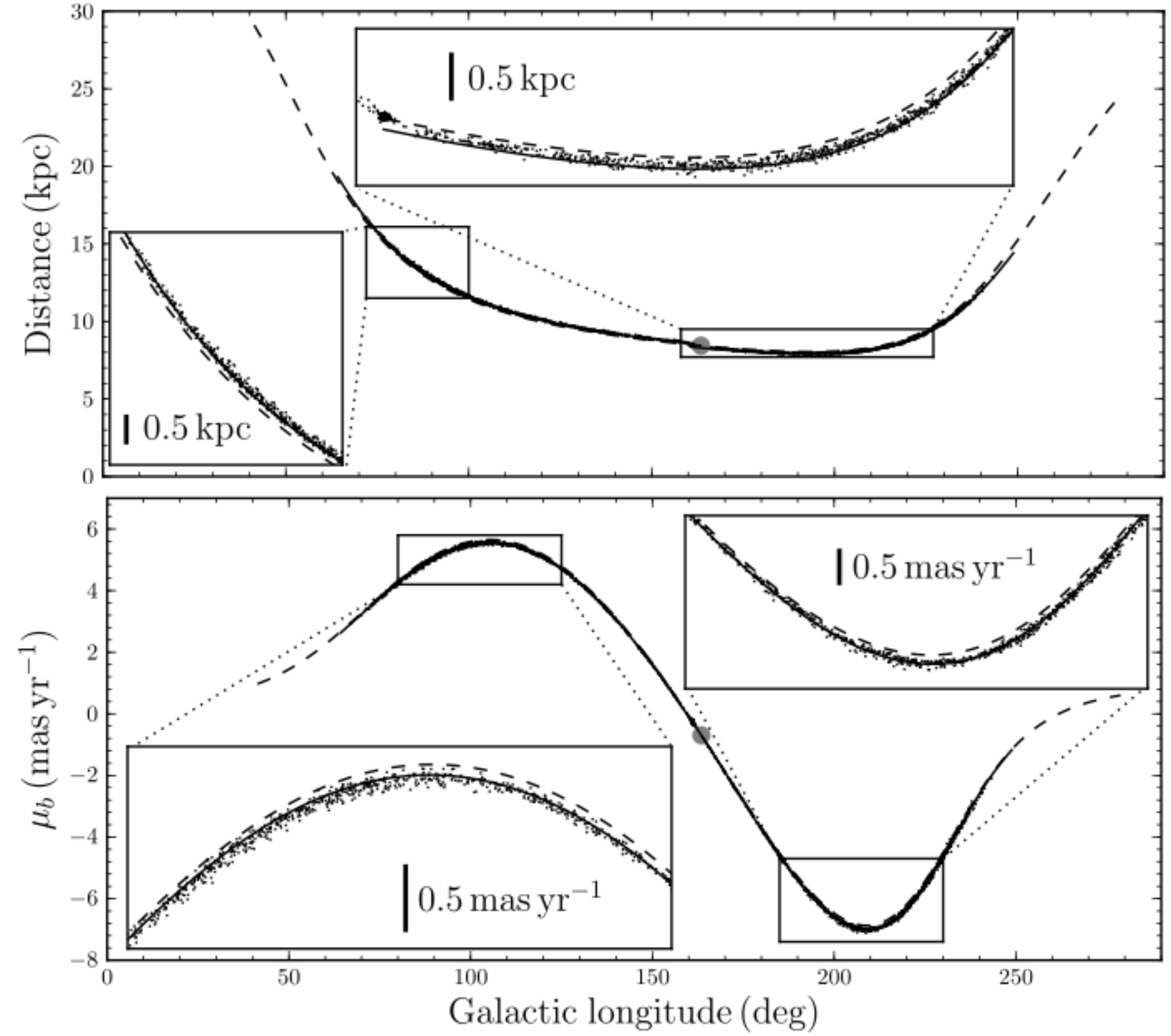
$$= p(\mathbf{\Omega})\, p(\theta - \mathbf{\Omega}\, t_s)\, p(t_s)$$

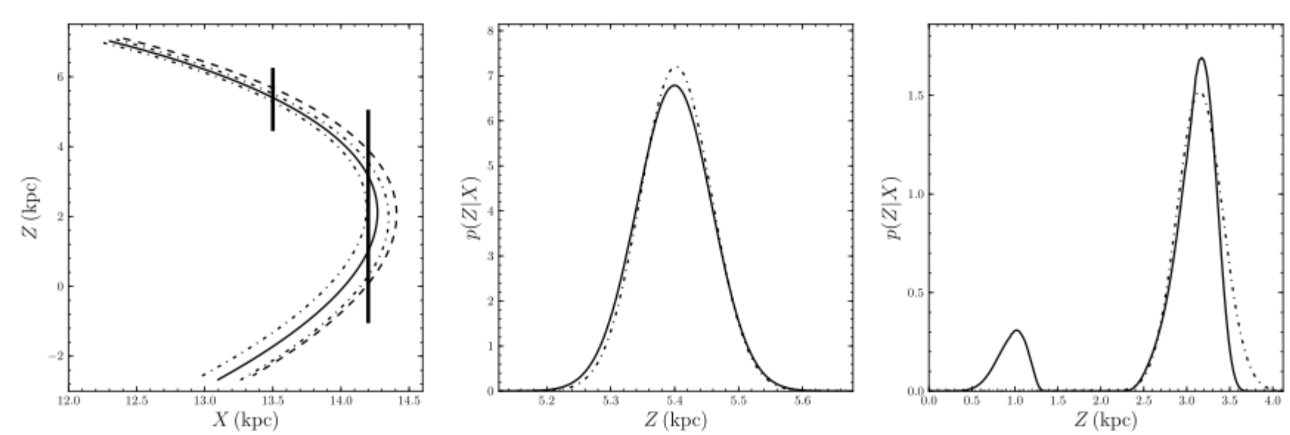
- Constant distribution of stripping times
- Mean offset in frequency from progenitor with small spread
- small initial angle spread

Bovy (2014)

Transforming the stream model to configuration space







galpy: A Python Library for Galactic Dynamics

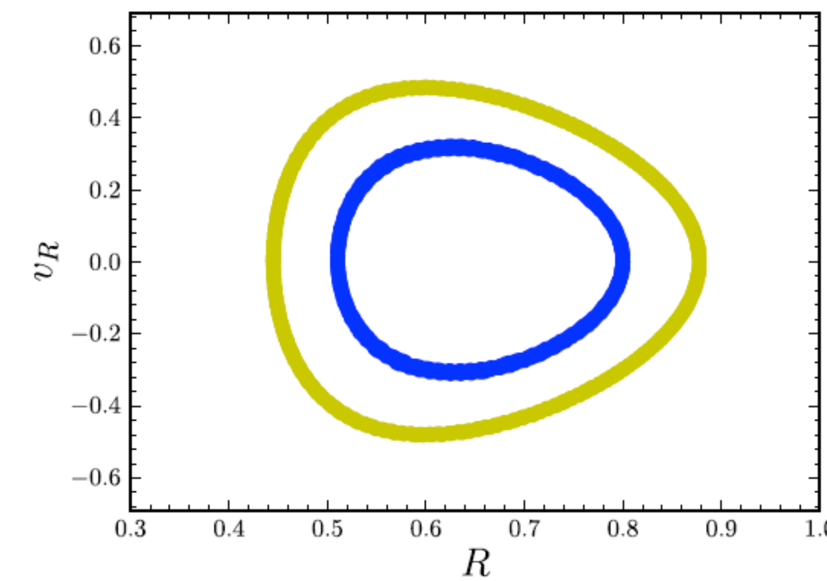
Jo Bovy (IAS)   https://github.com/jobovy/galpy

