Fast wavefront reconstruction with wavelet regularization for MCAO

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Joint work with Tapio Helin

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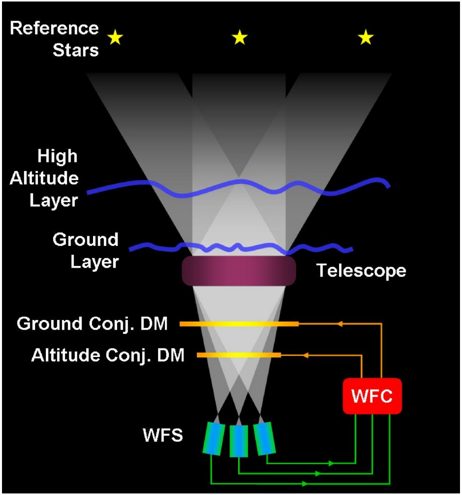
Garching, Germany

Outline

- An MCAO System
- Wavelet-based iterative method as an alternative to MVM
- Numerics: speed estimates and quality results

Multi Conjugate Adaptive Optics (MCAO)



(Source: ESO)

MCAO system:

- **several guide stars**
  each with assigned WFS
- **several deformable mirrors**
  conjugated to different altitudes

Goal:

- good quality over field of view

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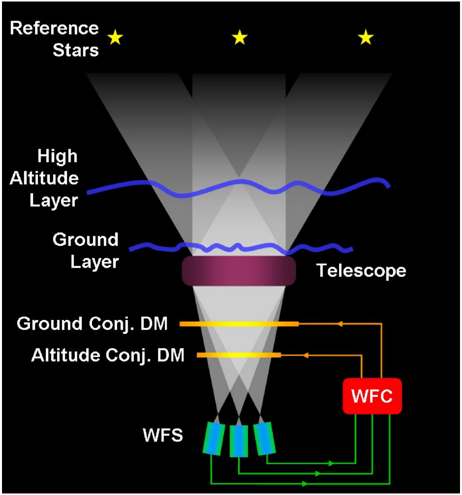


(Source: ESO)

MCAO system:

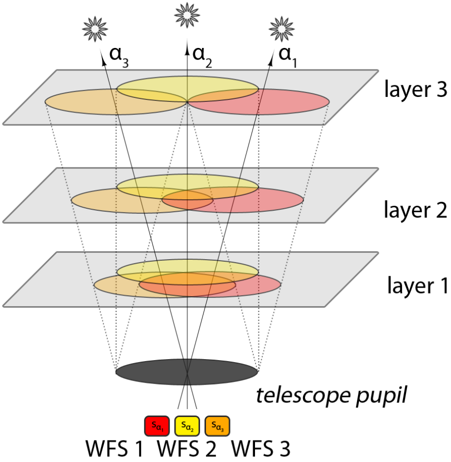
- **6 LGS, 3 NGS (tip/tilt)**
  each with assigned WFS
- **3 deformable mirrors**
  conjugated to different altitudes

Goal:

- good quality over field of view

MCAO – A Two Step Method

1. **Atmospheric tomography**



WFS measurements $\rightarrow$ layers

MCAO – A Two Step Method

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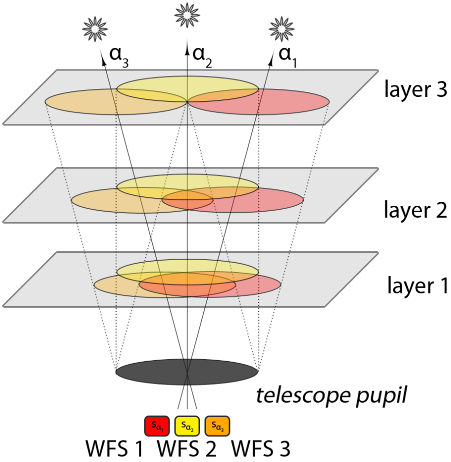
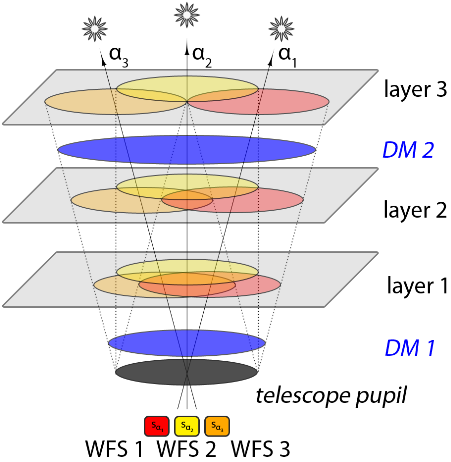


WFS measurements → layers

2. Determine mirror shapes



layers → DM shapes

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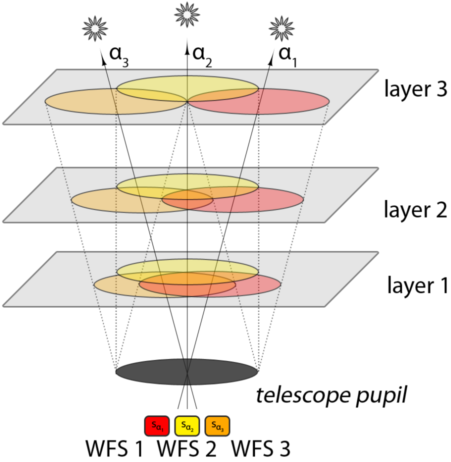

