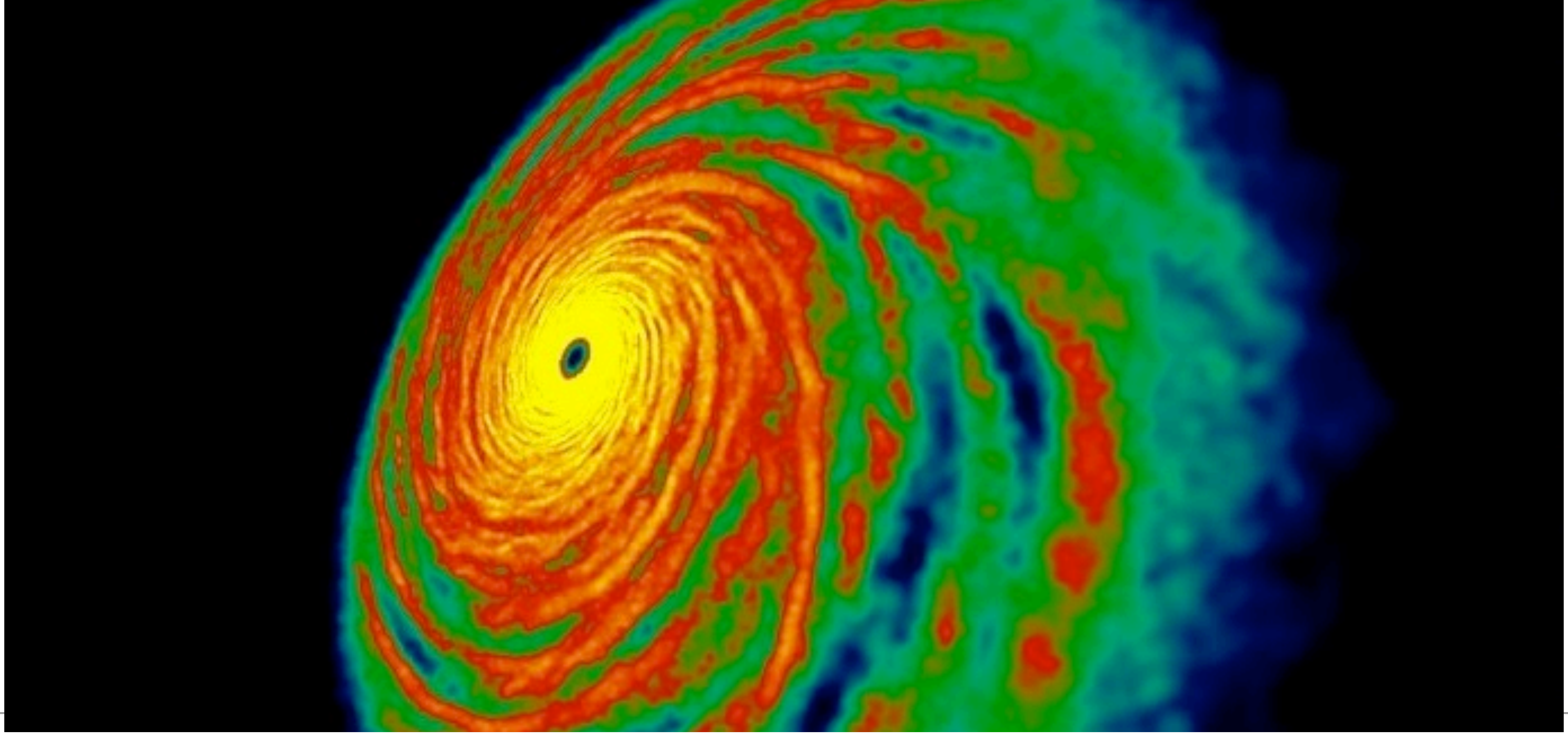

Spiral structure and gravitational instabilities in protostellar discs

Giuseppe Lodato - Università degli Studi di Milano

Herbig Ae/Be stars: the missing link in star formation - Santiago del Chile - 8 April 2014

Gravitational instabilities in protostellar discs

- Conditions for instability
- Dynamics of self-gravitating discs:
  - Self-regulation
  - Local vs global behaviour
- Numerical uncertainties - convergence, fragmentation
- Observations of density waves in protostellar discs

Linear stability criterion

- Linear dispersion relation

$$(\omega - m\Omega)^2 = c_{\rm s}^2 k^2 - 2\pi G \Sigma |k| + \kappa^2$$

- Well known axisymmetric instability criterion:

$$Q = \frac{c_{\rm s}\kappa}{\pi G \Sigma} < \bar{Q} \approx 1$$

- Equivalent form of the instability criterion

$$\frac{M_{\rm disc}(R)}{M_\star} \gtrsim \frac{H}{R}$$

- Need the disc to be cold and/or massive

- What are the masses and aspect ratio in actual protostellar discs?

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Are protostellar discs linearly unstable?

- Midplane temperature for irradiated discs (Chiang & Goldreich 1997, Chiang & Youdin 2009) gives:

$$\frac{H}{R} \simeq 0.02 \left(\frac{R}{\mathrm{AU}} \right)^{2/7}$$

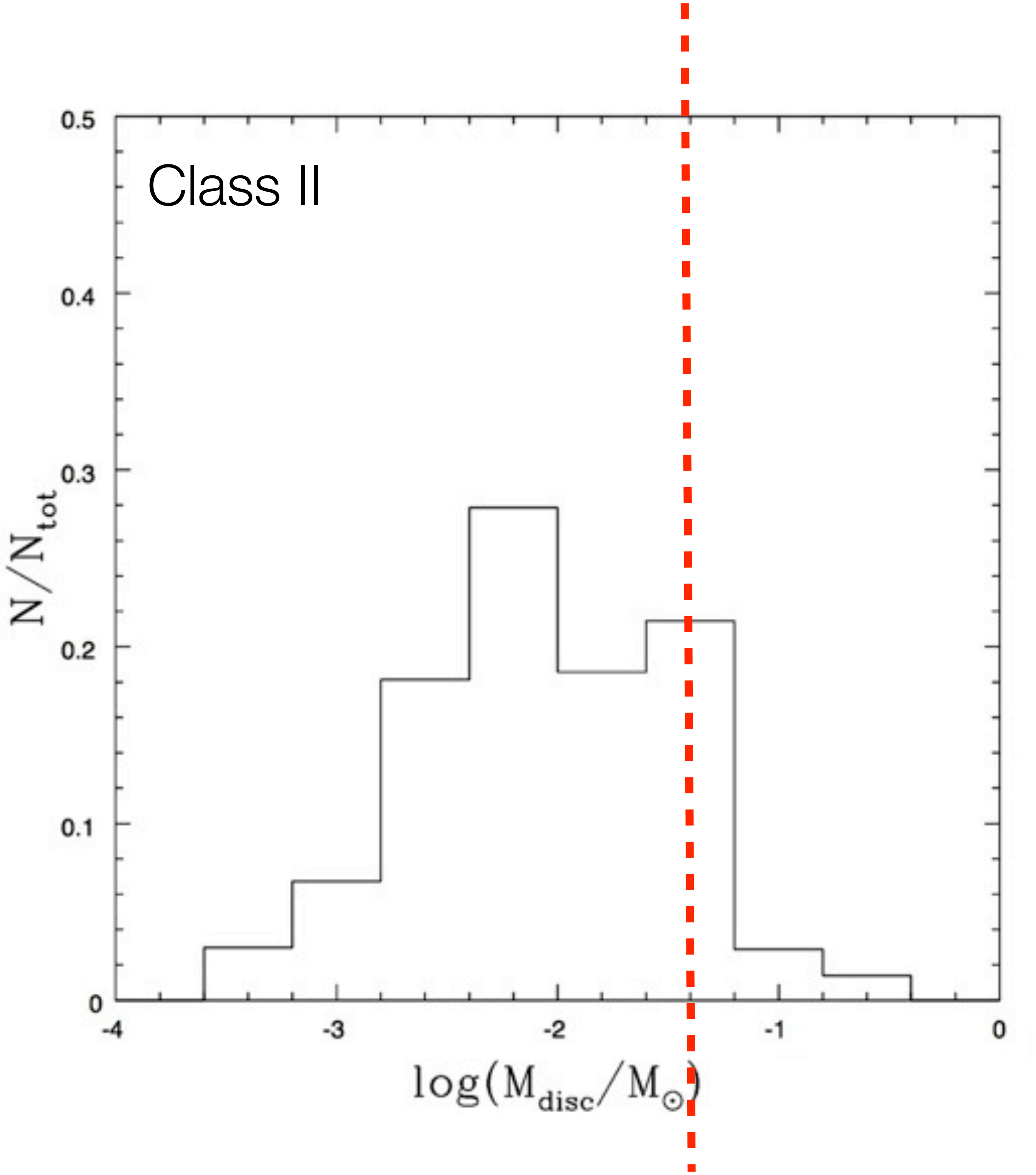
- Therefore *H/R* varies from **0.02** at 1AU to **0.06** at 100 AU

- Need disc masses of order 5% of the stellar mass to be unstable

- Protostellar disc masses difficult to measure (see Hartmann et al 2006)

Are protostellar discs linearly unstable?

- Disc masses in Taurus and Ophiucus by Andrews and Williams (2005, 2007)
- Disc masses might be underestimated significantly (Hartmann et al 2006)
- Uncertainties in dust opacities
- If density profile steep, most of the mass might be hidden in optically thick inner parts (Hartmann 2009)



Are protostellar discs linearly unstable?

Lodato, Dolci, Manara, Ricci, in prep.

- Class II (T Tauri) discs are relatively evolved. Can we infer the masses at early stages?
- Simple (simplistic?) approach:
- Take all objects with measured M and Mdot
- Apply similarity solutions (Lynden-Bell & Pringle 1973)
- Find “initial’ disc mass and evolutionary timescale
- Masses from Andrews & Williams
- Mdots from the literature
- Not all measurements consistent with similarity solutions (see also Jones, Alexander & Pringle 2012)

$$M_0 = M_{\rm d}(t)\left(\frac{t_{\rm d}}{t_{\rm d}-t}\right)^{1/2(2-\gamma)}$$

$$t_{\rm d} = \frac{M_{\rm d}(t)}{2(2-\gamma)\dot{M}(t)}$$

Are protostellar discs linearly unstable?

- Very preliminary results
- Limited sample
- Inhomogeneous analysis of Mdot measurements
- Do similarity solutions really apply, at least in an averaged sense?

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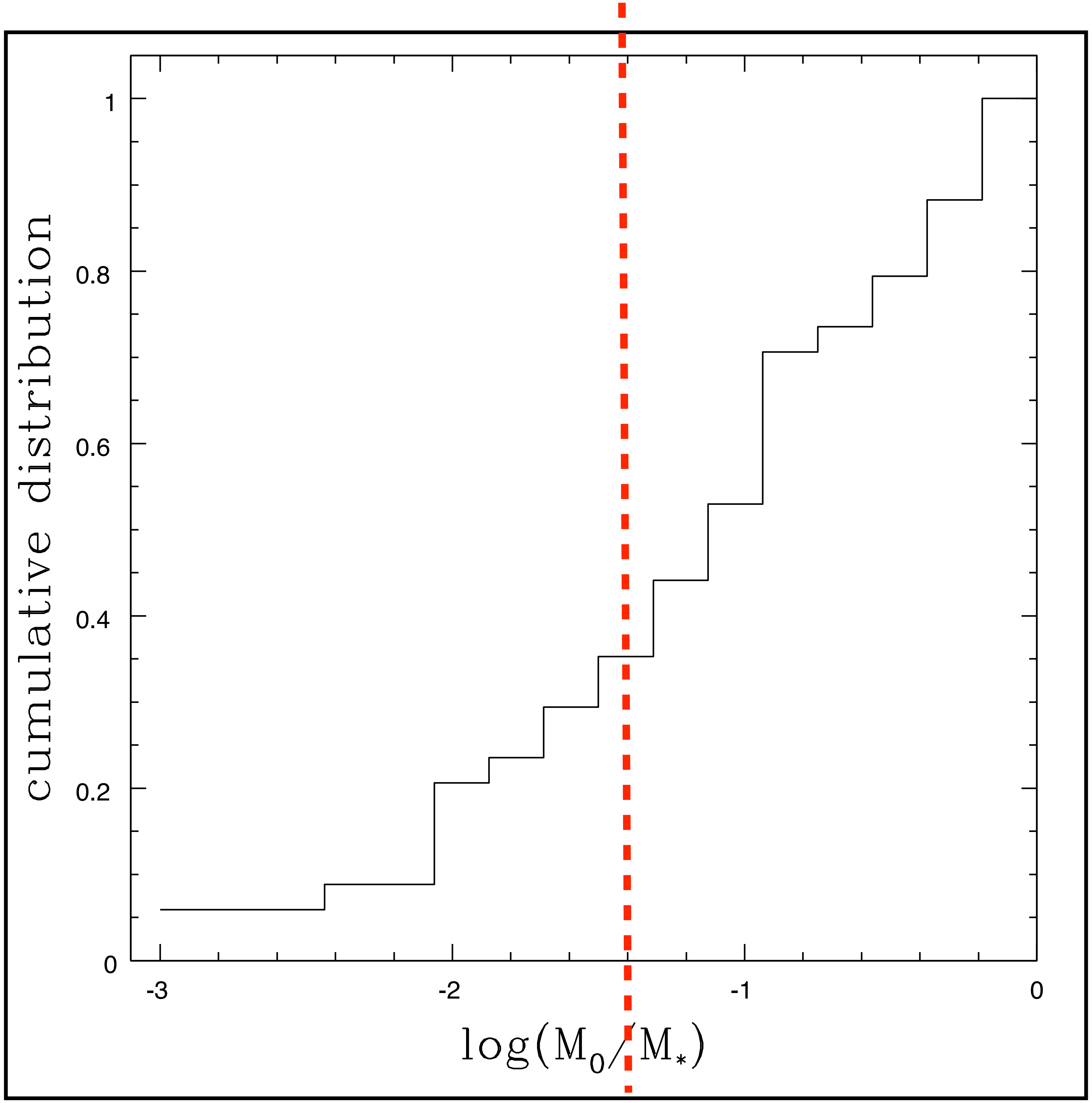


Gravitational instabilities in protostellar discs

- Conditions for instability
- Dynamics of self-gravitating discs:
    - Self-regulation
    - Local vs global behaviour
- Numerical unceretainties - convergence, fragmentation
- Observations of density waves in protostellar discs

Non linear evolution of GI

- Investigated numerically in the last decade by several authors (Laughlin & Bodenheimer 1994, Laughlin et al 1998, Pickett et al 2000, Boss 2000, Gammie 2001, Mayer et al 2002, Lodato & Rice 2004, 2005, Mejia et al 2005, Boley et al 2006)
- Early simulations used an isothermal or polytropic equation of state (Laughlin & Bodenheimer 1994, Mayer et al 2002)
- Starting from Gammie (2001) it has become clear that the evolution is strongly dependent on the cooling time t_{cool}
- Introduce a cooling parameter as the ratio of cooling to dynamical timescale

$$\beta = t_{\rm cool}\Omega$$

Thermal self-regulation of GI

- Role of cooling time clear if one thinks at the form of the stability parameter *Q*