Galactic Dynamics in python

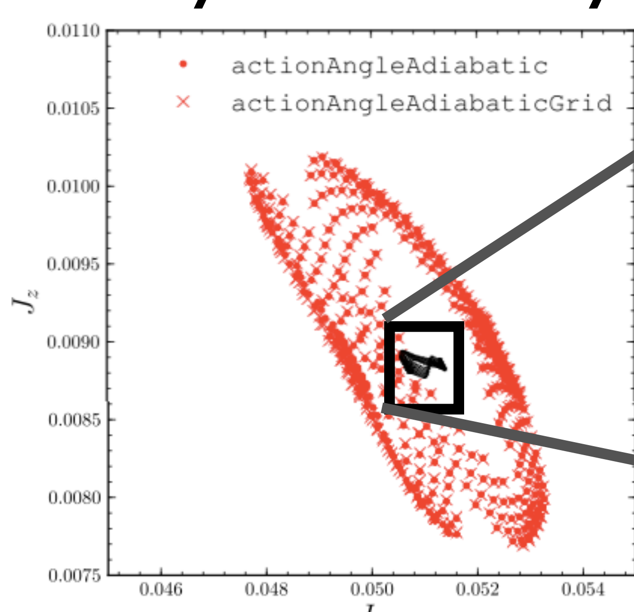
build passing | coverage 100% | C coverage 99% | docs latest | pypi v1.0 | license New BSD

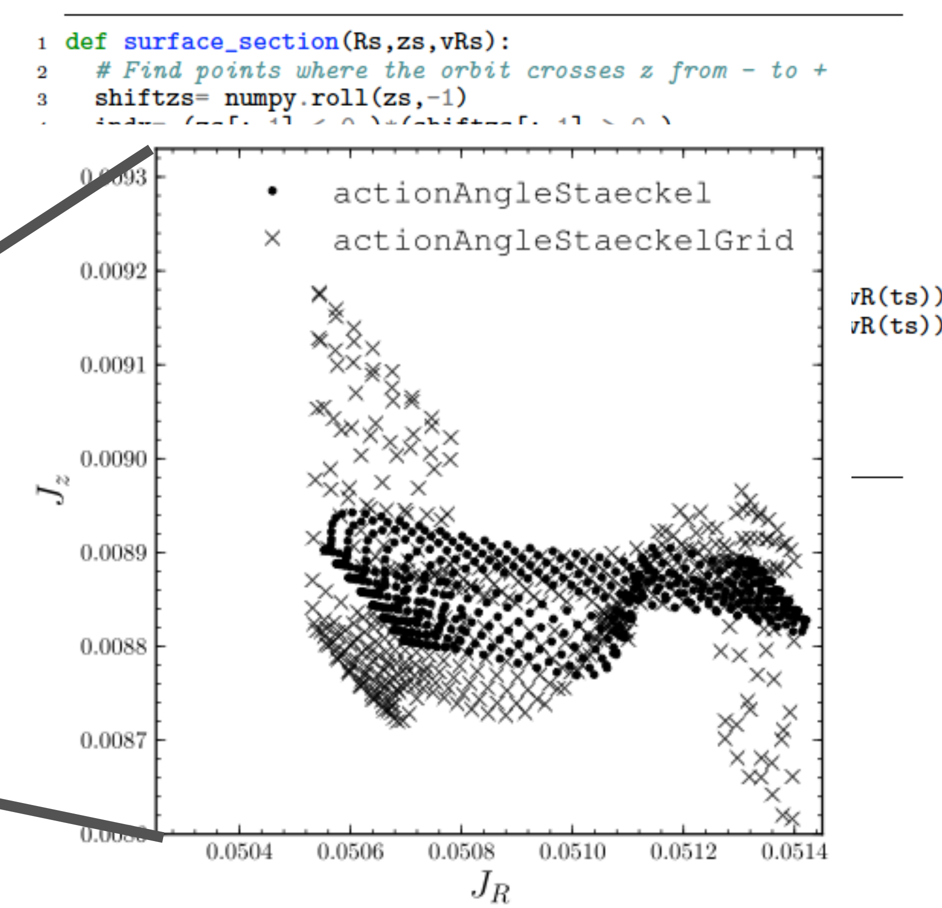
See Bovy (2015, ApJS) and online documentation

- *galpy*: general-purpose Galactic dynamics package; 23,000 lines + 11,000 lines of test code + 20,000 lines of documentation; test coverage of 99.6%

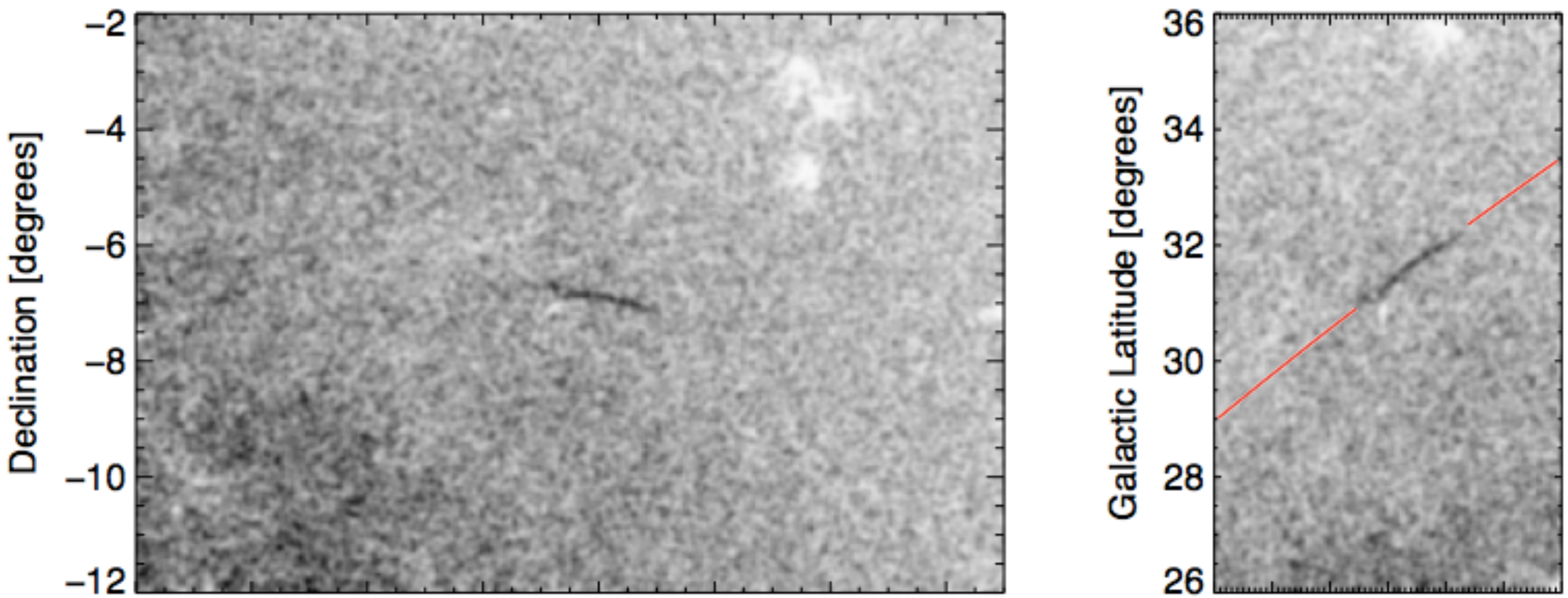


- Large variety of potentials, incl. a MW potential (`galpy.potential.MWPotential2014`)
- Fast orbit integration in variety of potentials, steady-state kinematics of disk galaxies (e.g., asymmetric drift), non-axisymmetric dynamics, all sorts of action-angle coordinates, this talk's stream model, and much more



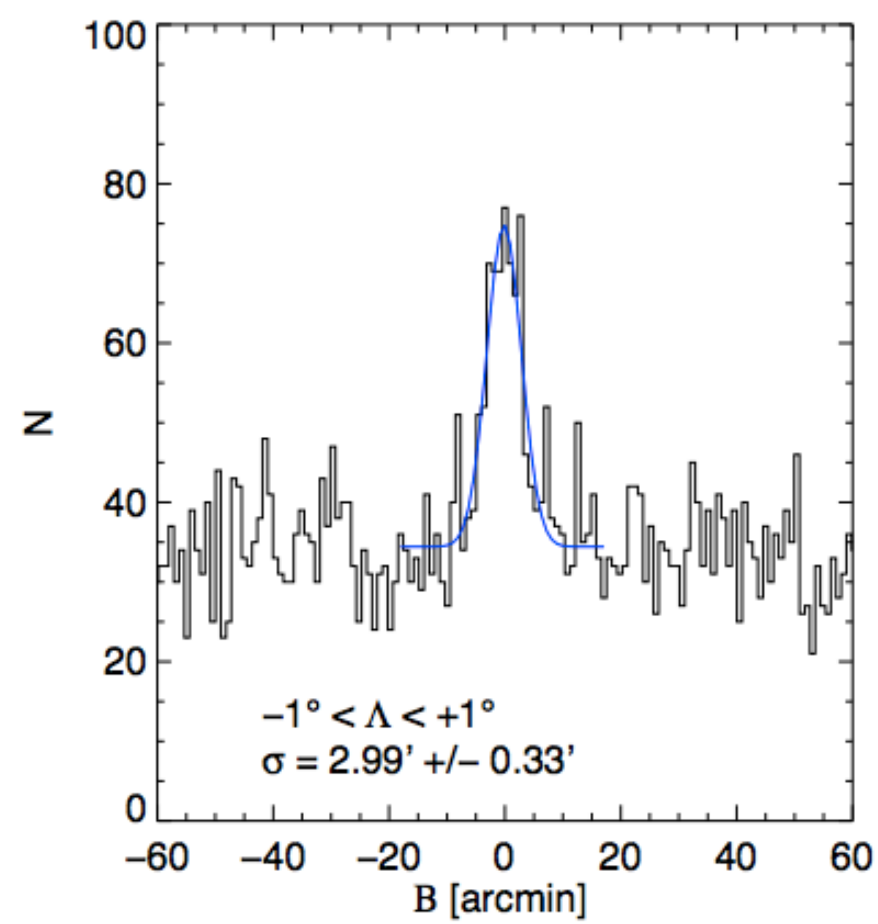


Application: orbit of the Ophiuchus stream

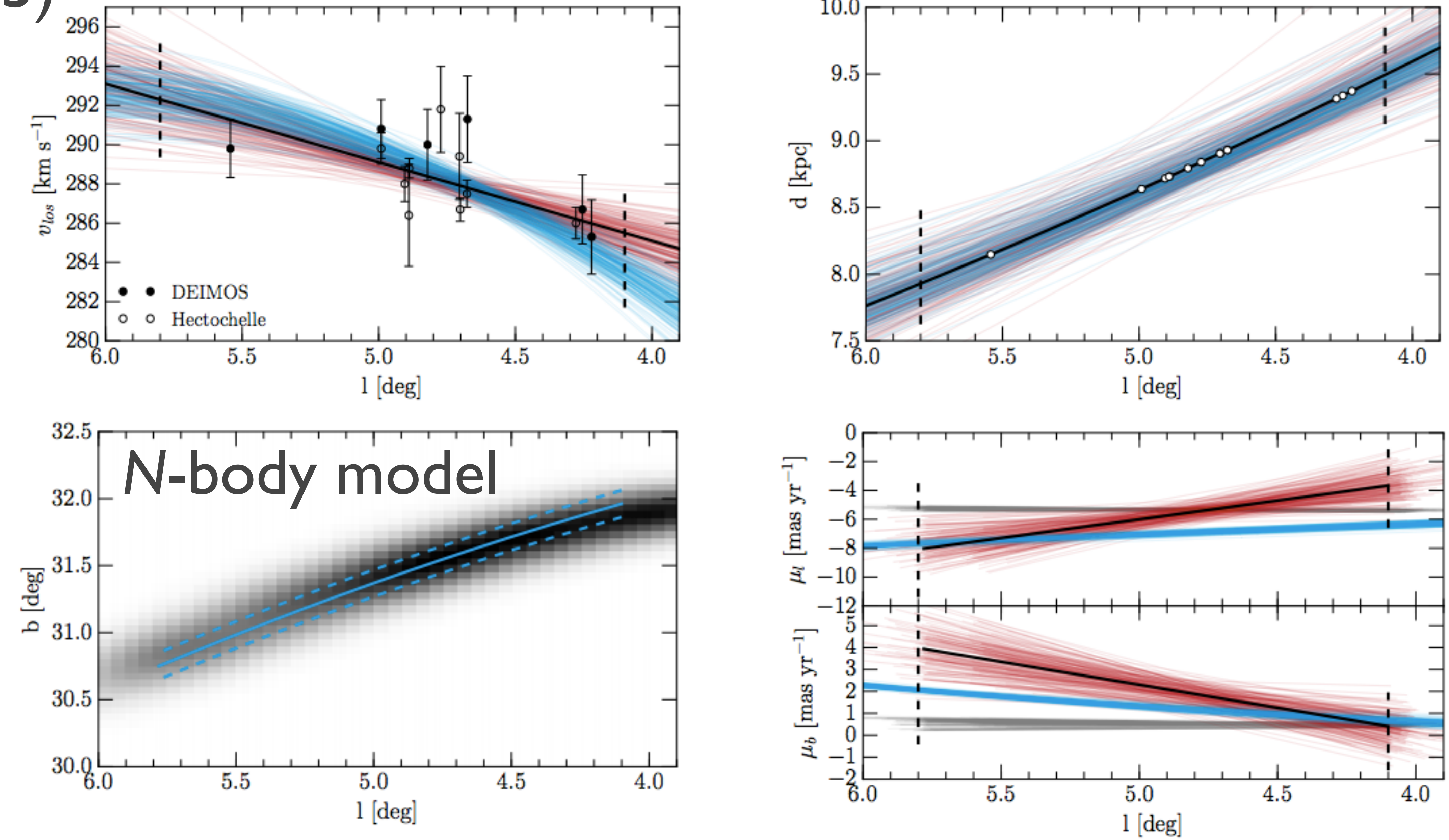


Bernard et al. (2014)