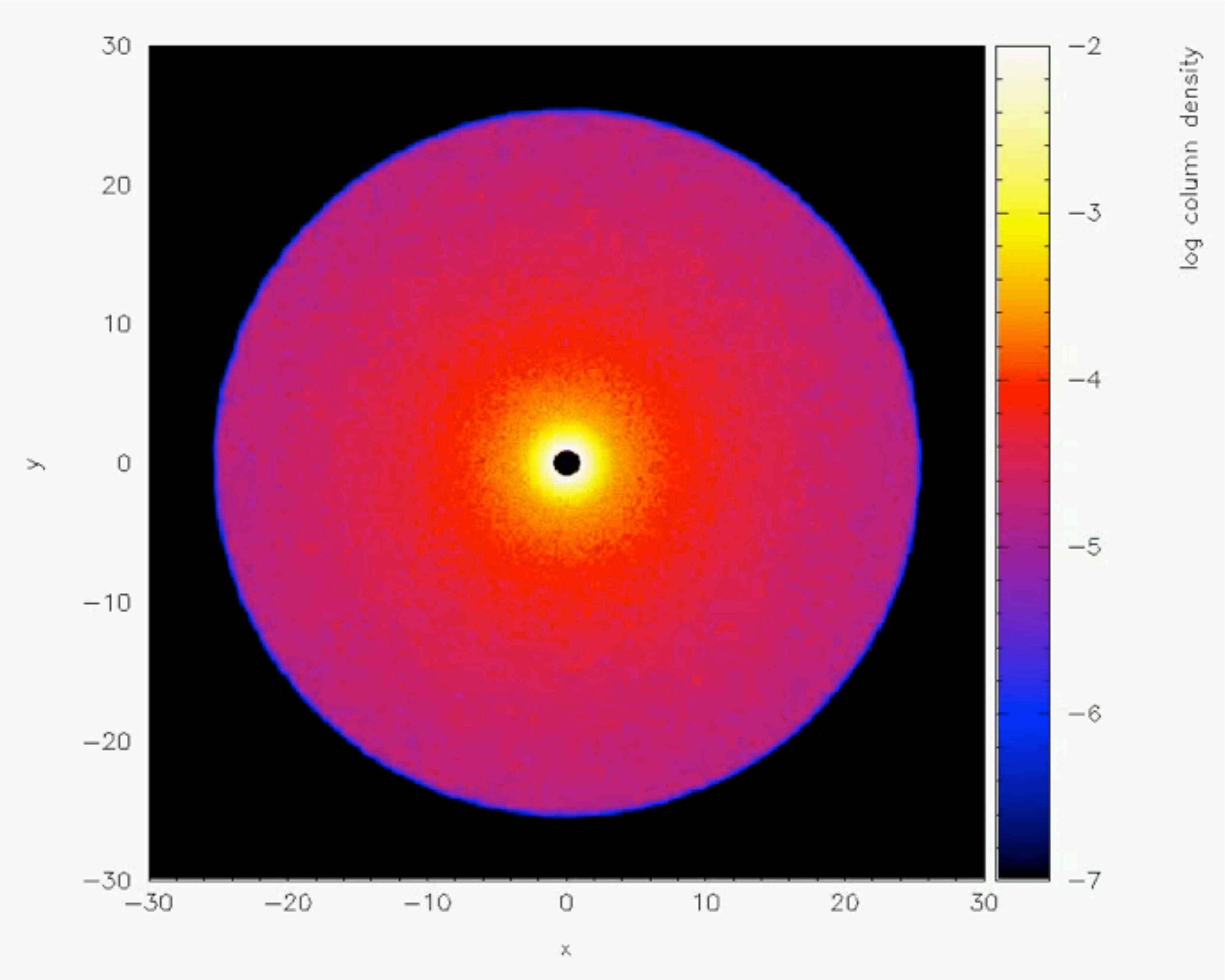
$$Q = \frac{c_{\rm s}\kappa}{\pi G\Sigma} \propto T^{1/2}$$

- Development of the instability feeds energy back onto the equilibrium and stabilizes the disc
- Works as an effective thermostat for the disc
- Expect the disc to stay close to marginal stability *Q ~ 1 (Paczynski 1977)*
- Self-regulated discs models can be constructed (Bertin 1997, Bertin & Lodato 1999)

Long cooling time: self-regulation

Cossins, Lodato &
Clarke (2009)
$\beta = 6$

Long cooling time: self-regulation



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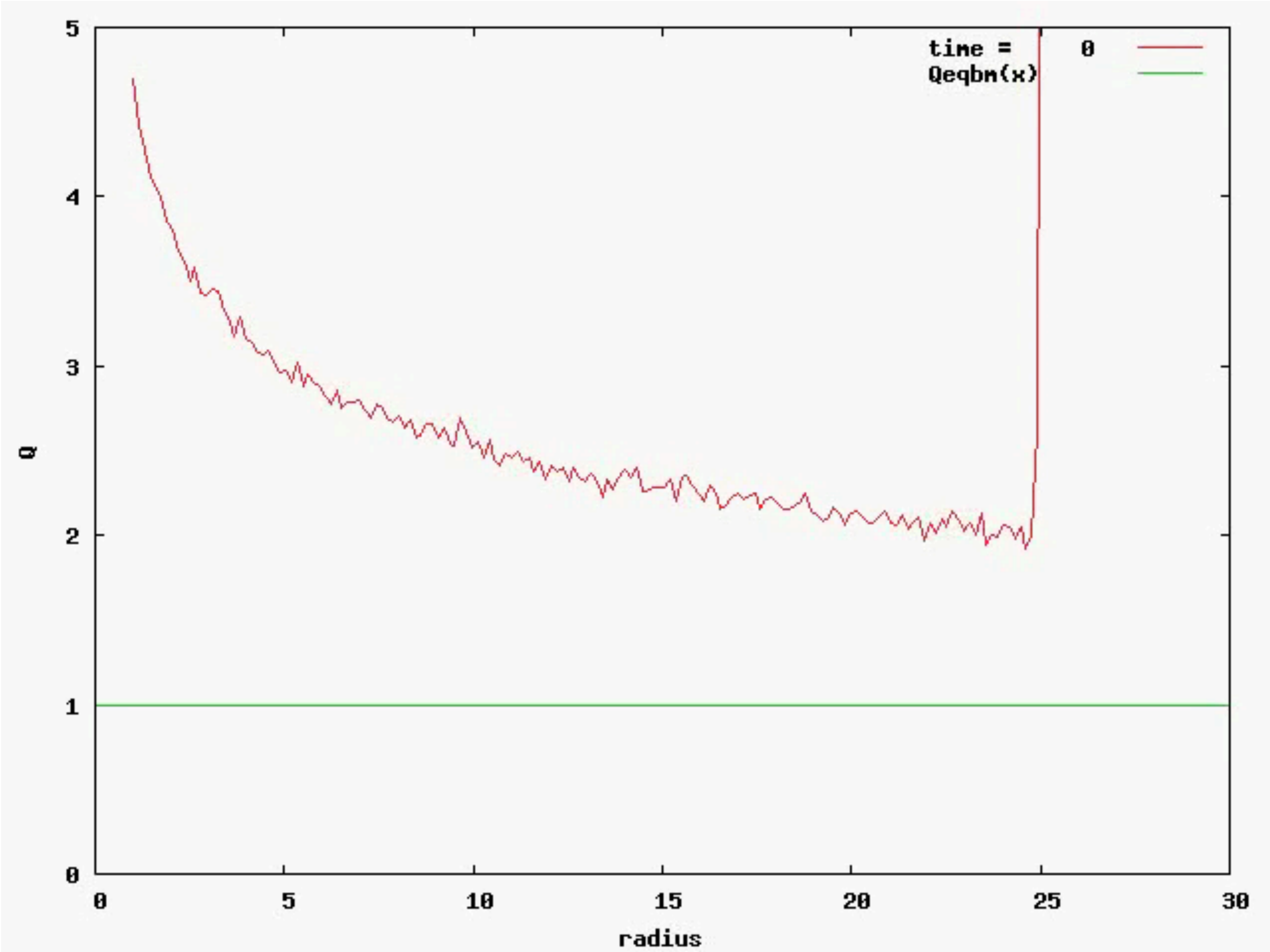

Cossins, Lodato &
Clarke (2009)
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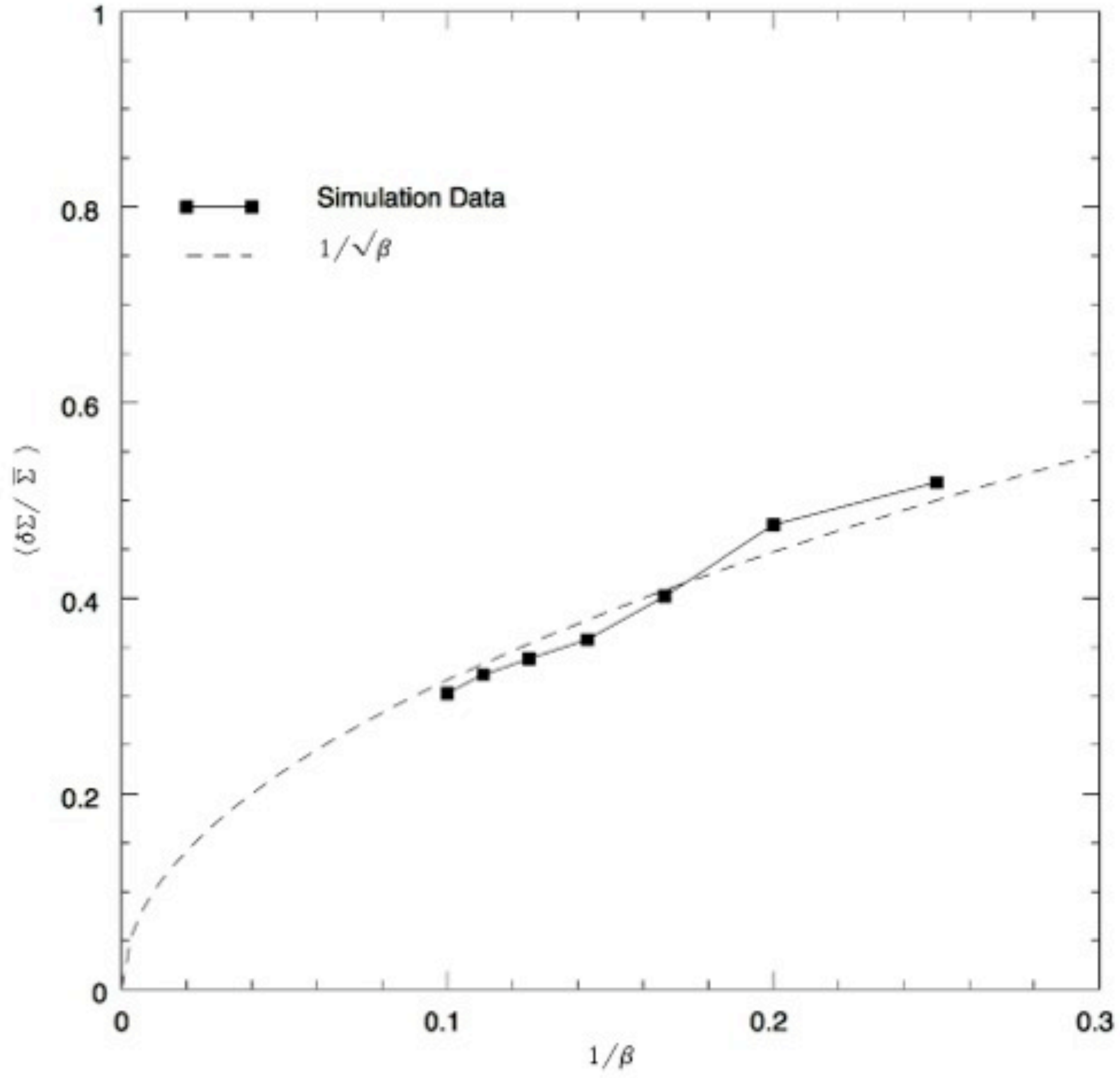
Thermal saturation of GI

Cossins, Lodato & Clarke 2009

- Self-regulation is established through thermal saturation of the spiral waves.
- Amplitude of density perturbation must be related to cooling rate
- We find that:

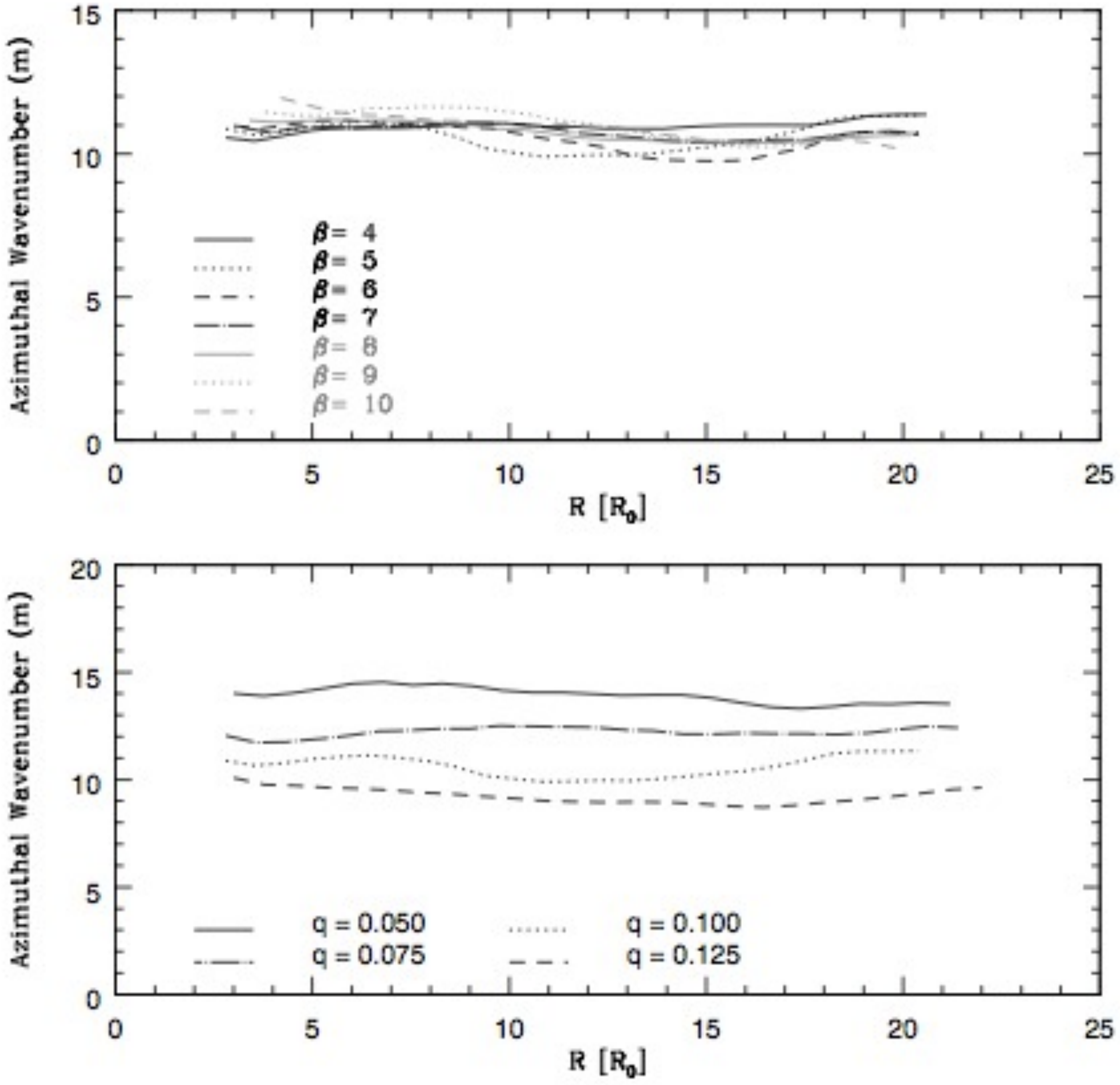
$$\frac{\Delta\Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}}$$