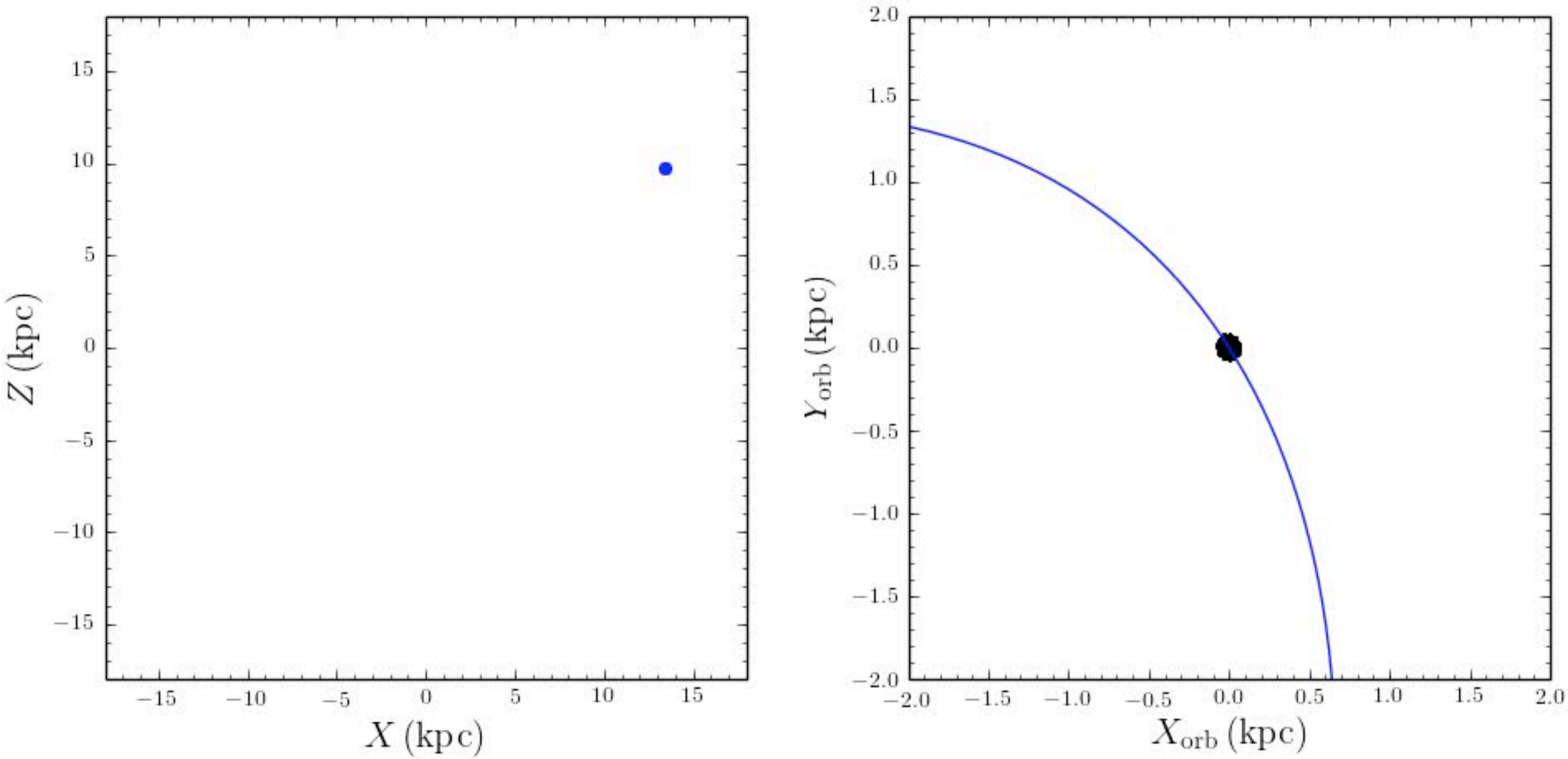
- Thin, very short stream discovered in Pan-STARRS imaging (Bernard et al. 2014)
- ~9 kpc away, almost exactly above the Galactic center
- Now have good measurements of proper motions along the stream and line-of-sight velocities of members



Application: orbit of the Ophiuchus stream (Sesar, Bovy, et al. 2015)

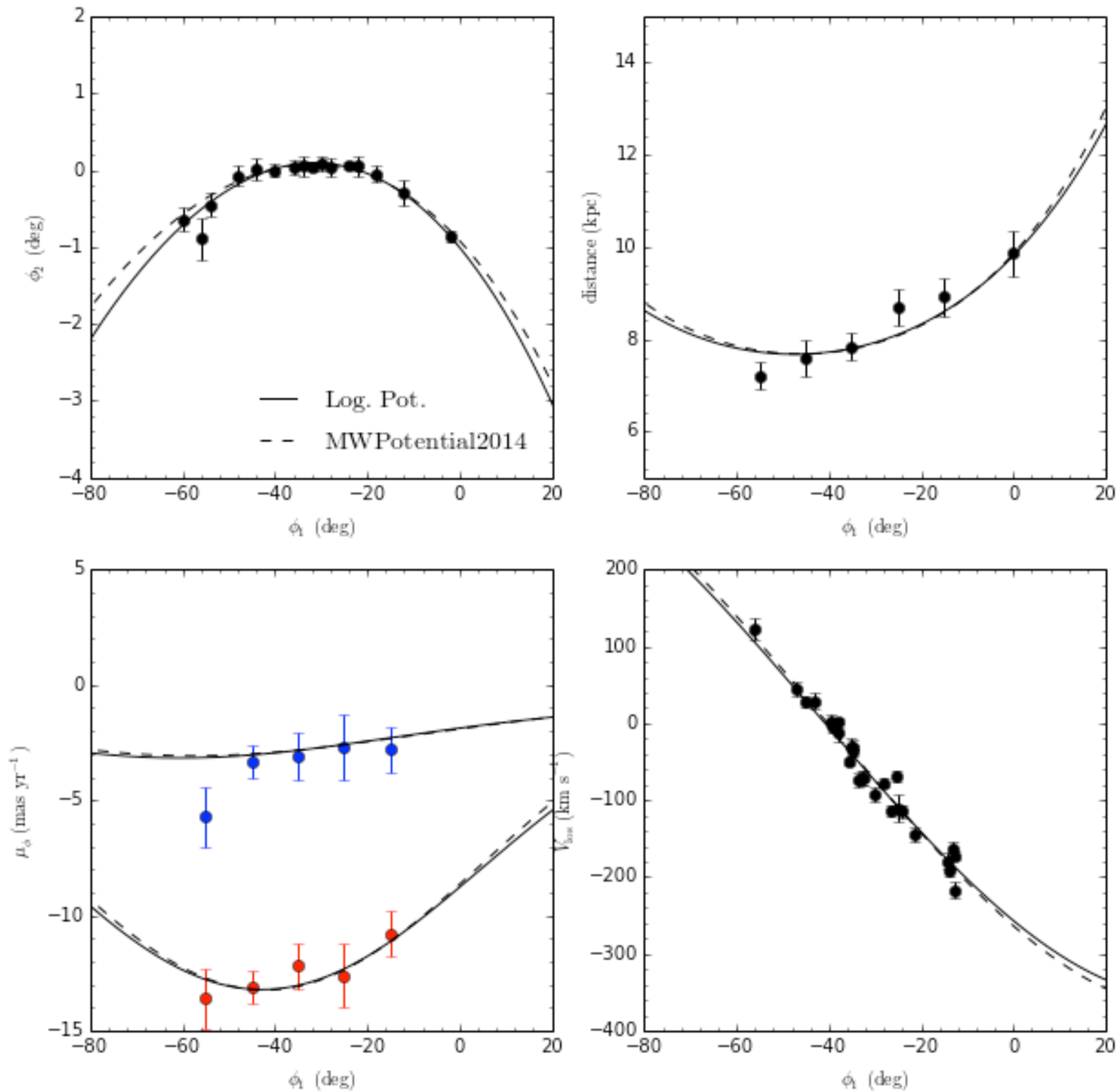


- Orbit fit in MW-like potential (MWPotential2014 in *galpy*)
- Eccentric orbit (e~0.65) near pericenter
- Stream model can reproduce these features, but only if $t_{disrupt} \lesssim$ 0.5 Gyr, perhaps due to interaction with massive satellite; or triaxiality