- Natural if consider that energy content of waves is proportional to the square of the perturbed fields

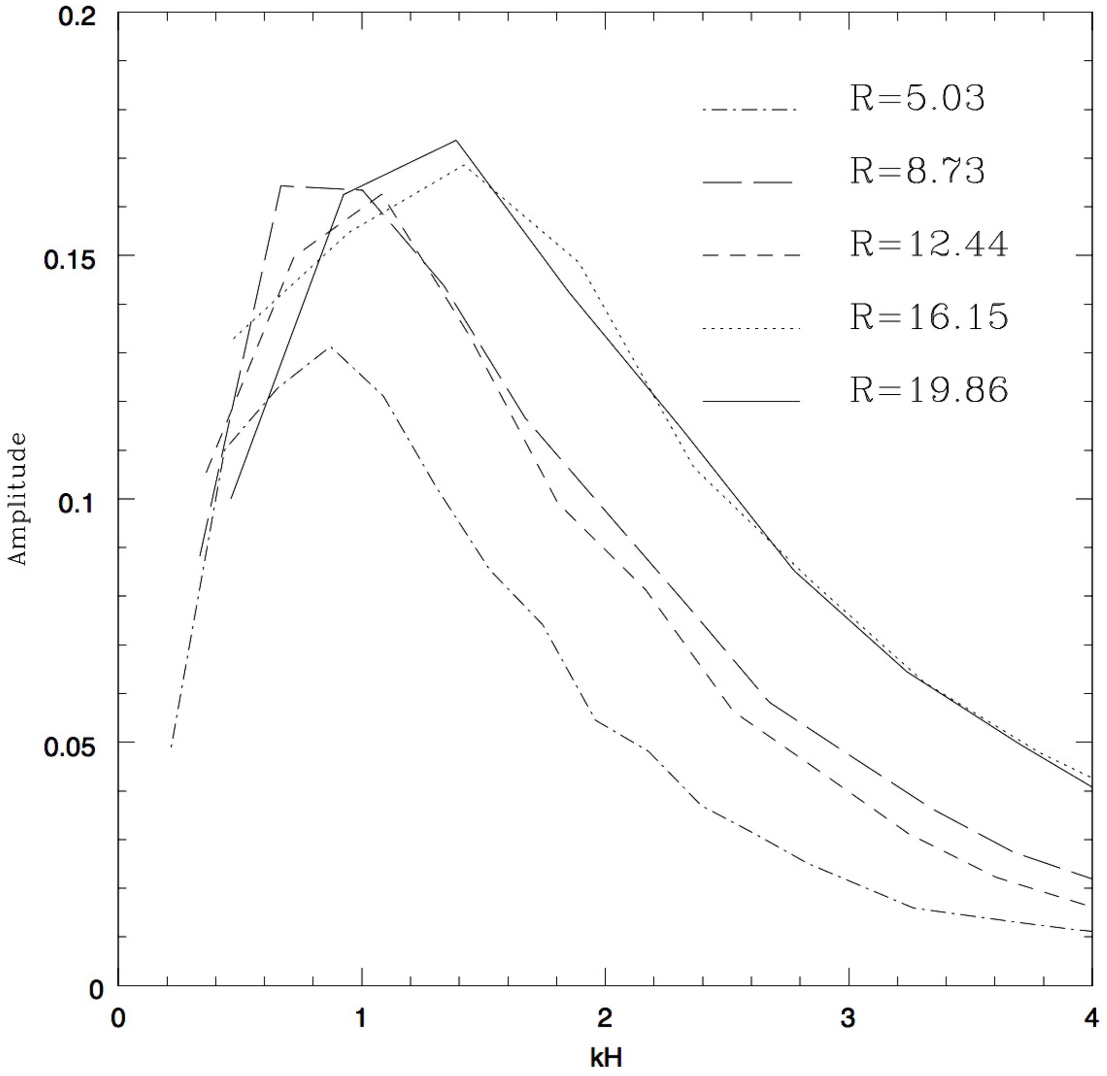


Spectrum of excited modes

Azimuthal structure: massive discs charachterized by small *m*



Radial structure: at all radii, *k* peaks at roughly *~1/H*

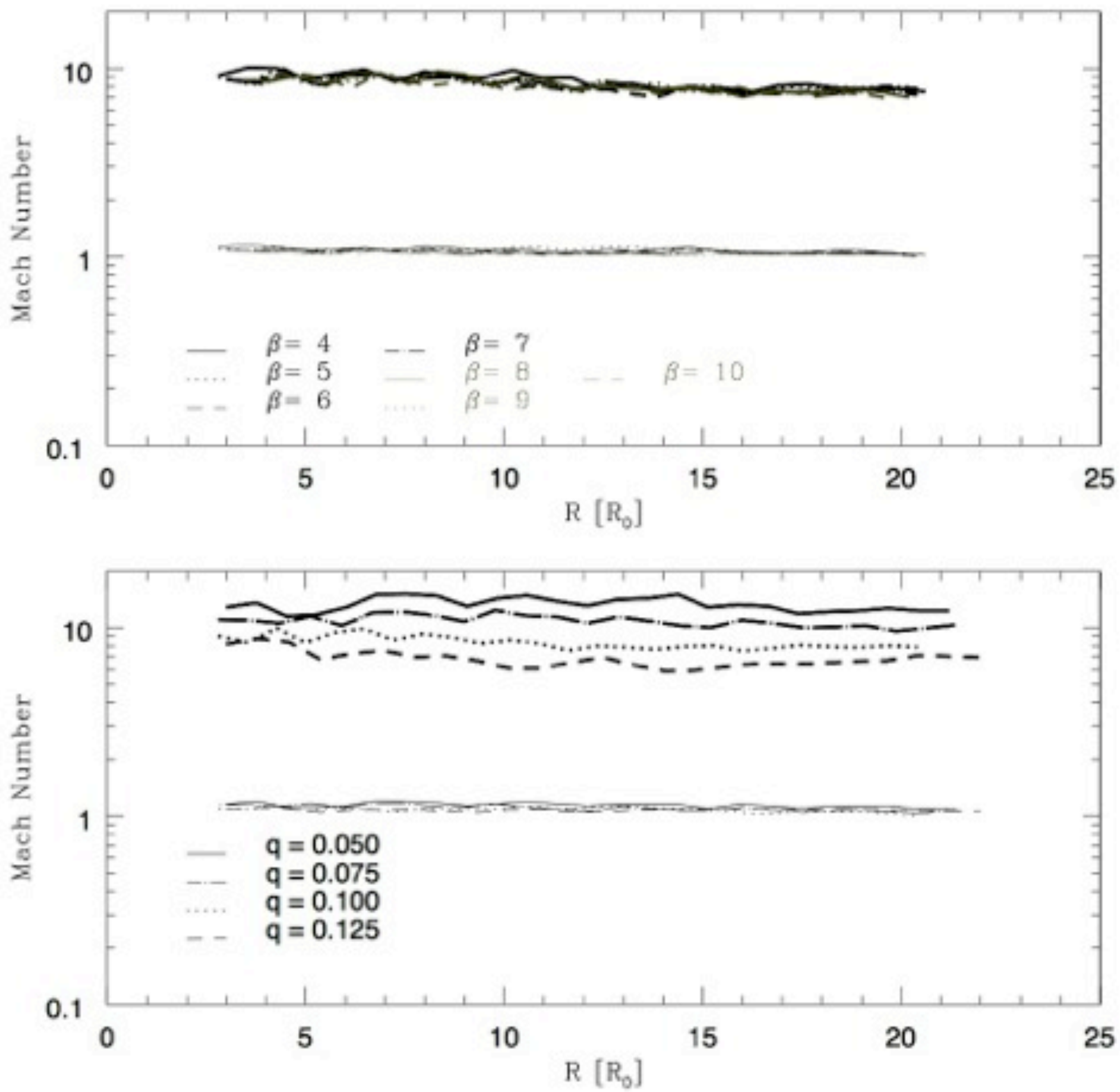


Sonic condition for spiral waves

Cossins, Lodato & Clarke 2009

- We have computed the pattern speed of the underlying spiral structure and its Mach number
- The Doppler-shifted Mach number is very close to unity, independently on radius, cooling rate, and disc mass.
- Density jump for almost sonic shocks also directly leads to

$$\frac{\Delta\Sigma}{\Sigma} \approx \frac{1}{\sqrt{\beta}}$$



Local vs global behaviour

Lodato & Rice 2004

- Can the evolution of self-gravitating discs be described within the standard, local, α-like prescription?
- Can compute gravitational + Reynolds stresses directly from simulations and compare with expectations from standard α-theory (LR04, see also Boley et al. 2006)
- **The disc adjusts so as to deliver the viscosity needed to stay in thermal equilibrium**

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- Can compute gravitational + Reynolds stresses directly from simulations and compare wit also Boley et al. 2006)
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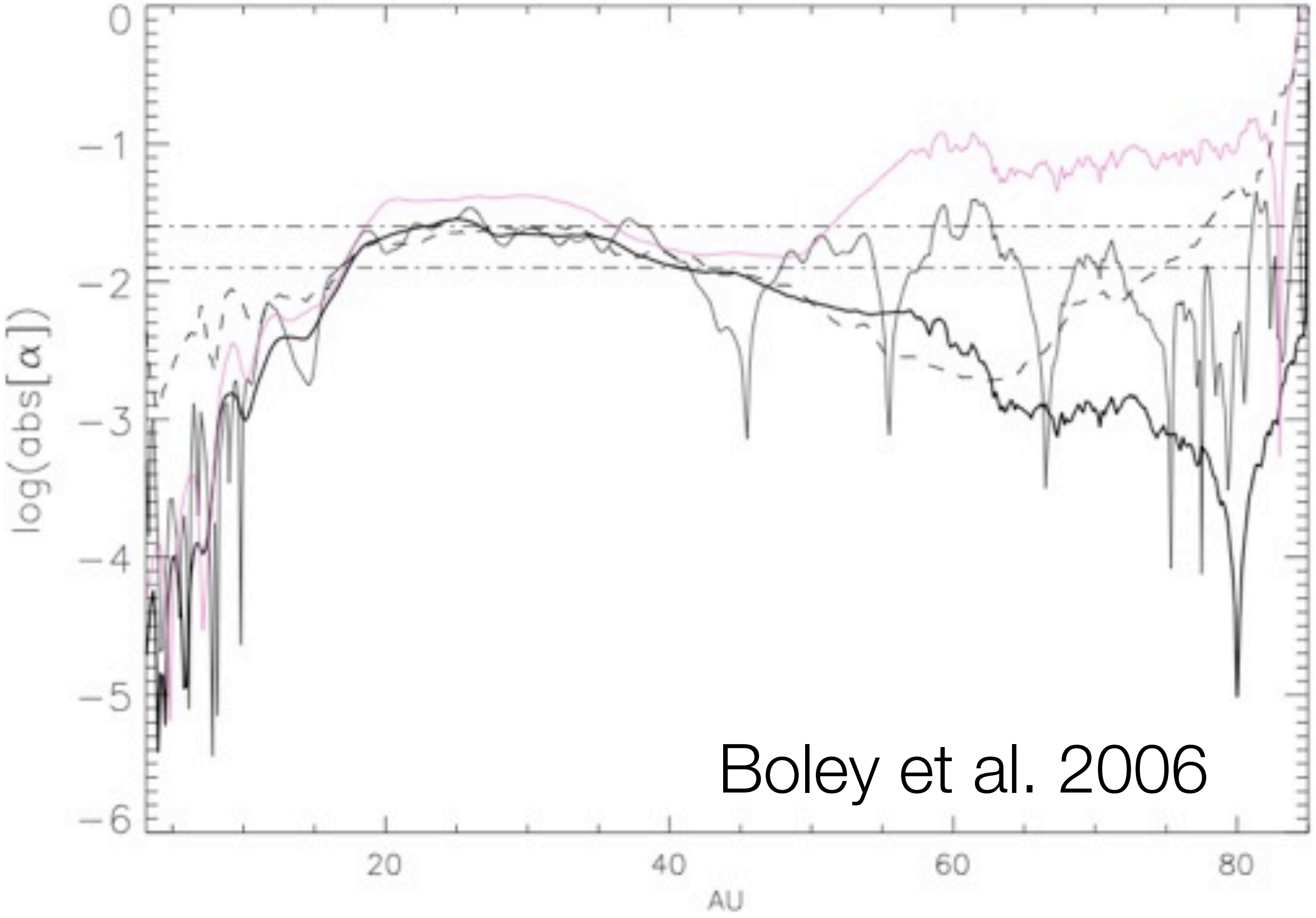


Boley et al. 2006

Local vs global behaviour

Cossins, Lodato & Clarke 2009

- Can the evolution of self-gravitating discs be described within the standard, local, α-like prescription?
- Described in detail by Balbus & Papaloizou (1999), recently discussed extensively by Cossins et al (2009)
- Relation between energy and angular momentum densities in a density wave

$$\mathcal{E} = \Omega_{\rm p}\mathcal{L} \longrightarrow \dot{\mathcal{E}} = \Omega_{\rm p}\dot{\mathcal{L}}$$

- Relation between power and stress due to local (viscous) processes

$$\dot{\mathcal{E}}_\nu = \Omega\dot{\mathcal{L}}_\nu$$

- If density waves dissipate far from co-rotation, behaviour is non-local

Local vs global behaviour

Cossins, Lodato & Clarke 2009

- Degree of non-locality can be measured by

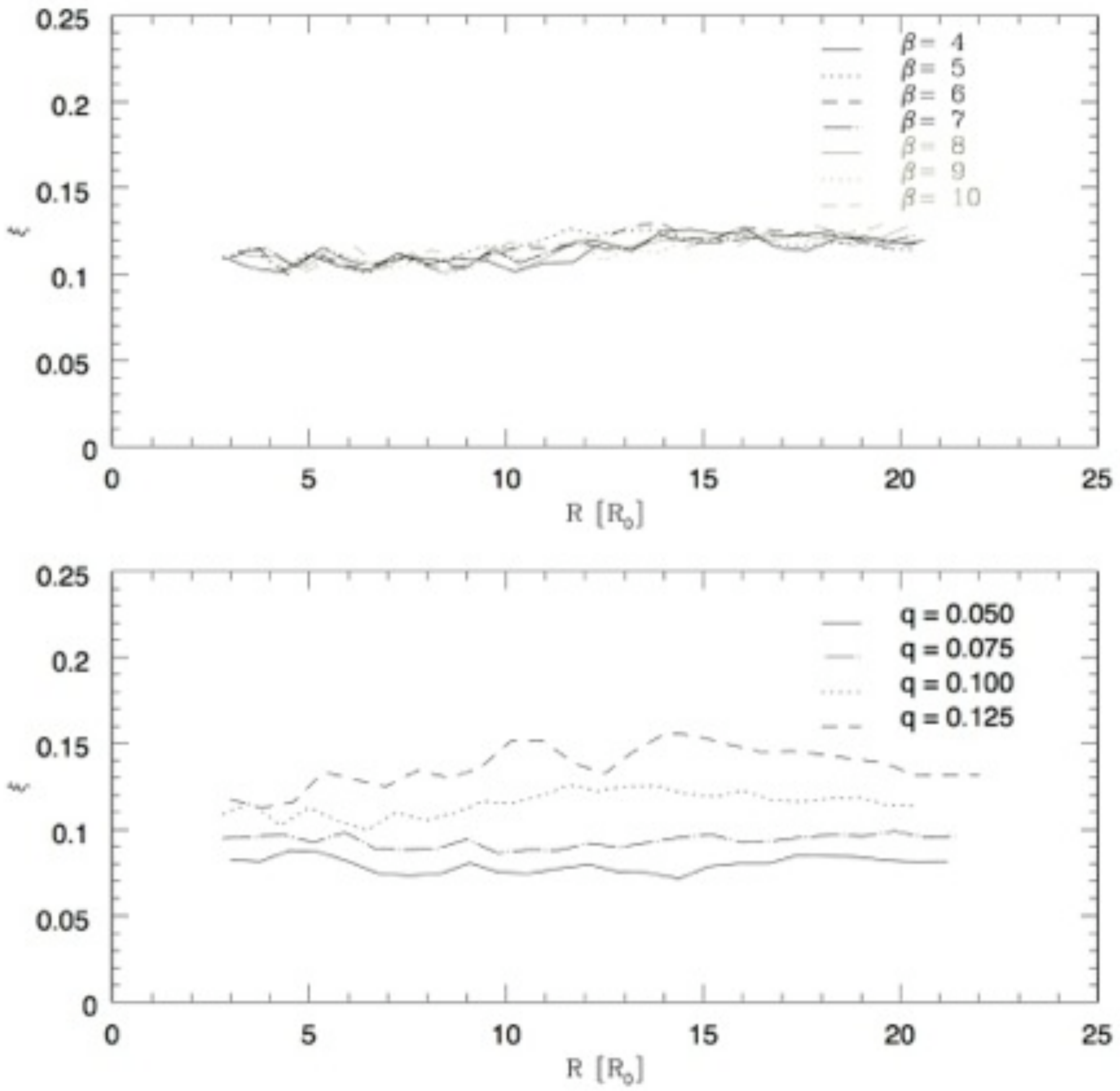
$$\xi = \left| \frac{\Omega - \Omega_{\rm p}}{\Omega} \right|$$