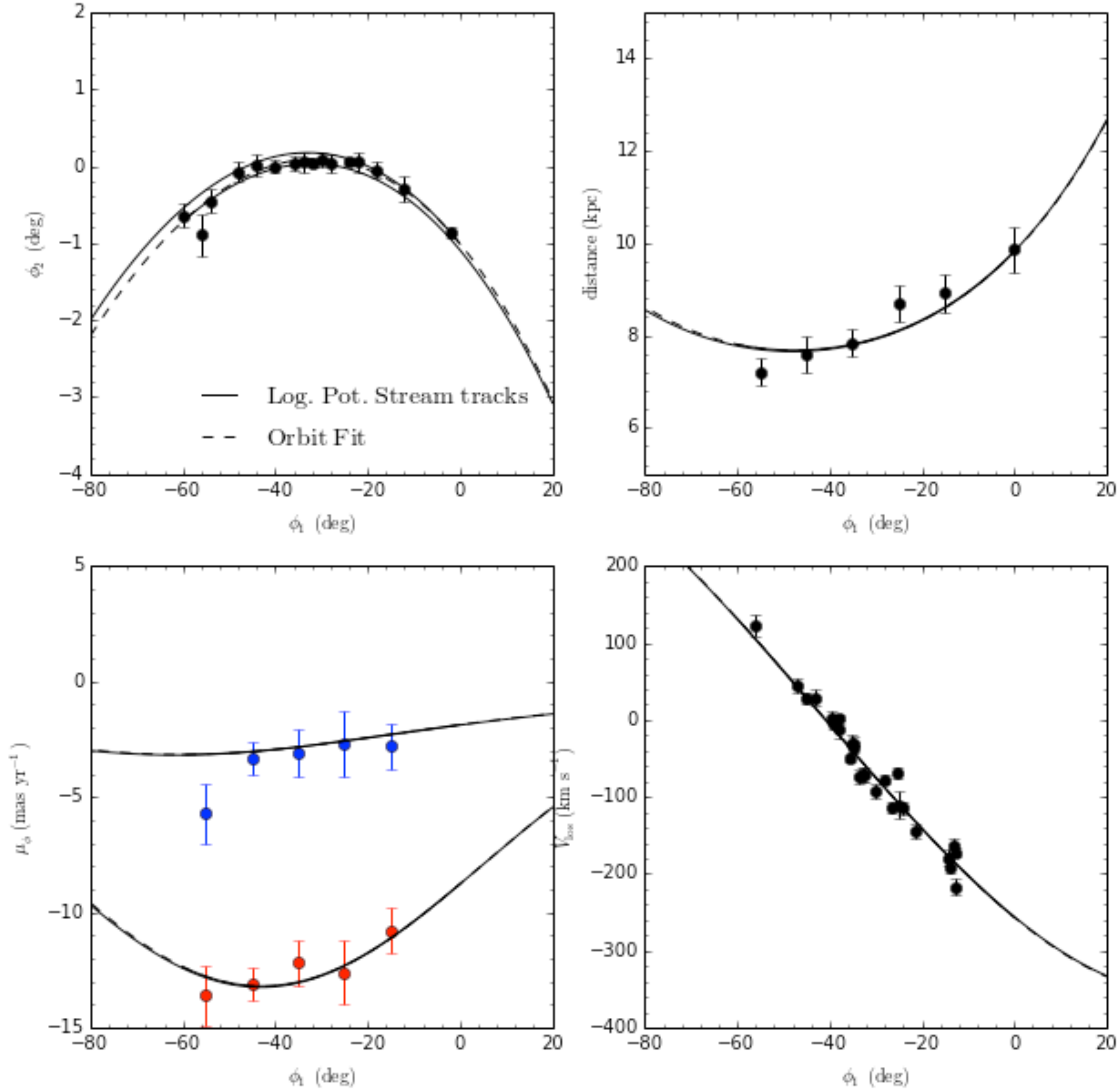


15
10
5
0
−5
−10
−15
Z (kpc)
−15
−10
−5
0
5
10
15
X (kpc)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Y_orb (kpc)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X_orb (kpc)