- Sonic condition for wave dissipation also tells us something about this:

$$\xi \approx \frac{c_{\rm s}}{v_\phi} = \frac{H}{R}$$

- To the extent that the disc is thin (*H*<<*R*), global behaviour should be negligible

- Possible to construct local, viscous models of disc evolution (Clarke 2009, Rafikov 2009)



Convergence of numerical results

- It is well known that for short cooling times the disc is subject to fragmentation

- Meru and Bate (2011) show that such simulations are not converged

- As resolution increases fragmentation appears to become effective for longer cooling times

Simulation by Peter Cossins

$\beta = 4$

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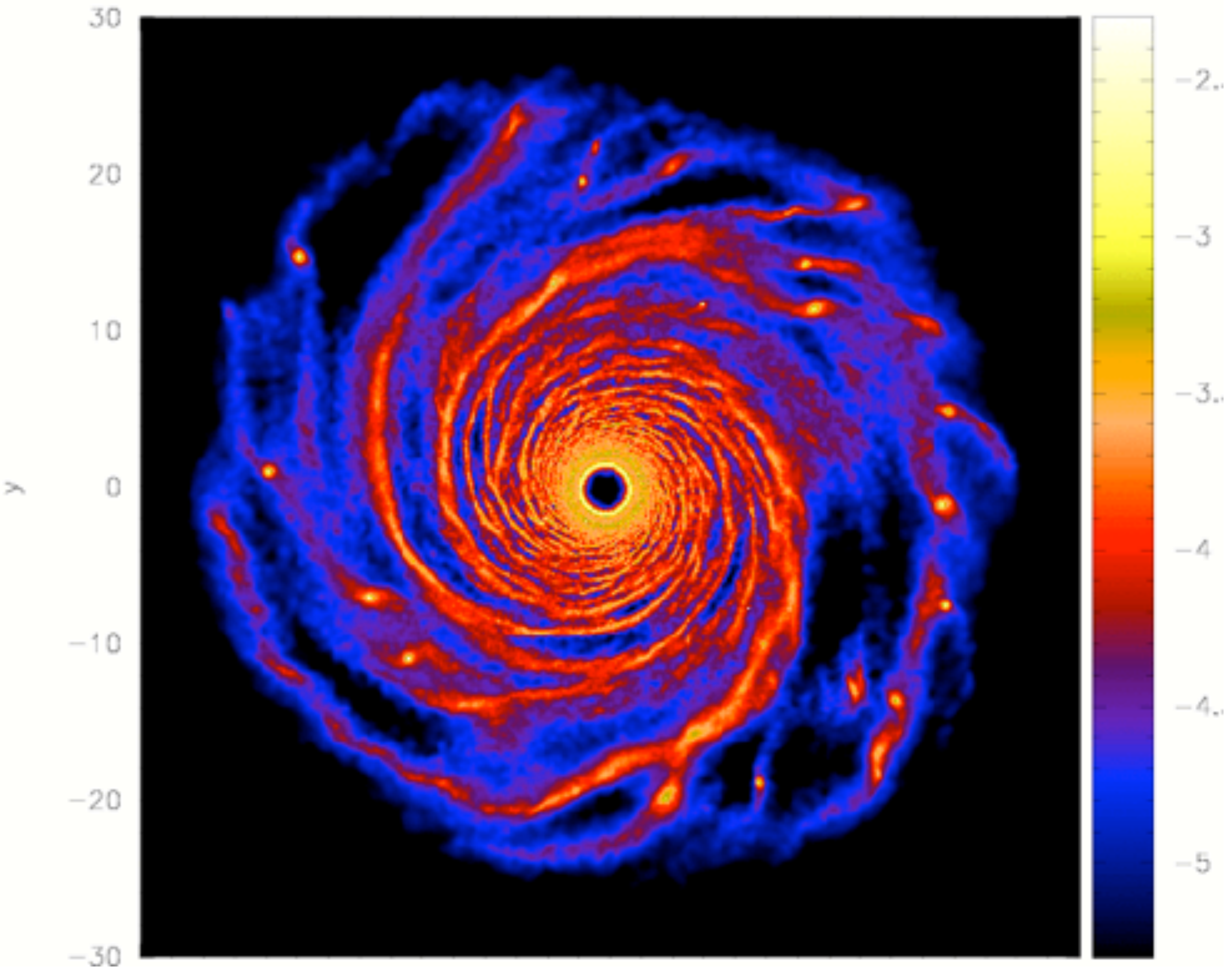


Simulation by Peter Cossins

$\beta = 4$

Convergence of numerical results

- Non-convergence most likely due to the effects of artificial viscosity in SPH simulations: additional viscosity (or any heating terms) weakens the gravitational instability (Lodato & Clarke 2011)
- Meru and Bate (2012): result converge at extremely high resolution. Fragmentation for $\beta < 20$
- Paardekooper (2012): non-convergence observed in 2D grid-based simulations. Fragmentation seen as a stochastic process
  - Essential to compute the likelihood of fragmentation in realistic protostellar discs (Hopkins and Christiansen 2013)

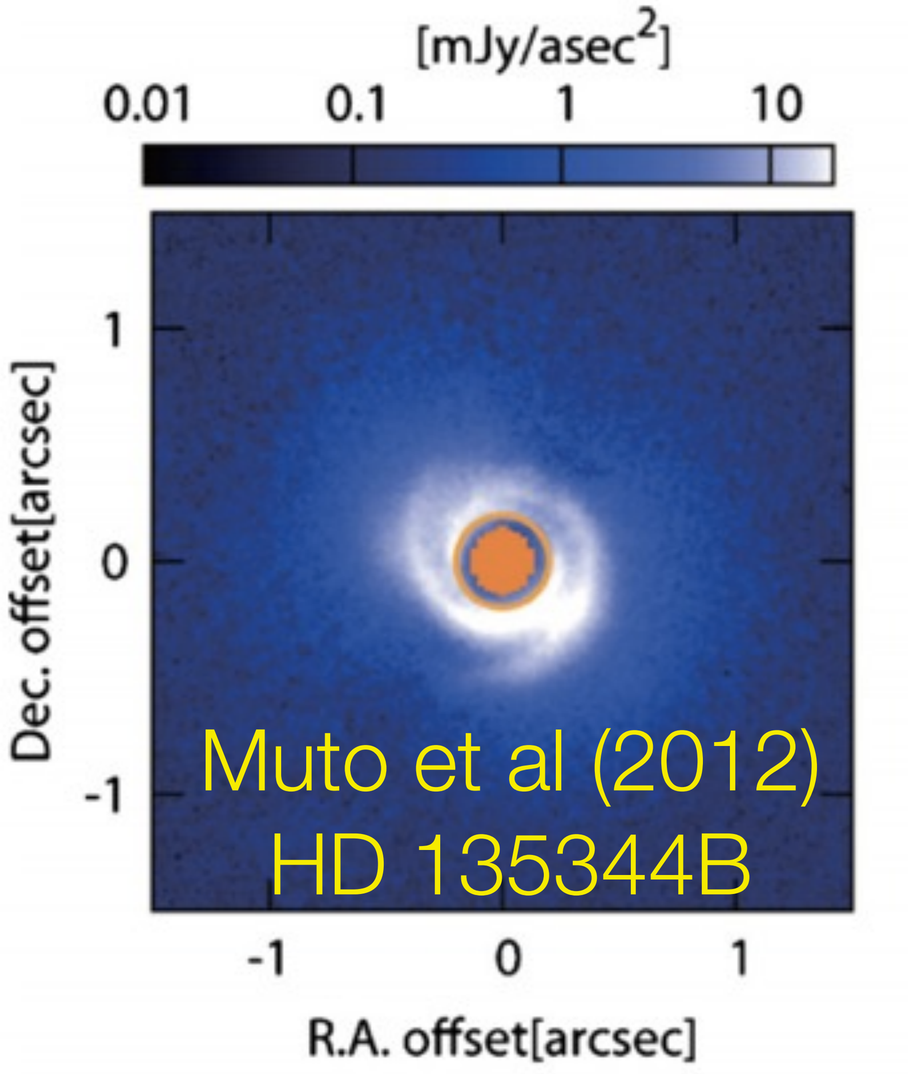
Work in progress by Young and Clarke

Convergence of numerical results

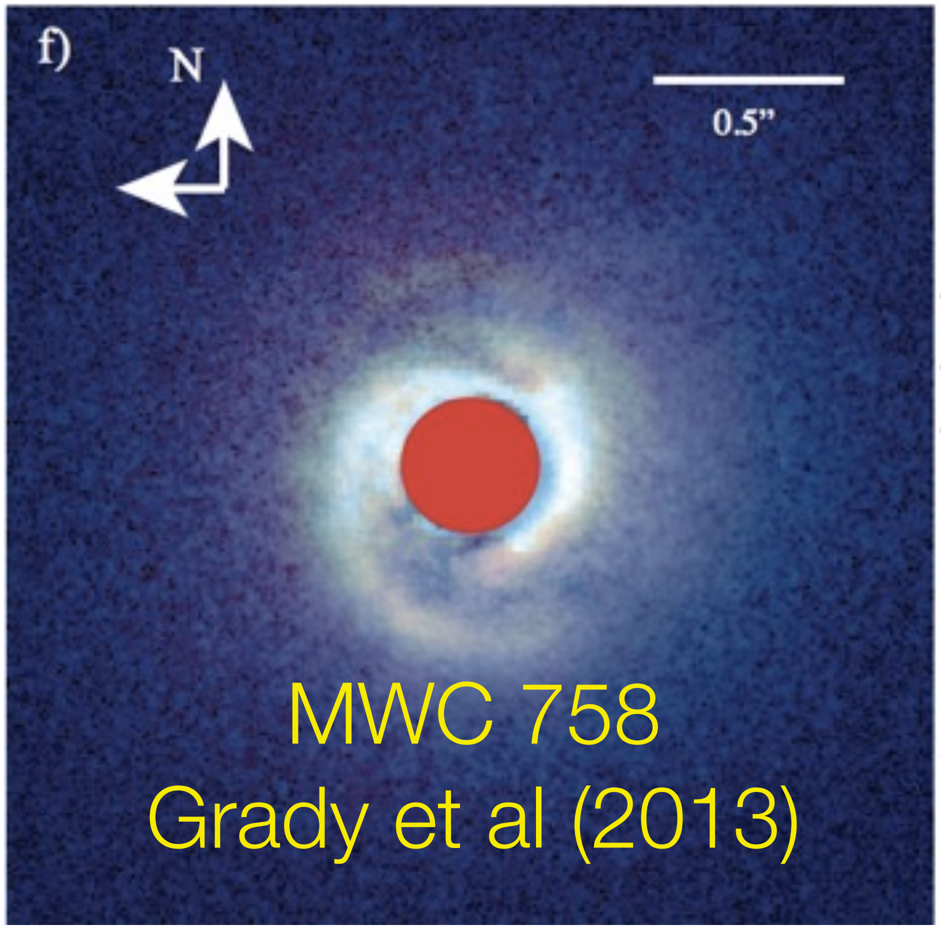
- What about convergence of results in the non-fragmenting limit?
- Is the stress and effective alpha converged?
- Michael et al (2012) (the Indiana group): convergence of measured stress observed in grid-based simulations
  - At low resolution alpha appears to be overestimated (more power in large scale structures ---> potentially more global transport)

Observing gravitational instabilities in Herbig Ae/Be stars

- Several discs with spiral structures observed in scattered light
- Most of these are relatively evolved systems (transitional discs): most likely the origin of the spiral is not due to self-gravity