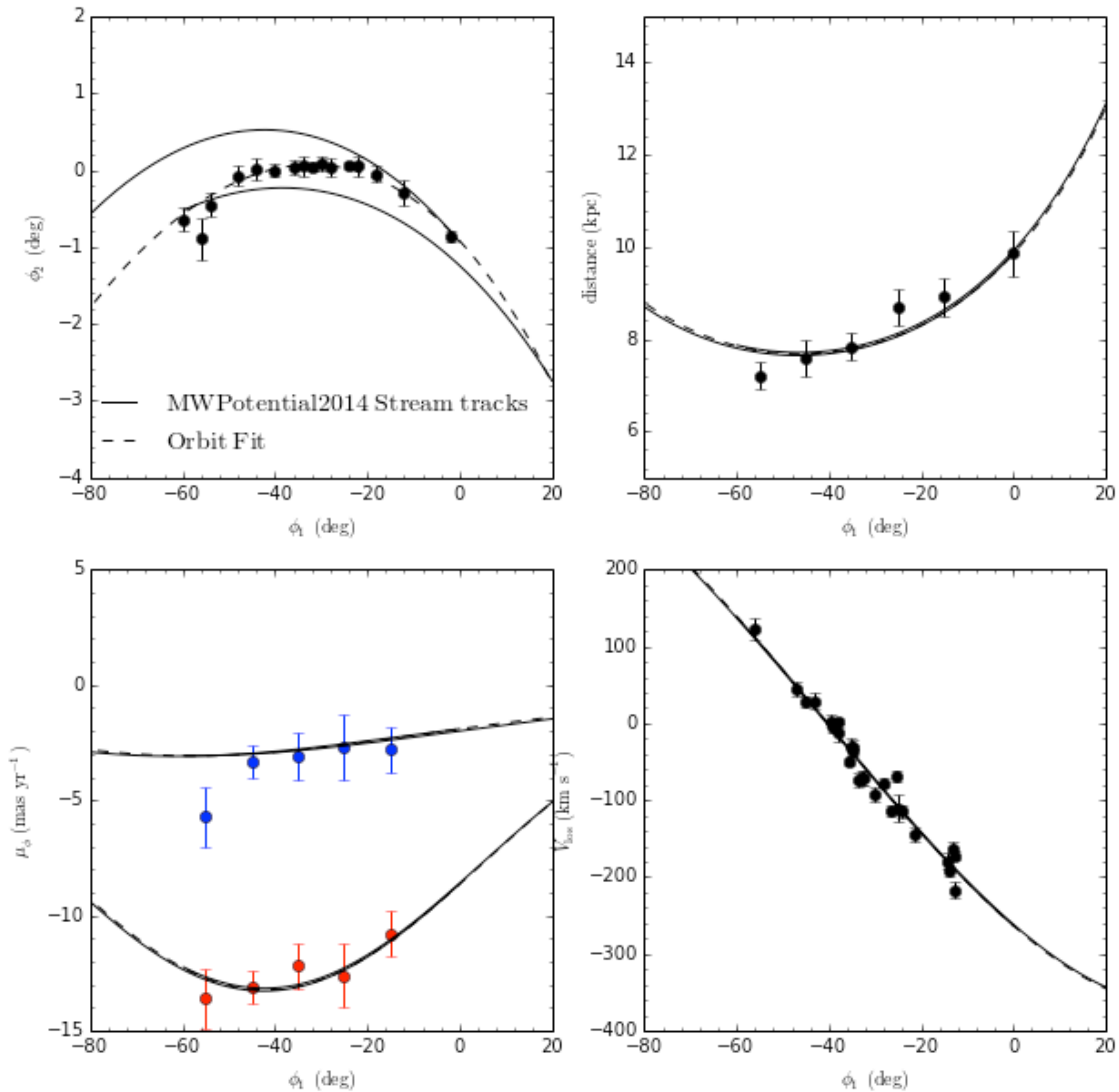
Application: GD1



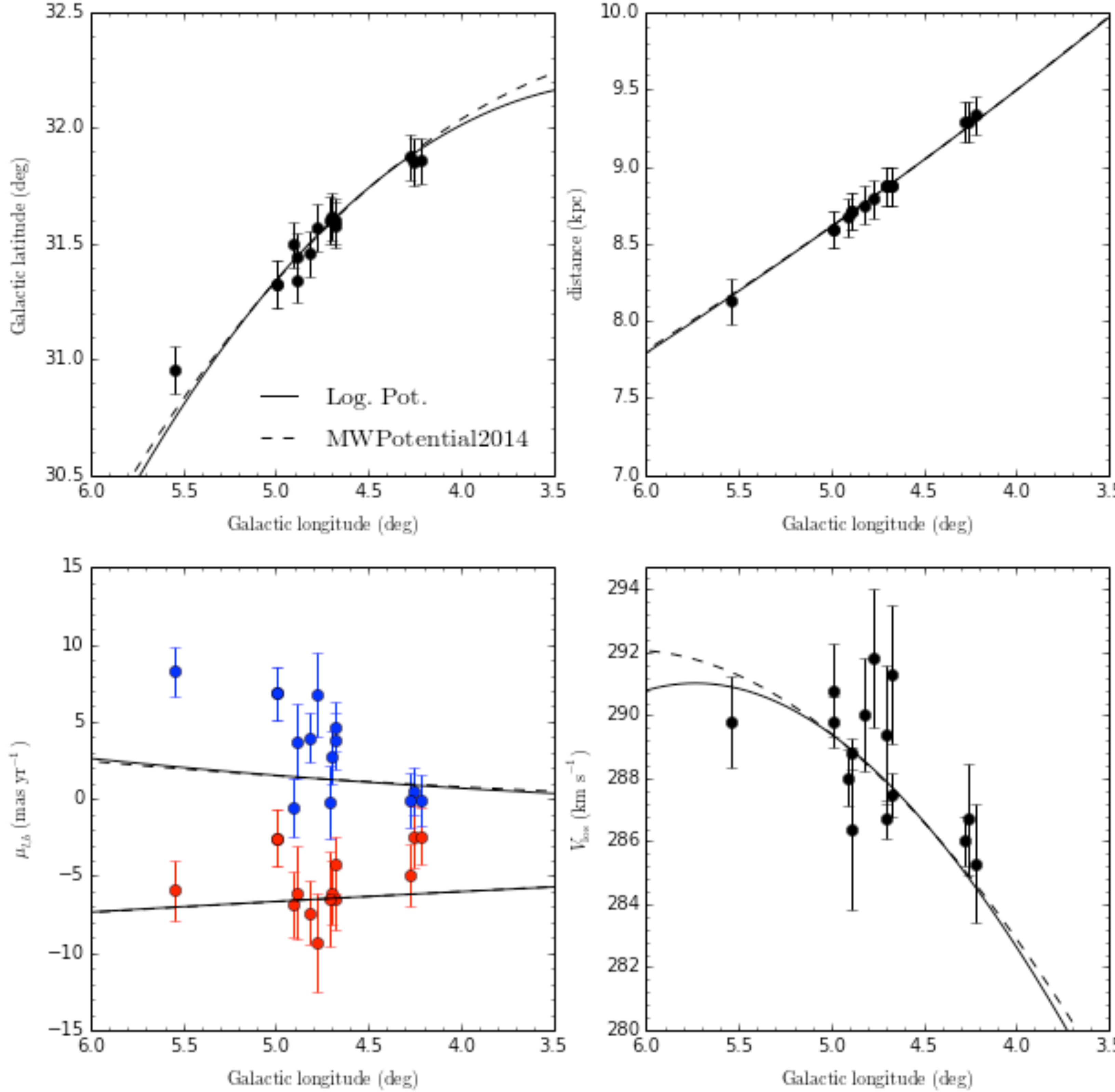
Application: GD1



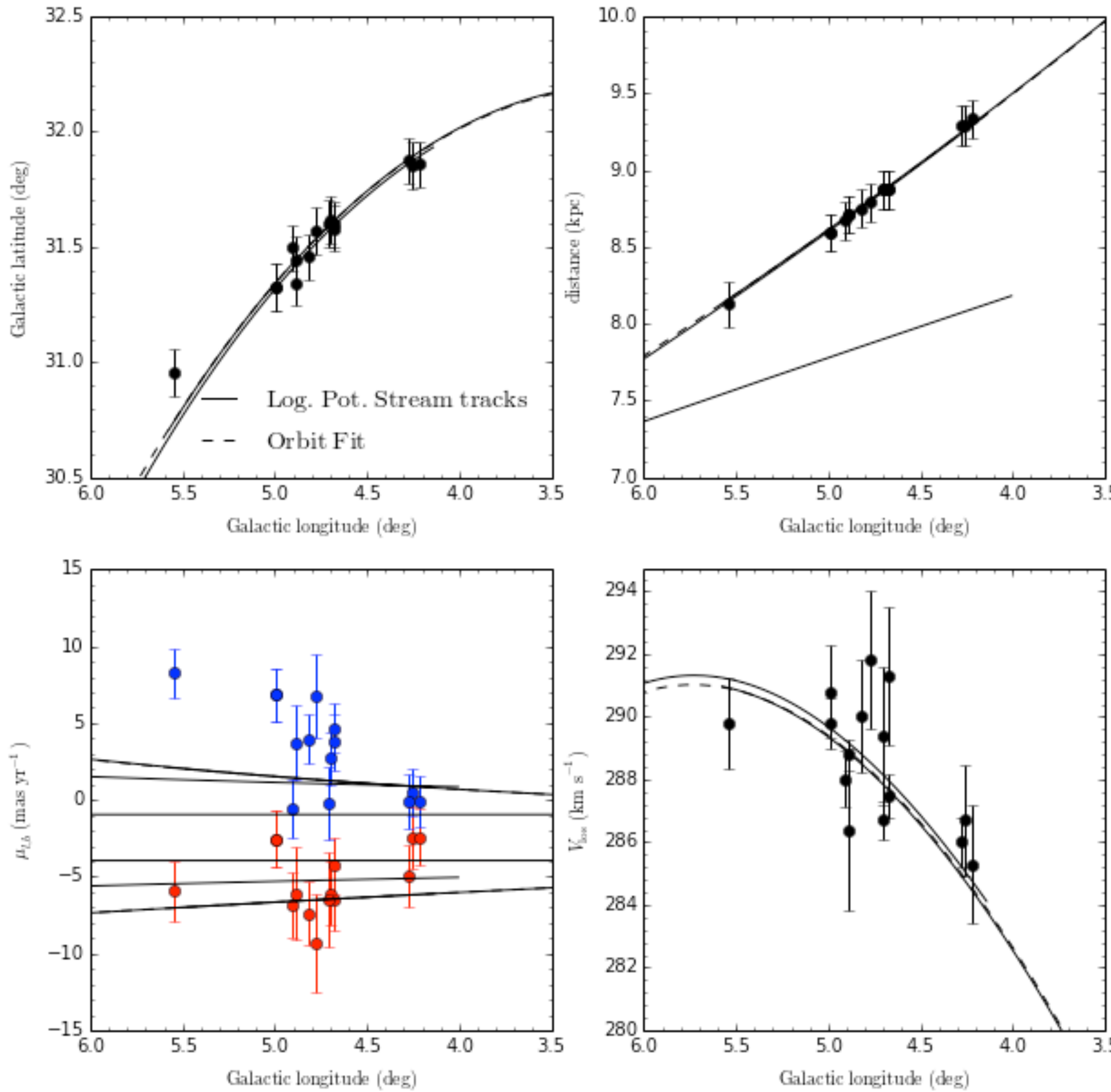
Application: GD1



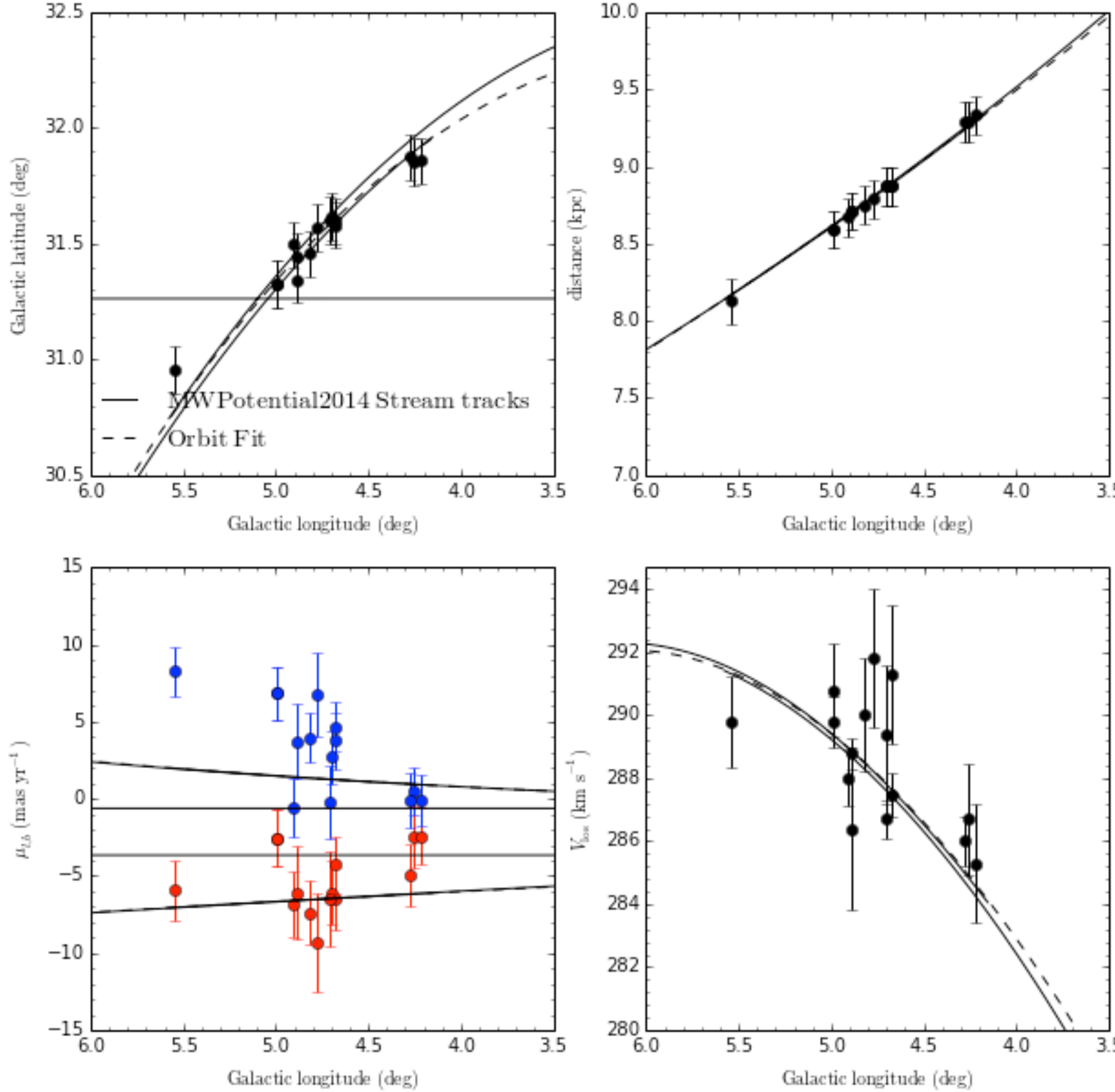
Application: Same for Ophiuchus



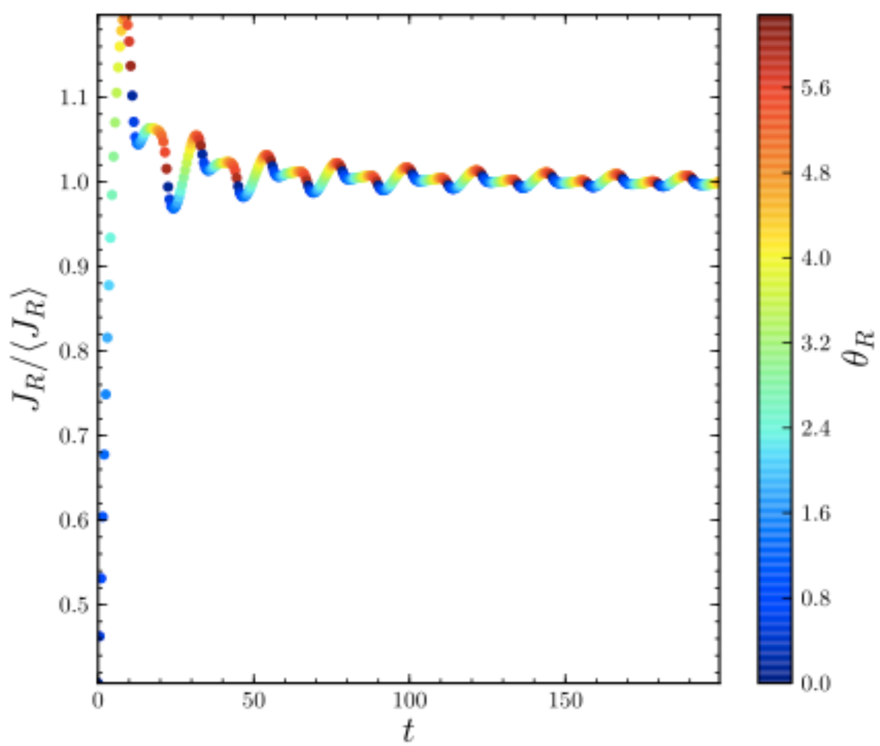
Application: Same for Ophiuchus