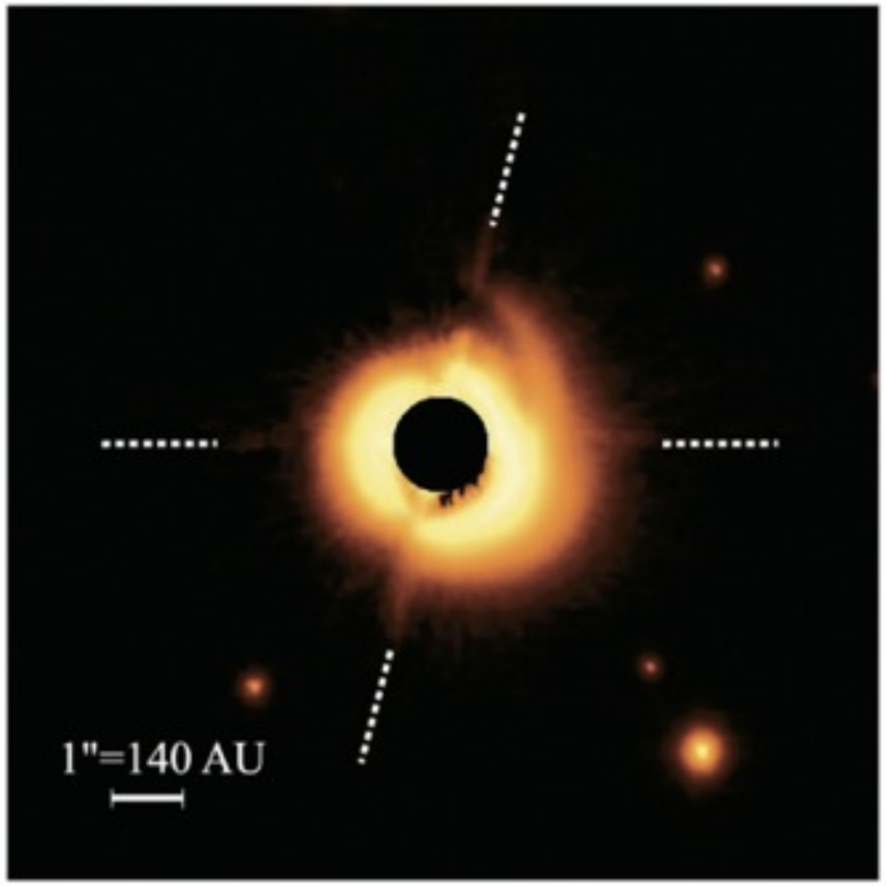
Muto et al (2012)
HD 135344B



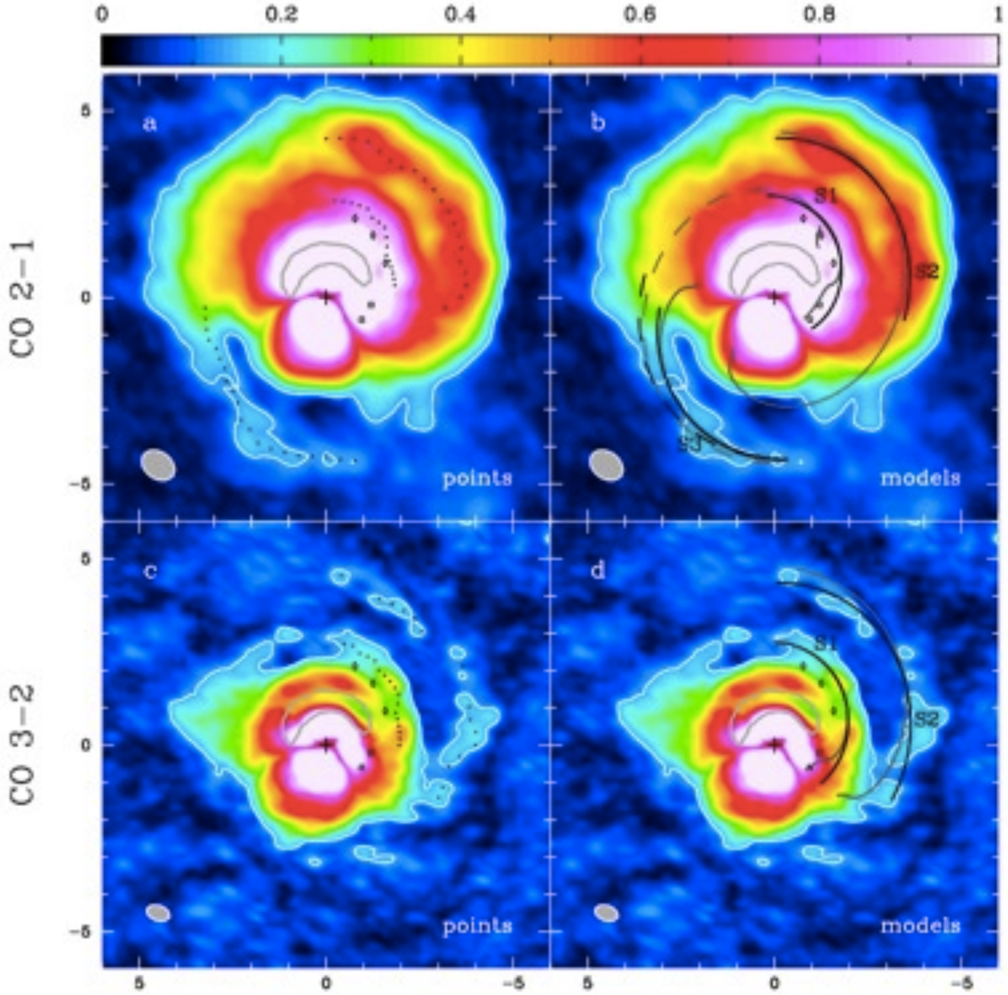
MWC 758
Grady et al (2013)

Observing gravitational instabilities in Herbig Ae/Be stars

- The case of HD 142527
- Christiaens et al (2014) estimate Q ~ 2, possibly marginally unstable



H-band: Fukagawa et al 2006



ALMA line emission: Christiaens 2014

Self-gravitating discs with ALMA

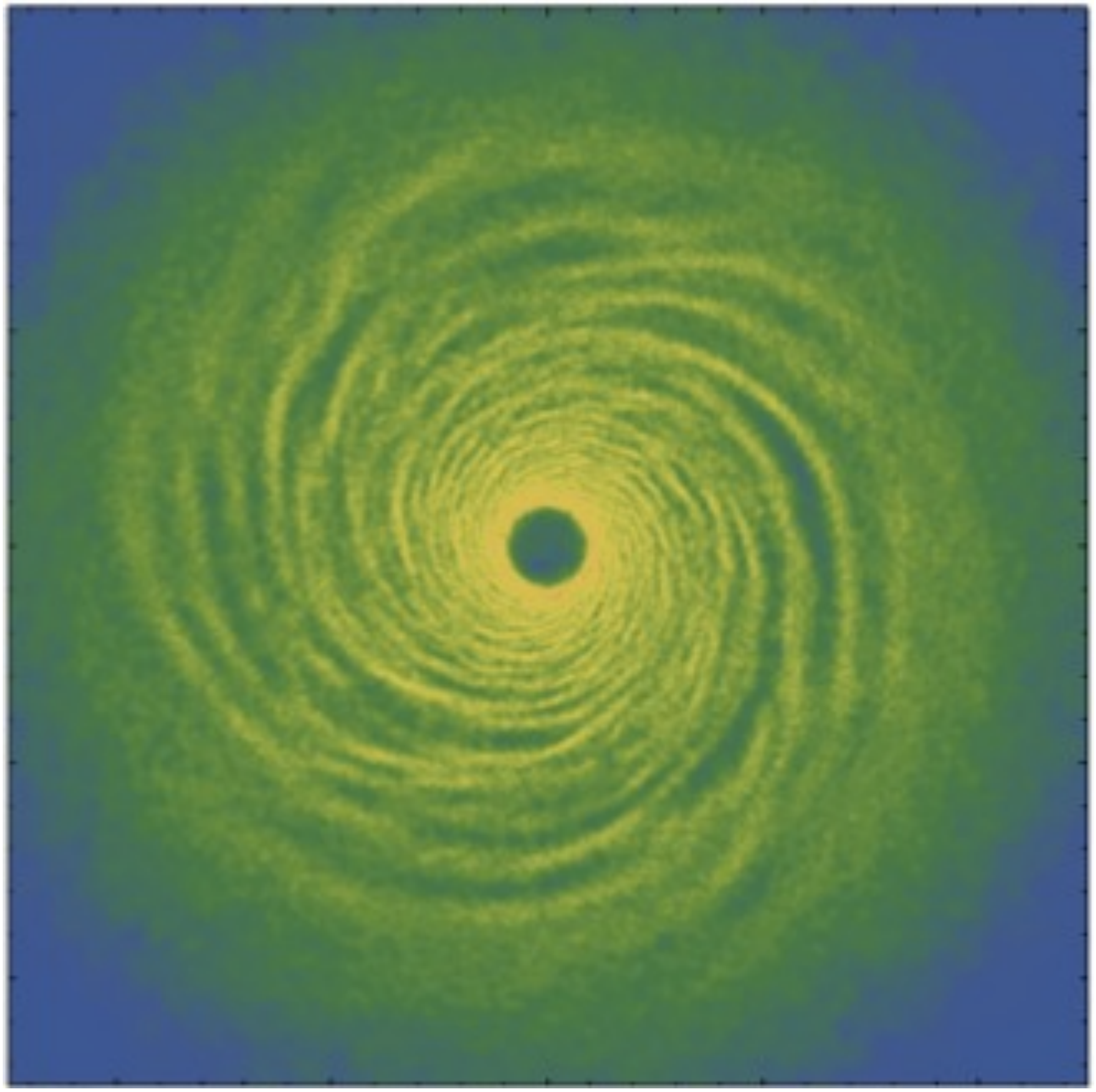
Dipierro, Lodato & Testi (in preparation)

- Is it possible to resolve the spiral structure from GI and derive some system parameters?
- Extend the work of Cossins, Lodato & Testi (2010)
- Consider some simulated discs with a variety of different parameters
  - Disc masses ($M_{disc}/M_{*}=0.1, 0.25$)
  - Stellar mass ($M_{*} = 0.3, 1, 3$Msun)
  - Distance
  - Inclination
  - Size (Outer radius at either 25AU, or 100AU)
- Assume a “standard” opacity law (maximum grain size + 1cm)
- Build an “atlas” of mock ALMA images in the dust continuum

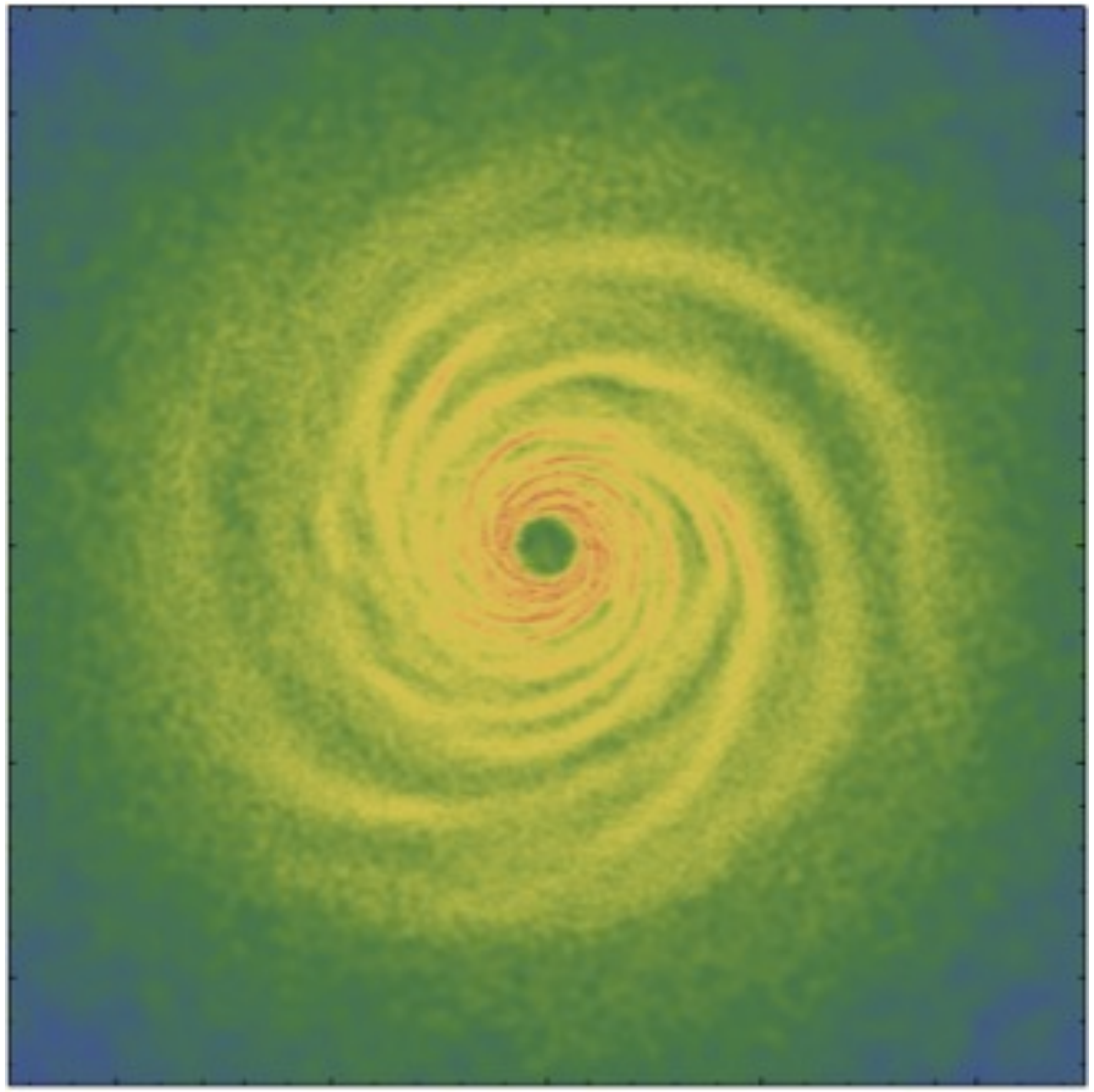
Self-gravitating discs with ALMA

Dipierro, Lodato & Testi (in preparation)

$M_{disc}/M_{*}=0.1$



$M_{disc}/M_{*}=0.25$



Assume a 25AU disc in TW Hya, at 220GHz

Self-gravitating discs with ALMA

Dipierro, Lodato & Testi (in preparation)

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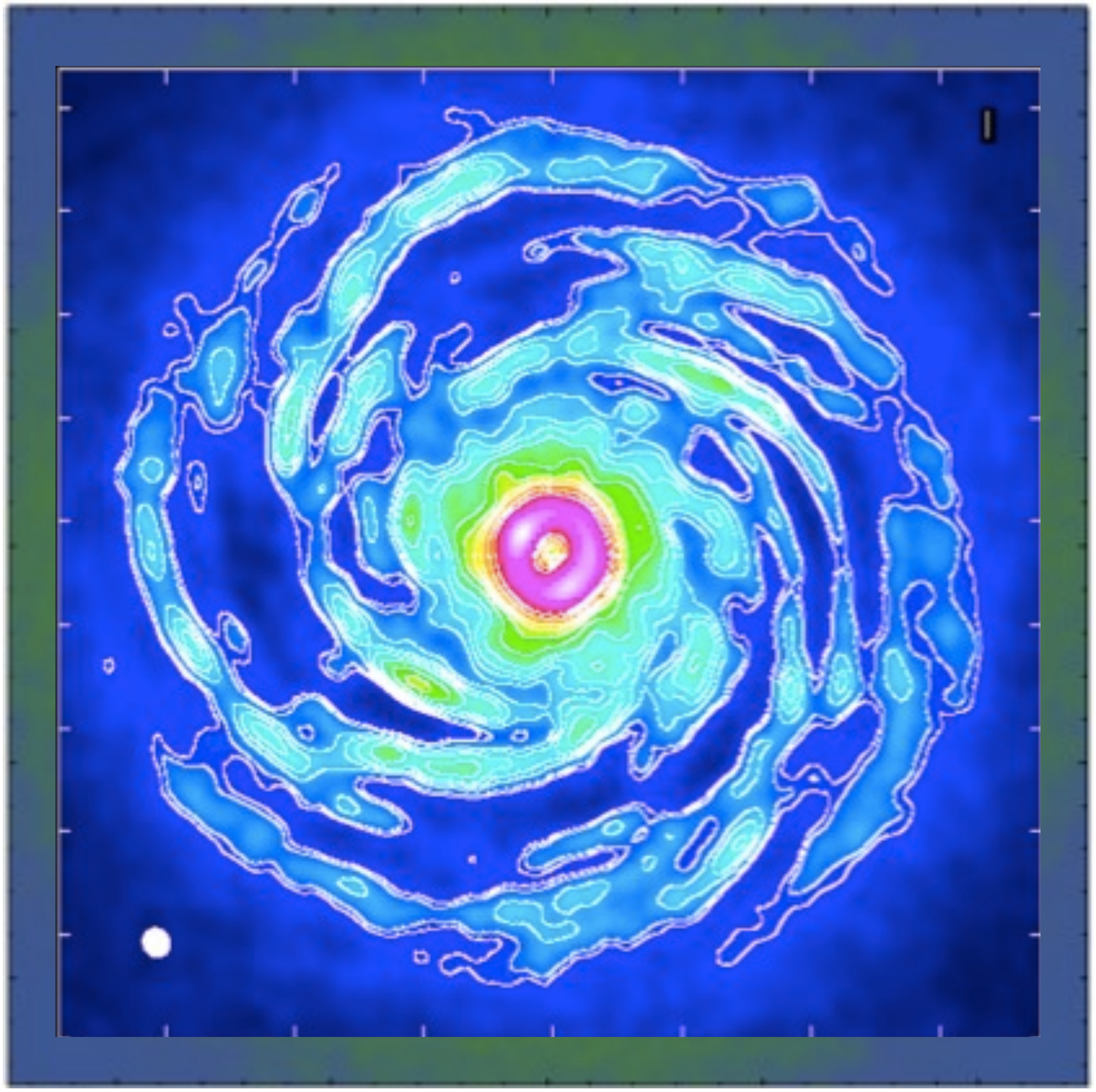

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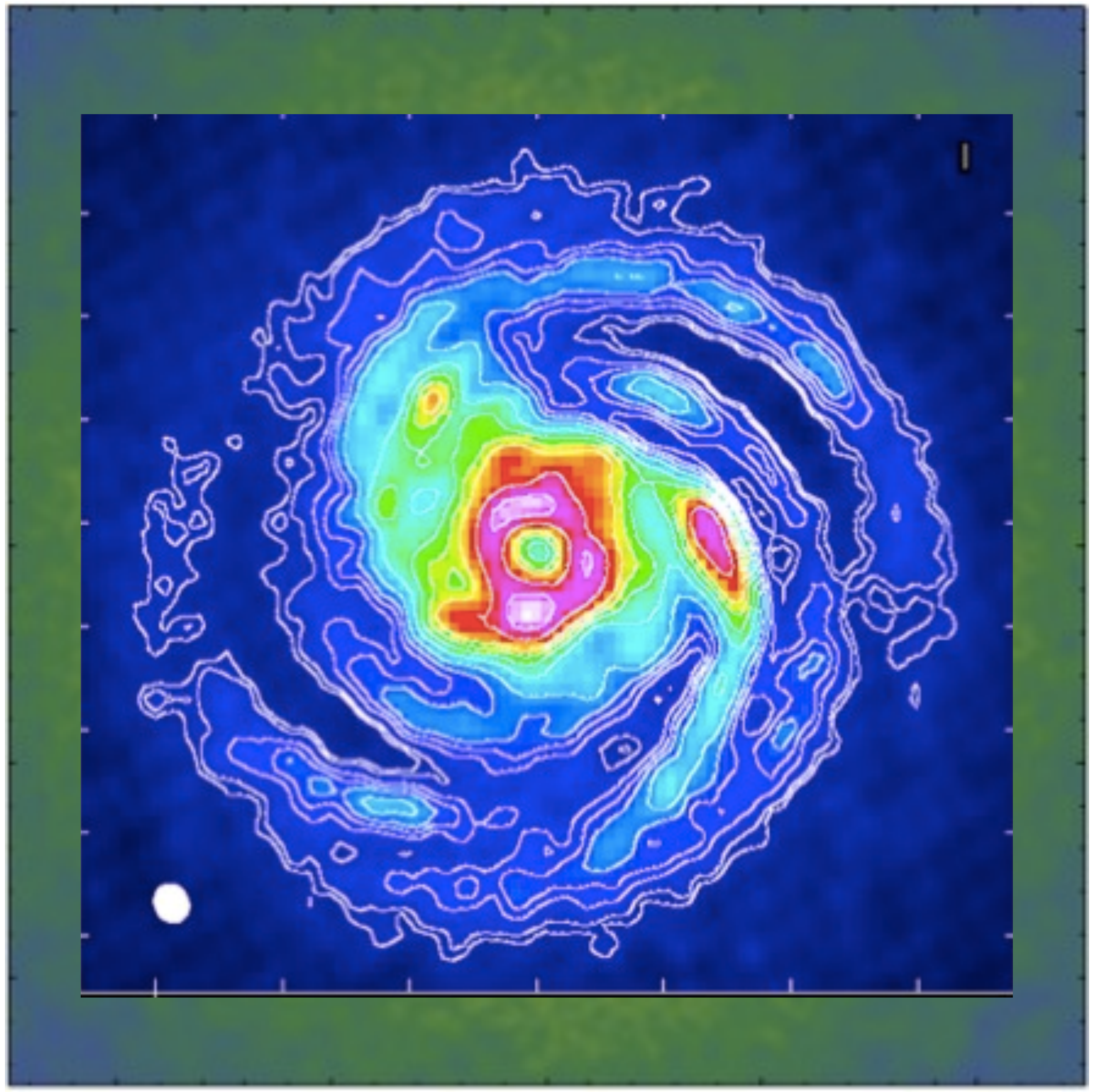

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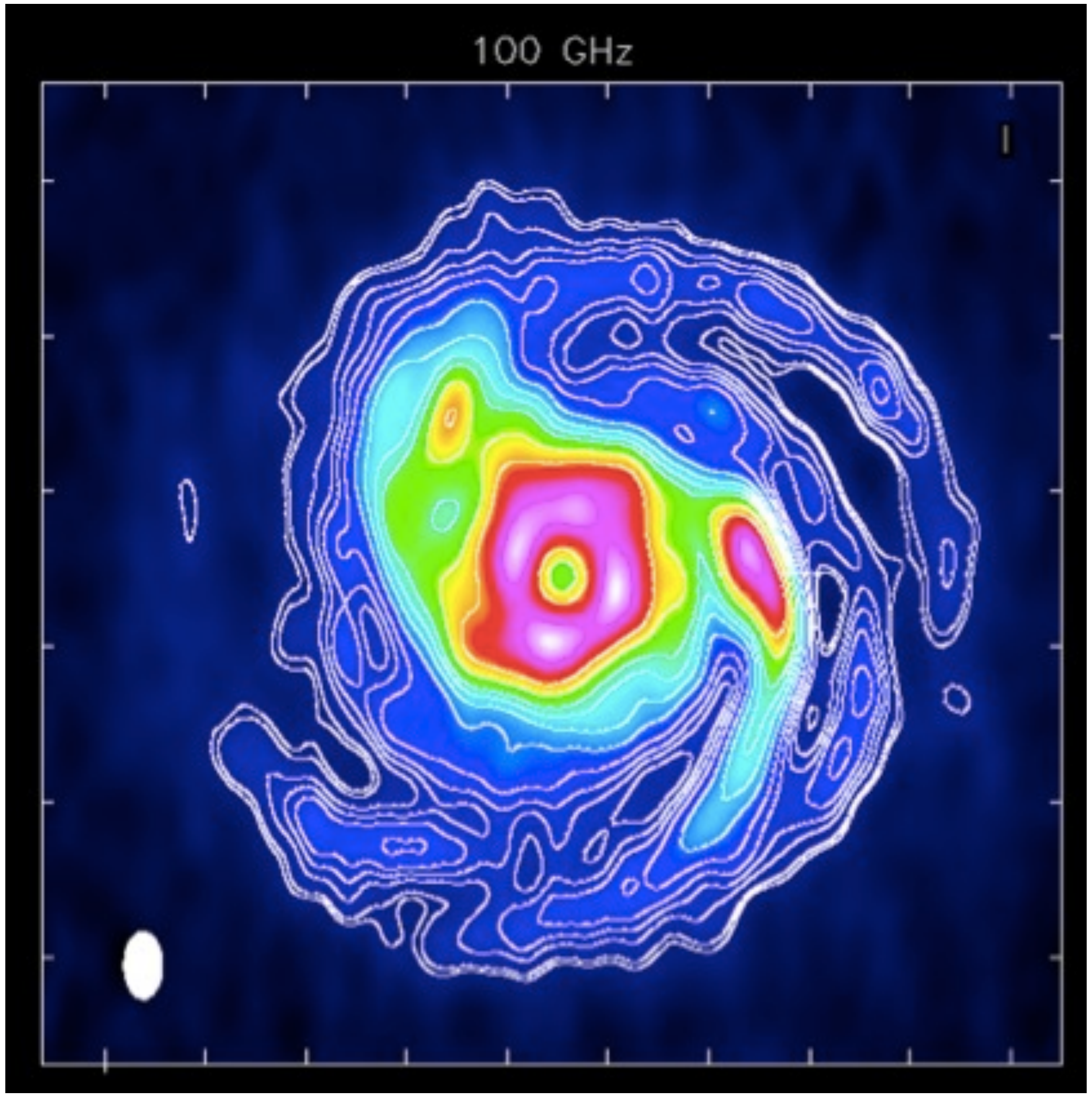
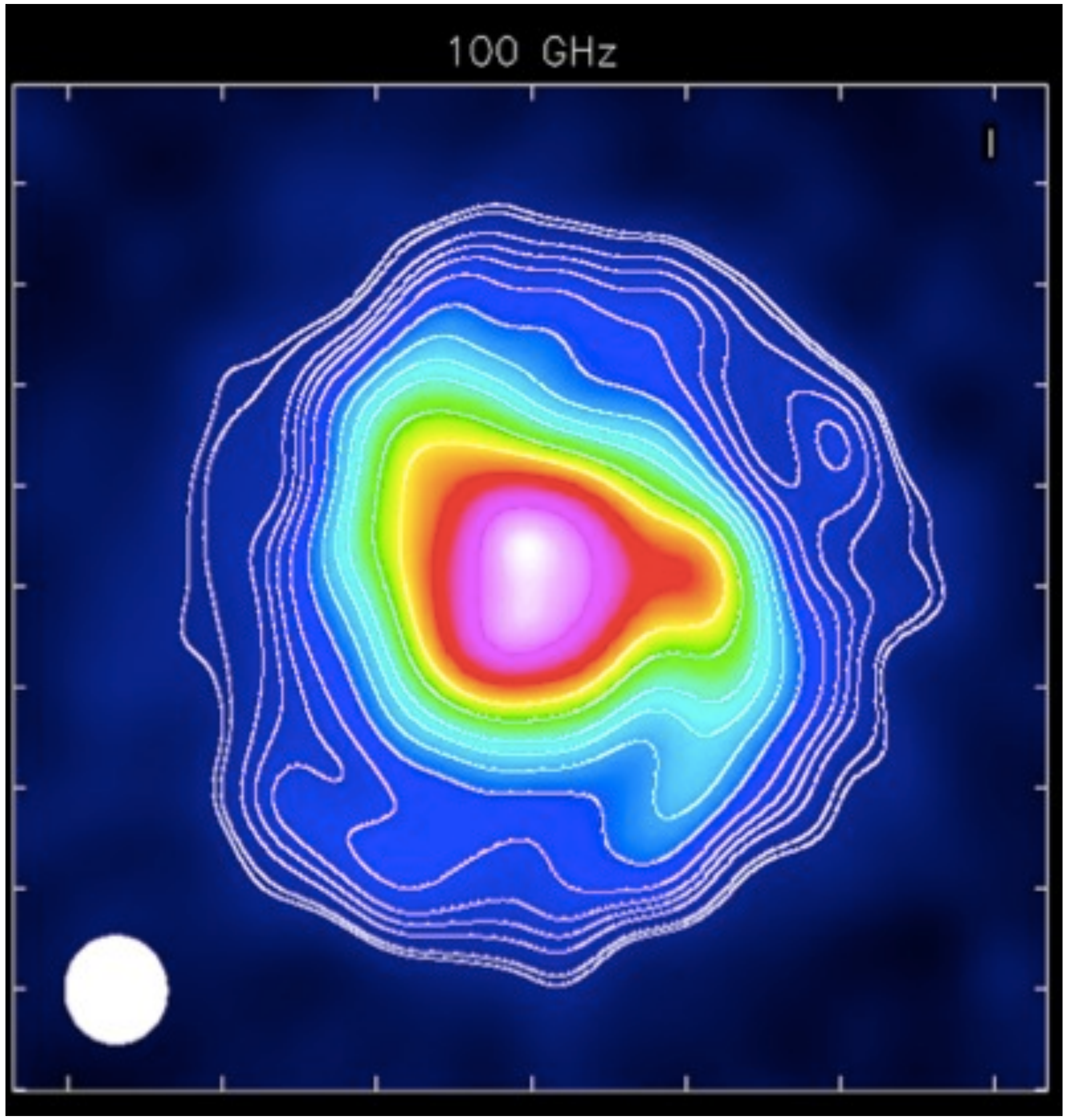
A 100AU massive disc in Taurus or Orion





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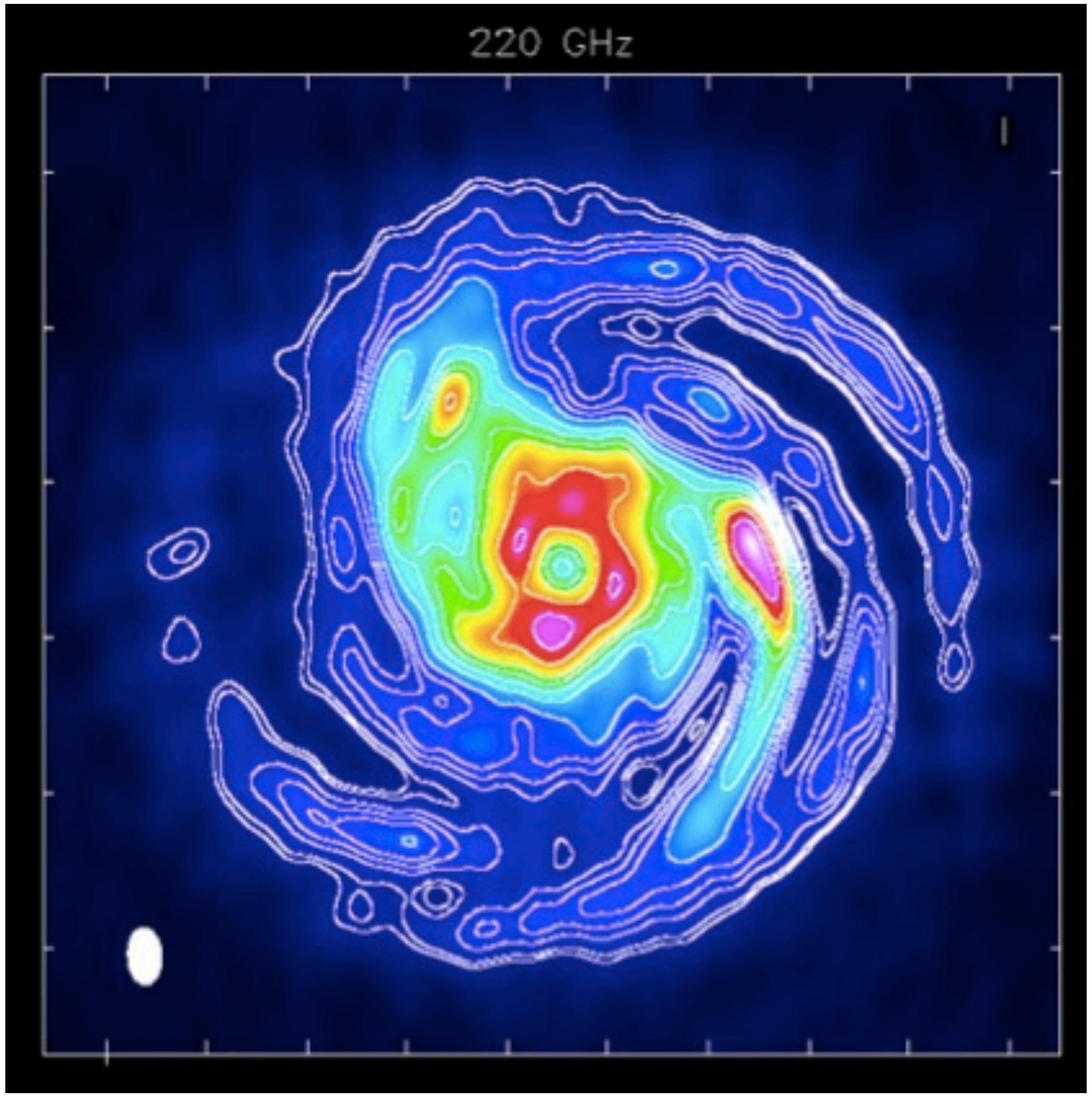
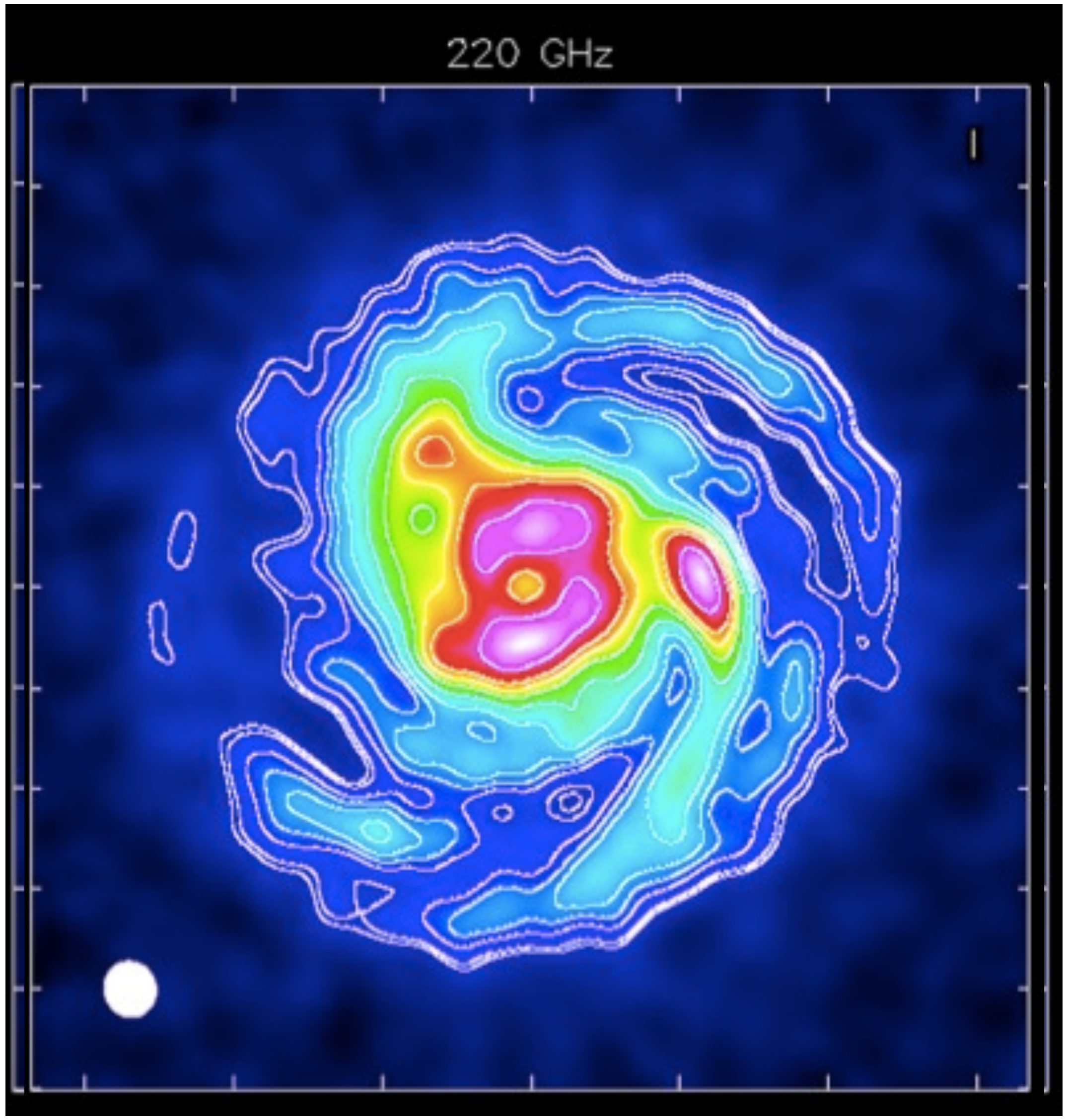