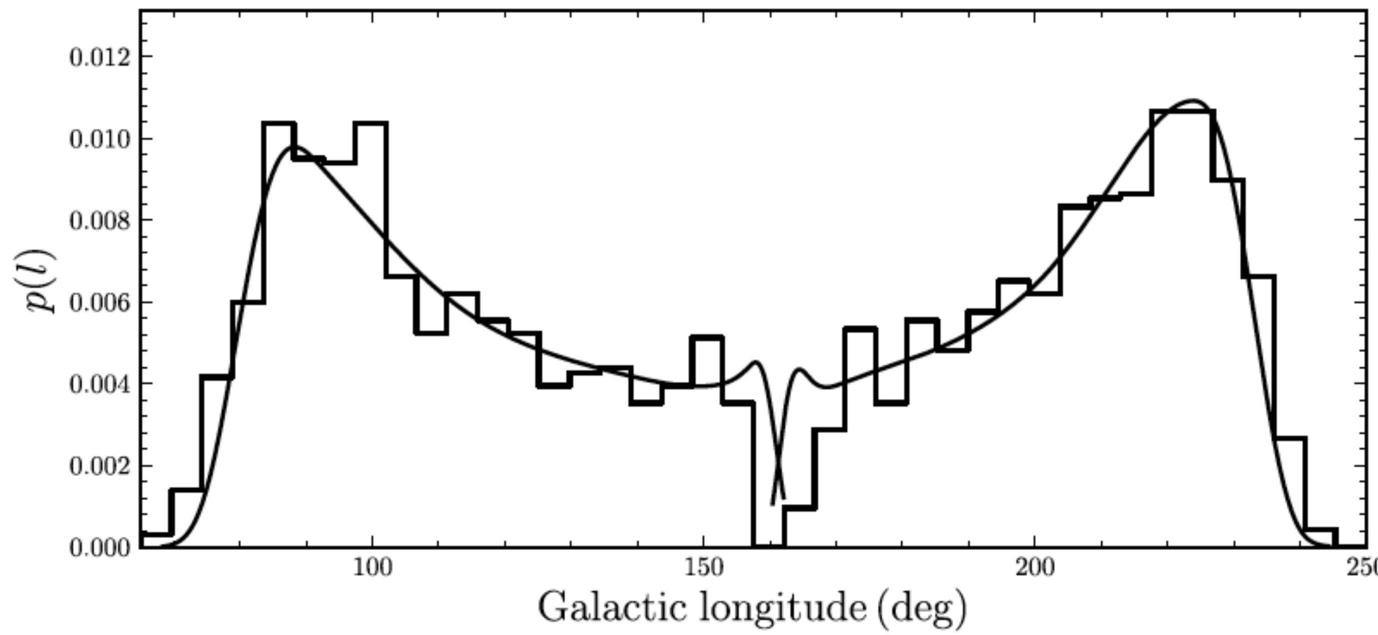


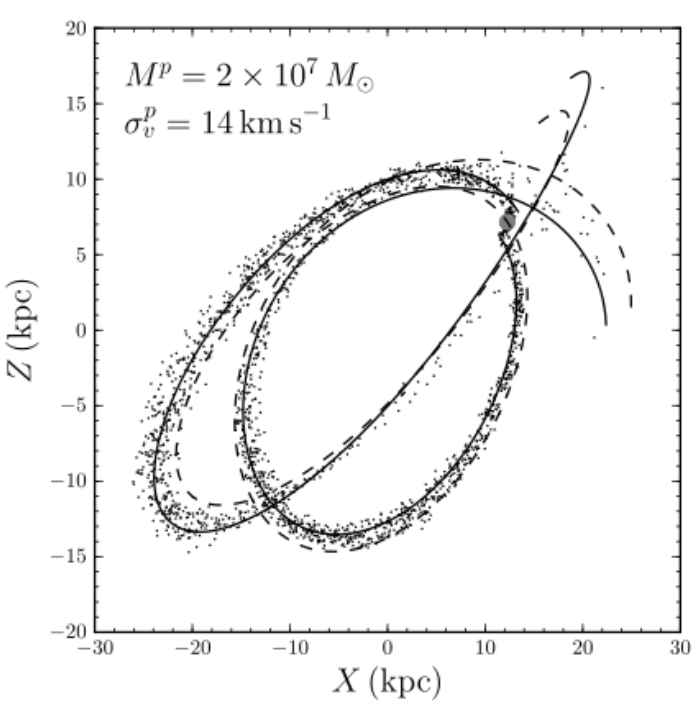
Application: Same for Ophiuchus

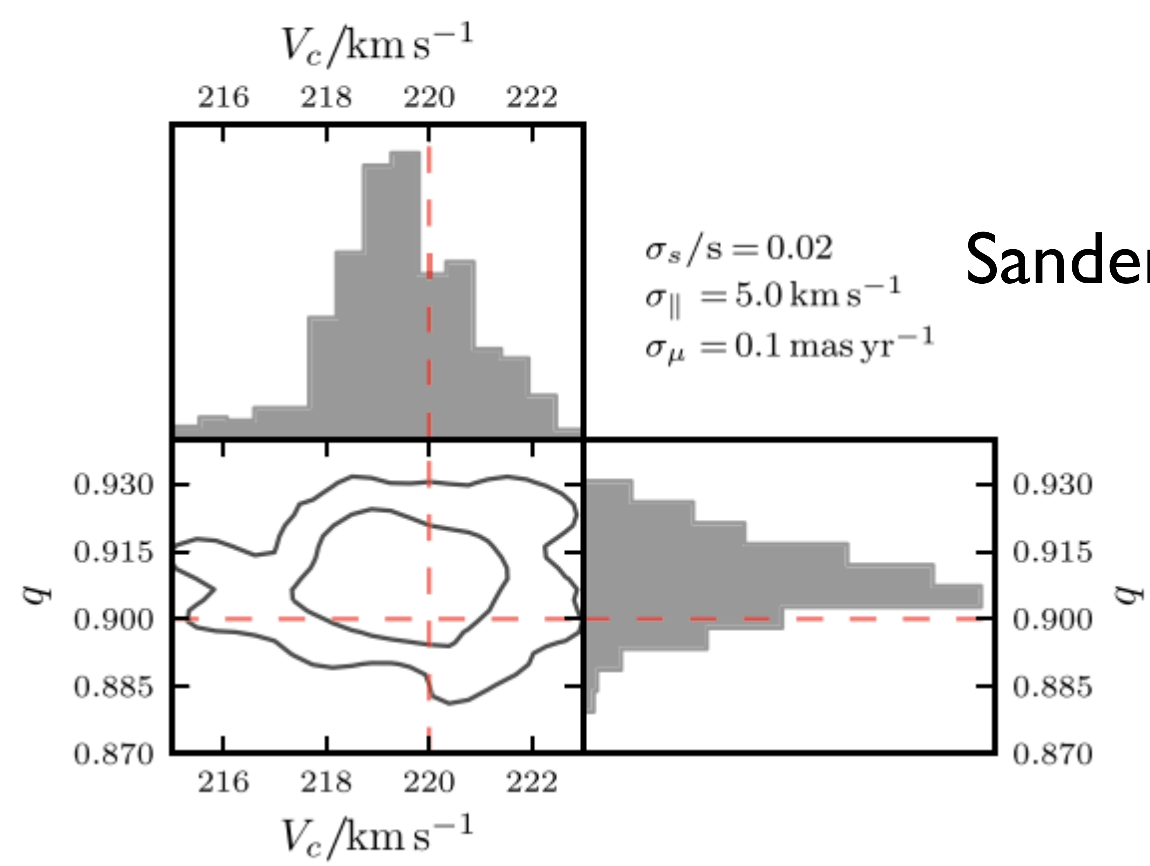


Conclusion









Sanders (2014)