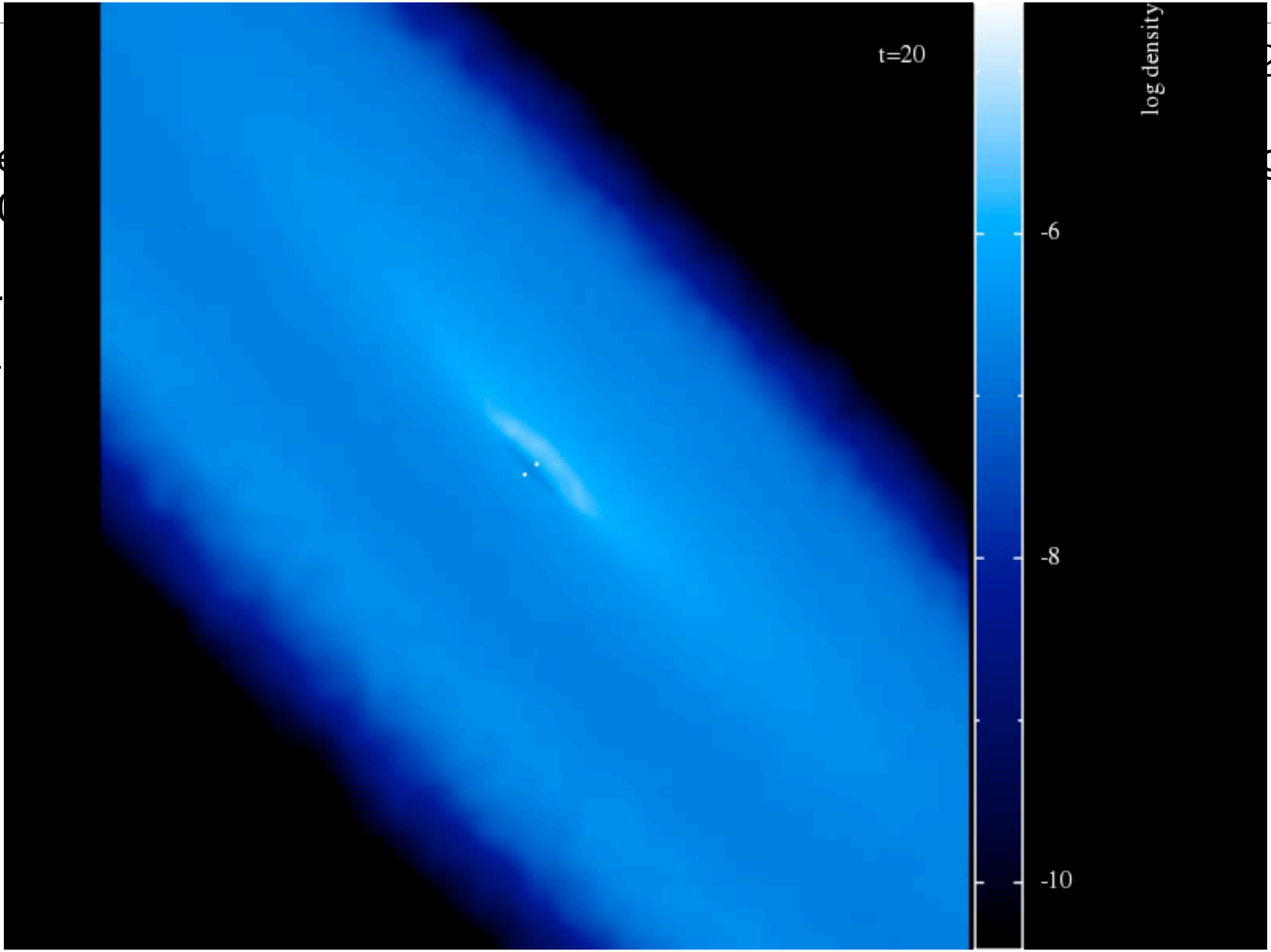


Completely off-topic: warped discs

Facchini & Lodato (2013), Lodato & Facchini (2013)

- Observations of warped disc can tell us a lot on disc internal physics (King et al. 2013)
- Interaction of a circumbinary disc with the binary can produce warps (and breaks for large misalignments)

Completely off-topic: warped discs



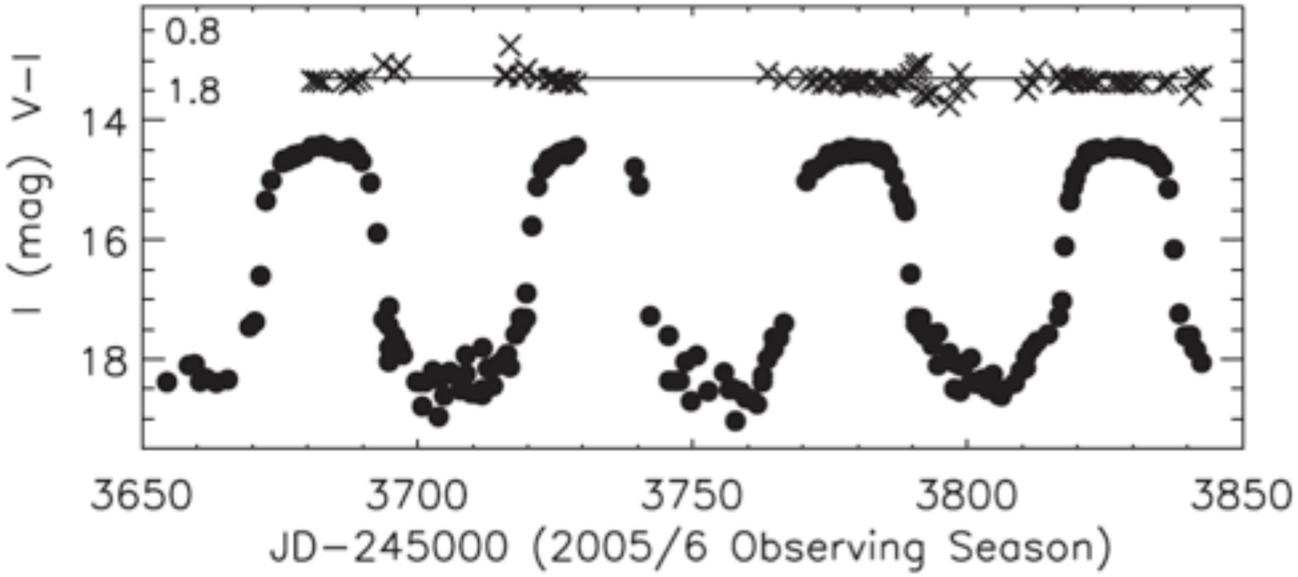
2013)

- Obse g et
  al. 2

- Inter
  brea

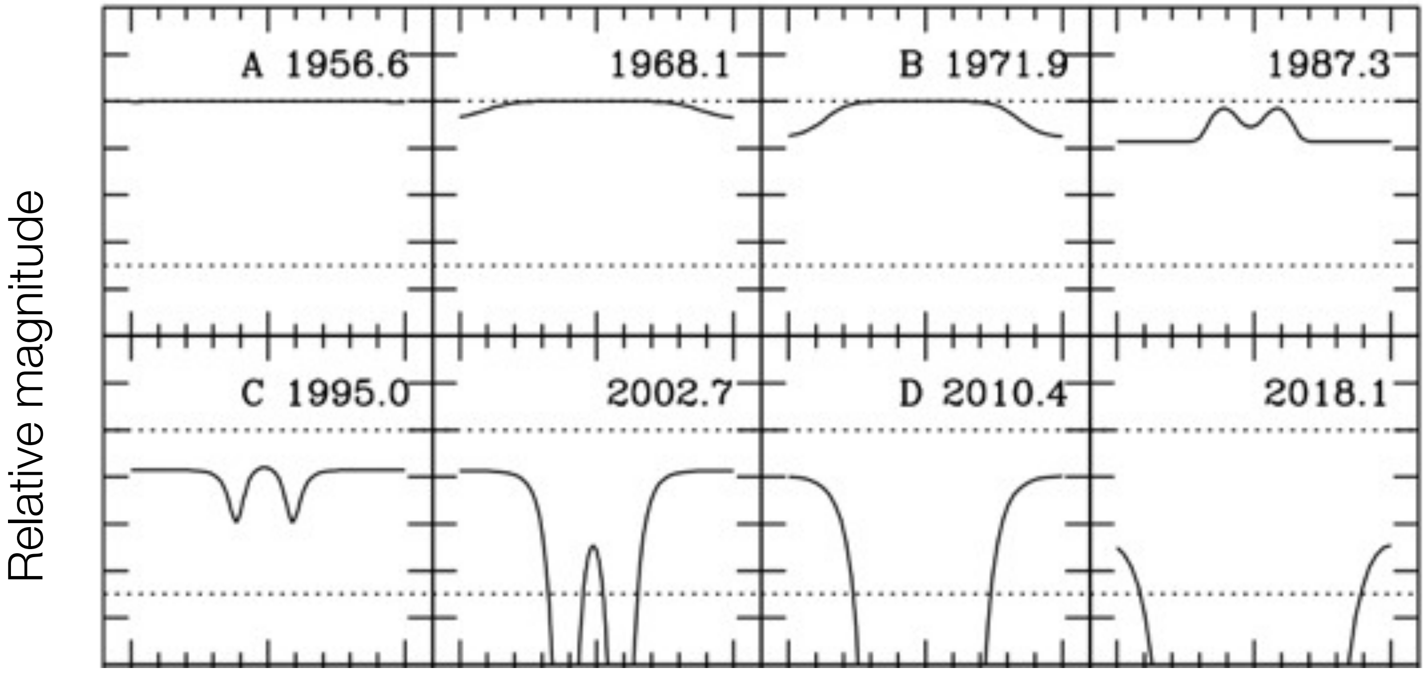
KH 15D: a peculiar binary system (Lodato and Facchini, 2013)

Herbst et al. (2010)

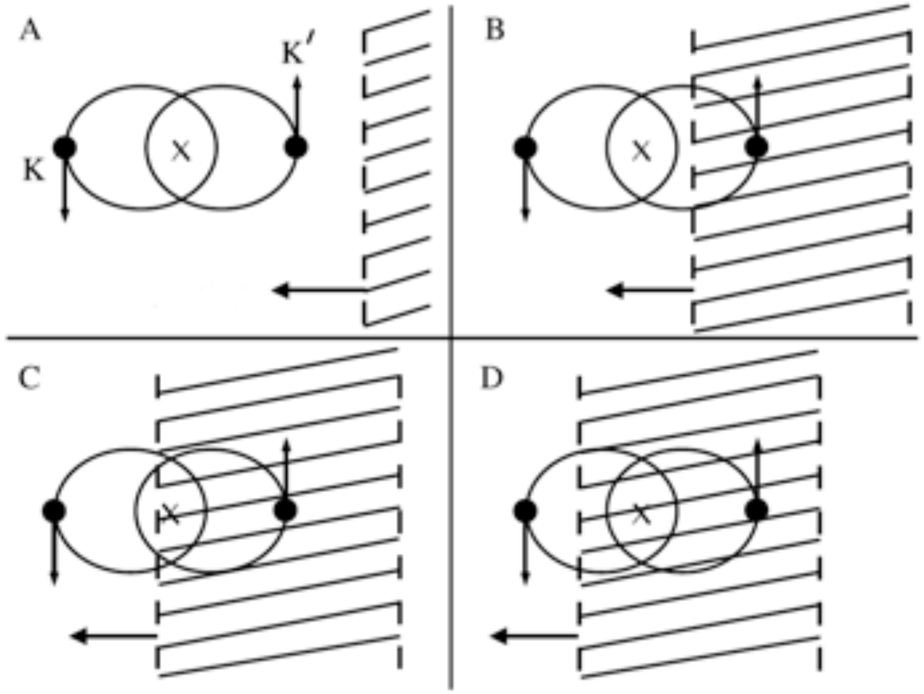


See Windemuth & Herbst (2013), arXiv: 1310.8126, for most recent observations.

Evolution of light curve explained by occultation due to a precessing narrow ring, confirmed by IR excess.



Chiang & Murray-Clay (2004)



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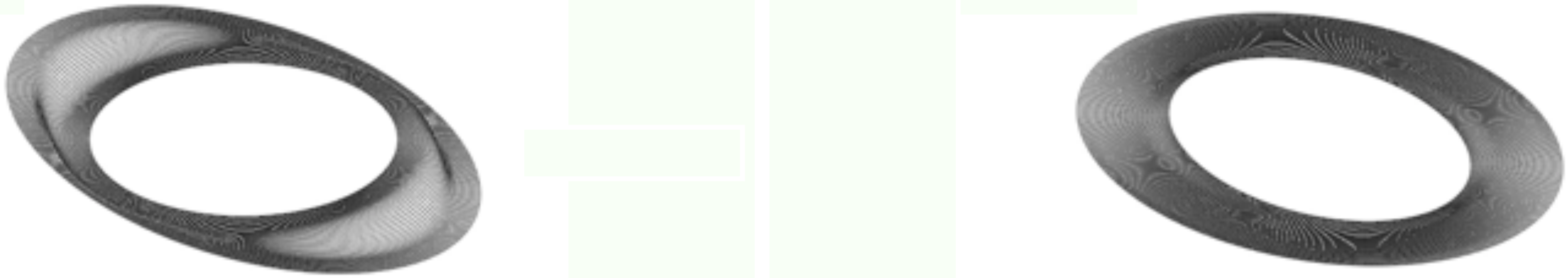
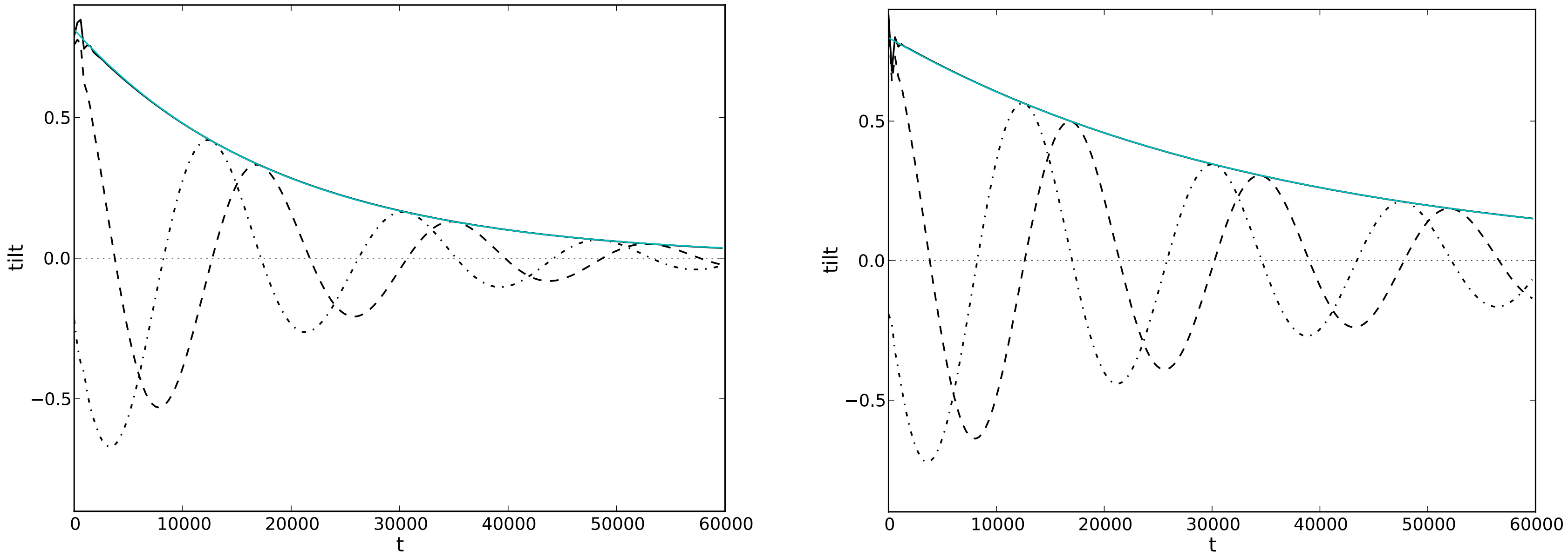


Condition: alignment timescale $t_{\rm align} > \dfrac{1}{\Omega_p}$



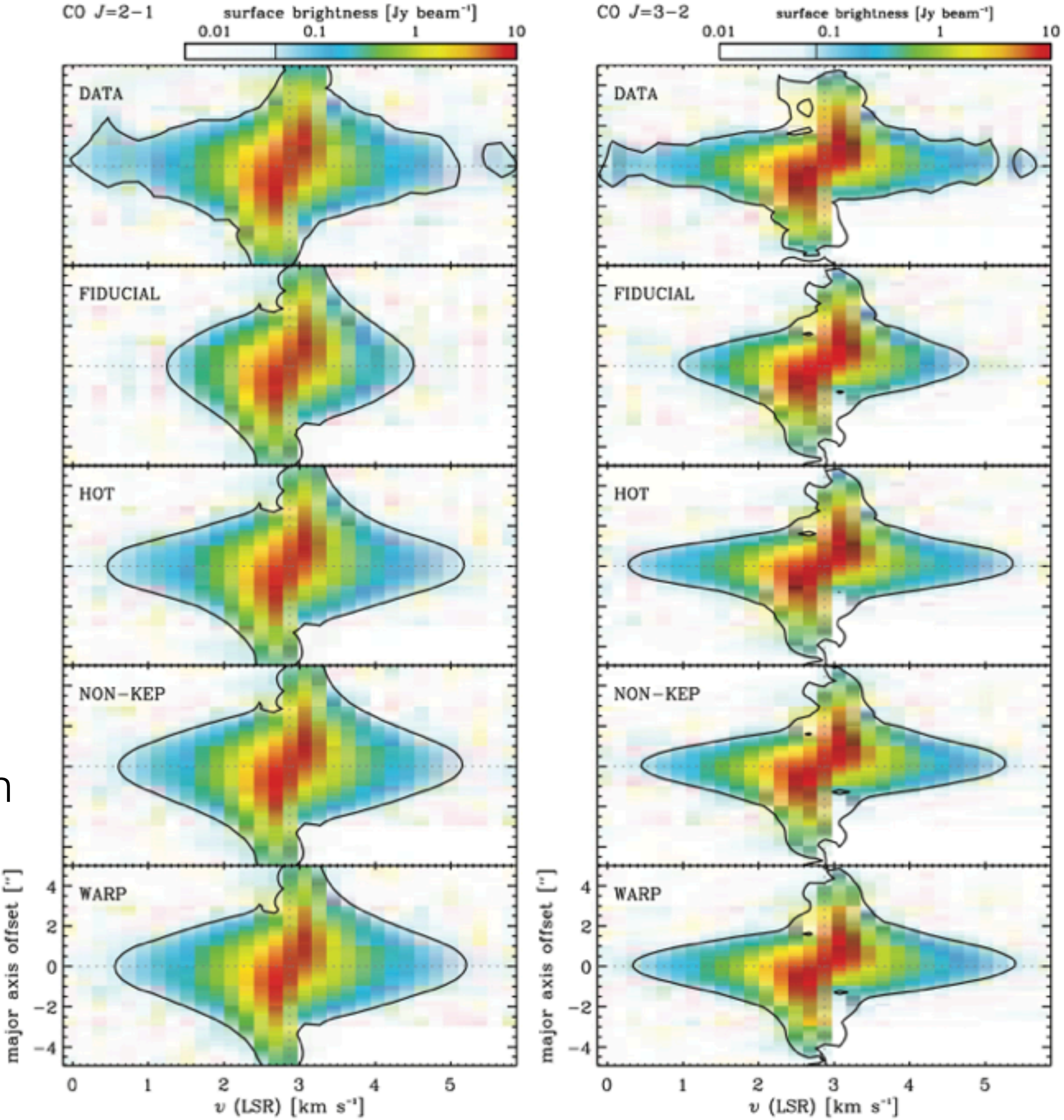
→ α (isotropic) < 0.05

We can set an upper limit for alpha!

cfr. King et al. (2013)

TW Hya

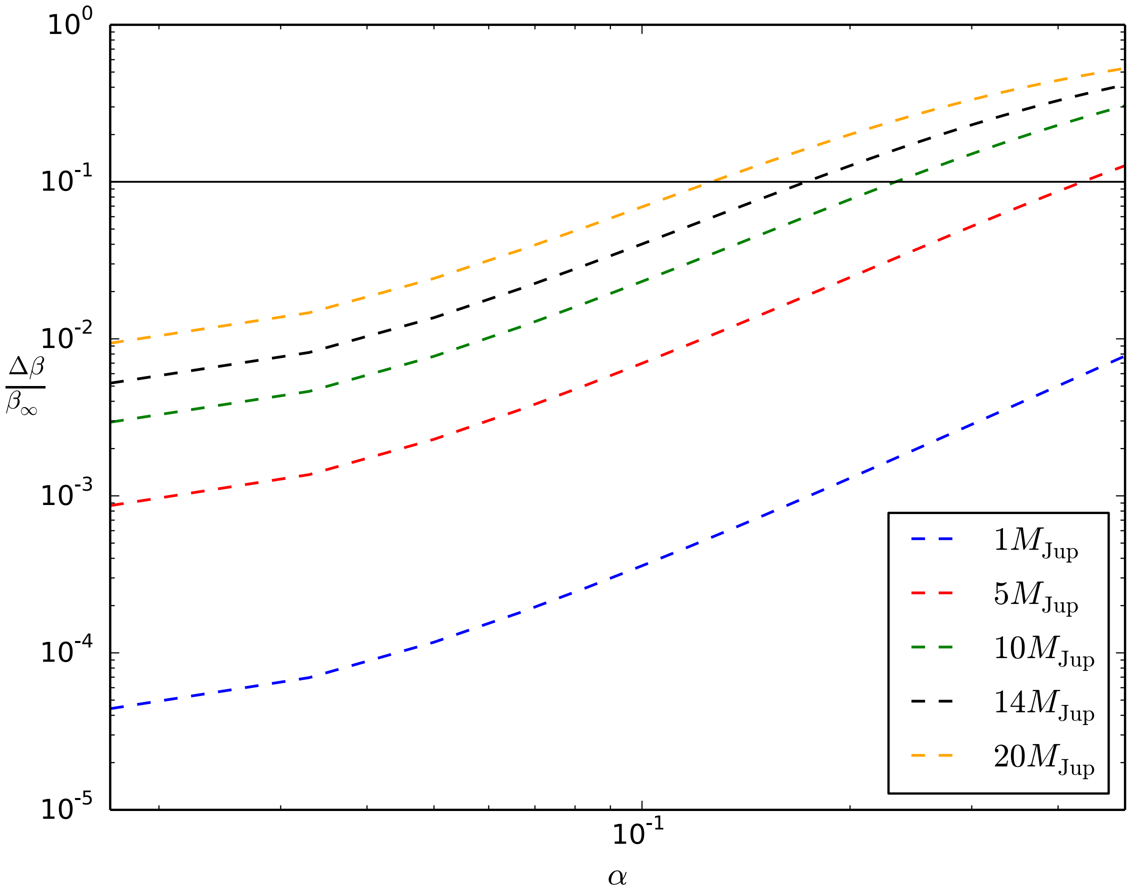
- d = 54 pc
- R_{hole} = 4 AU in submm
- Almost face-on, i = 7°
- Kinematics from ^{12}CO J=2-1 and 3-2 emission lines (Rosenfeld et al., 2012)
- Projected velocity too high in inner regions, $\Delta\beta \approx 4^\circ$



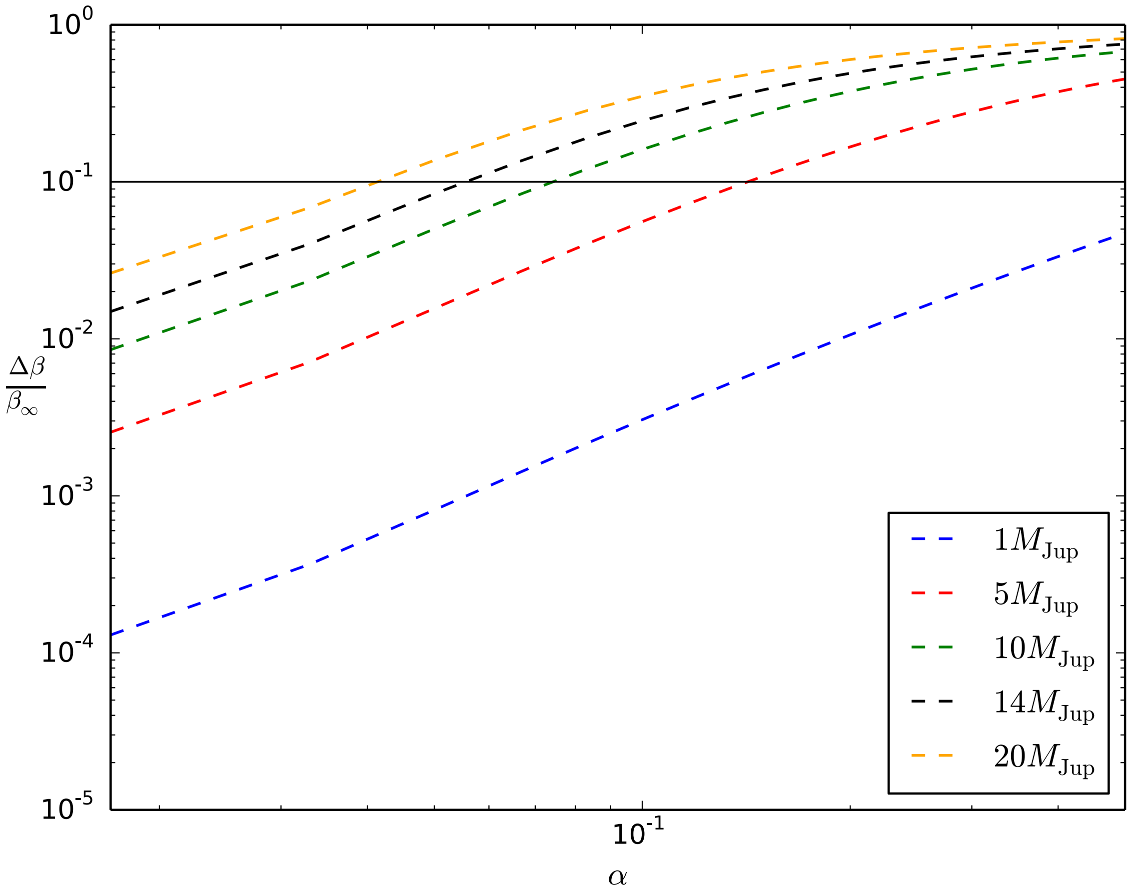
A planet in TW Hya?

$$R_{\rm hole} = R_{\rm Hill} + a = \sqrt[3]{\frac{M_{\rm p}}{3M_*}}a + a$$

3 unknown quantities: $M_{\rm p},\ \alpha,\ \beta_\infty$



q=0.26, p=0.93



q=0.53, p=0.93

Facchini, Ricci & Lodato (2014, submitted)

Conclusions

- Young protostellar discs are likely to be gravitationally unstable
- Self-regulated evolution of GI leads to sustained angular momentum transport for ~ 1 Myr, bringing the disc into the T Tauri phase
- Density waves dissipate when they become sonic
- Induced transport is local IF disc is sufficiently thin
- GI could lead to fragmentation: exact fragmentation conditions unfortunately strongly affected by numerical resolution
- ALMA will be very important not only to detect, but also to characterize the gravitational instability in young discs.