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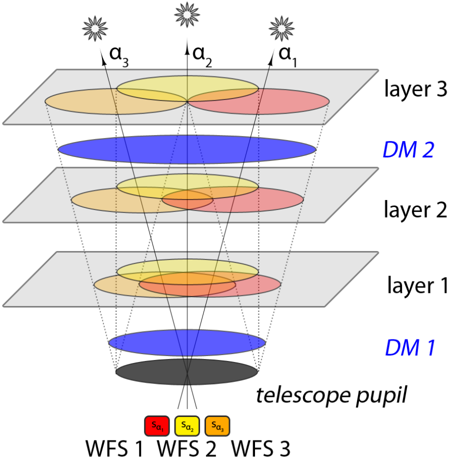

layers → DM shapes

Concept of the Approach

Standard approach:

- set up a system matrix $\mathbf{A}$ that maps DM commands to WFS measurements
- compute regularized inverse
- perform matrix-vector multiplication

$$\begin{array}{rcccl} \mathbf{A} & : & \dim \text{WFS} & \times & \dim \text{DM} \\ \text{E-ELT} & \sim & 60.000 & \times & 10.000 \qquad \text{Rec. time} \sim 2\text{ms} \end{array}$$

$\Longrightarrow$ high computational cost

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Proposed approach:

- set up a sparse system (using wavelets)
- use an iterative method (preconditioned conjugate gradient method)

$\Longrightarrow$ reduce computational cost

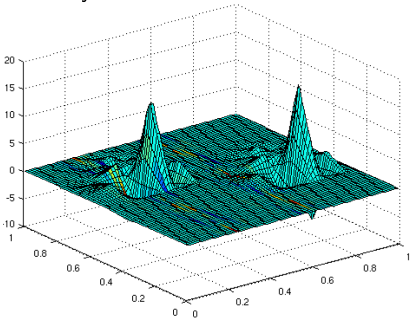
Concept of the Approach – Wavelets

Wavelet-based approach:

Concept: Use wavelets to represent the turbulence layers

Wavelets are

- a way to represent and analyze signals
- used in JPEG compression



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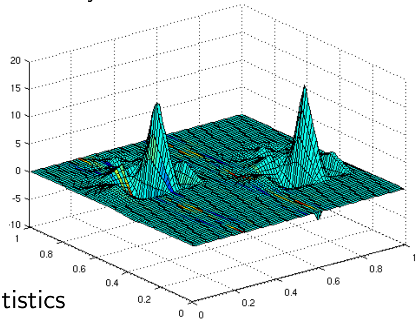


Why wavelets?

- good approximative properties
- efficient representation of atmosphere statistics
- discrete wavelet transform (DWT) is $\mathcal{O}(n)$

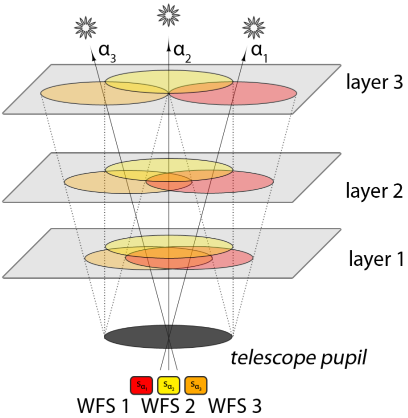
Wavelets of choice: Daubechies 3

Atmospheric Tomography Problem

To solve:

$$A\phi = s$$

atmospheric tomography operator ↗ ; turbulence layers ↑ ; noisy measurements ↖



We solve: statistics–regularized equation (maximum a-posteriori estimate)

$$(A^* C_\eta^{-1} A + \alpha C_\phi^{-1})\phi = A^* C_\eta^{-1} s$$

noise covariance ↗ ; turbulence covariance ↖

Atmospheric tomography operator:

layers $\rightarrow$ WFS measurements

$$A = \Gamma P$$

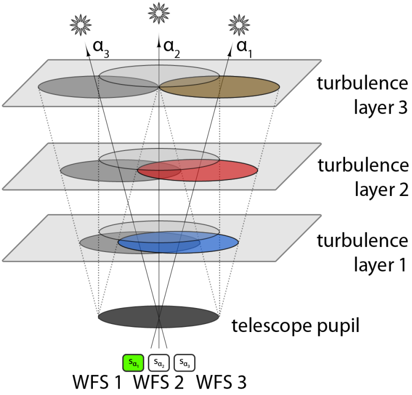
Projection operator:

layers $\rightarrow$ incoming wavefronts

$$P\phi = \varphi$$

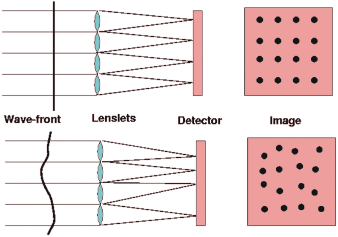
Componentwise:

$$\underbrace{\begin{bmatrix} P_{\alpha_1}^{(1)} & P_{\alpha_1}^{(2)} & P_{\alpha_1}^{(3)} \\ P_{\alpha_2}^{(1)} & P_{\alpha_2}^{(2)} & P_{\alpha_2}^{(3)} \\ P_{\alpha_3}^{(1)} & P_{\alpha_3}^{(2)} & P_{\alpha_3}^{(3)} \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} \phi^{(1)} \\ \phi^{(2)} \\ \phi^{(3)} \end{bmatrix}}_{\phi} = \underbrace{\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}}_{\varphi}$$



Shack-Hartmann operator:

incoming wavefront $\rightarrow$ WFS measurements



(Source: Tokovinin)

$$\Gamma_{\alpha_g} = \begin{bmatrix} \Gamma^x \\ \Gamma^y \end{bmatrix},$$

$$(\Gamma^x \varphi)_i = \frac{1}{|\Omega_i|} \int_{\Omega_i} \frac{\partial \varphi(x,y)}{\partial x} d(x,y),$$

$$(\Gamma^y \varphi)_i = \frac{1}{|\Omega_i|} \int_{\Omega_i} \frac{\partial \varphi(x,y)}{\partial y} d(x,y)$$

Block Shack-Hartmann operator:

incoming wavefront in all directions $\rightarrow$ WFS measurements of all sensors

$$\Gamma\varphi = s \qquad \Gamma = \begin{bmatrix} \Gamma_{\alpha_1} & & \\ & \Gamma_{\alpha_2} & \\ & & \Gamma_{\alpha_3} \end{bmatrix}$$

Turbulence Covariance Operator:
layers $\to$ layers

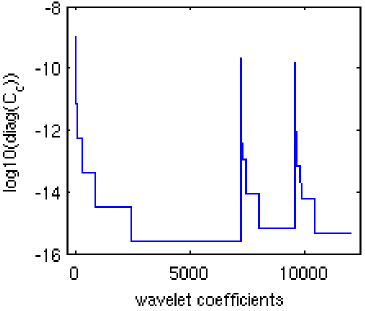
Kolmogorov power law for each layer:

$$C_\phi = \begin{bmatrix} C_\phi^{(1)} & & \\ & C_\phi^{(2)} & \\ & & C_\phi^{(3)} \end{bmatrix}, \qquad C_\phi^{(\ell)} = c^{(\ell)} \mathcal{F}^{-1} M \mathcal{F}$$

$$(Mf)(\xi) = \xi^{-11/3} f(\xi)$$

In wavelet domain:

- $C_\phi \rightsquigarrow C_c$
- C_c ... a *diagonal* matrix of weights w.r.t. wavelet coefficients



Atmospheric Tomography Problem – Discretization

Sparse discretizations

- atmospheric tomography operator $A = \Gamma P$ in *bilinear basis*
- turbulence covariance operator C_ϕ^{-1} in *wavelet basis*
- couple via: DWT

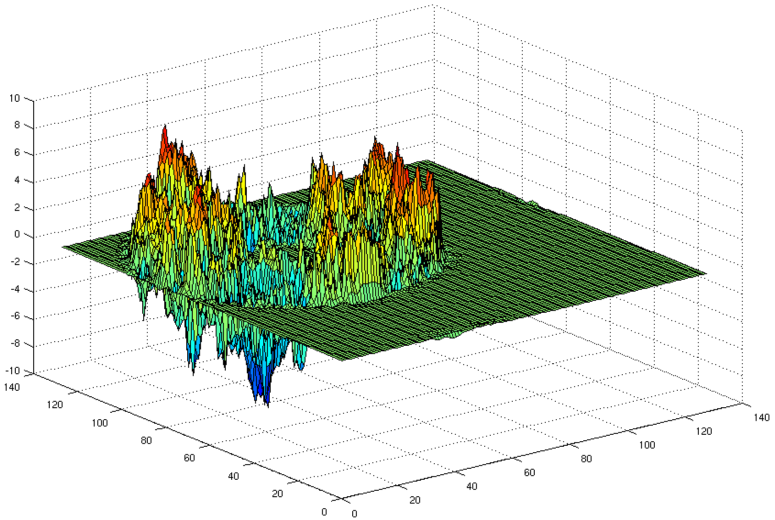
We solve: statistics–regularized equation

$$(W A^* C_\eta^{-1} A W^{-1} + \alpha C_c^{-1}) c = W A^* C_\eta^{-1} s$$

W ↗	A ↑	C_c^{-1} ↑	c ↖
discrete wavelet transform $\mathcal{O}(n)$	atmospheric tomography bilinear basis (sparse)	diagonal operator wavelet basis	wavelet coefficients

Discrete Wavelet Transform – Example

DWT for Daubechies 3 $\longleftrightarrow$ convolution with highpass, lowpass filters (6 numbers)



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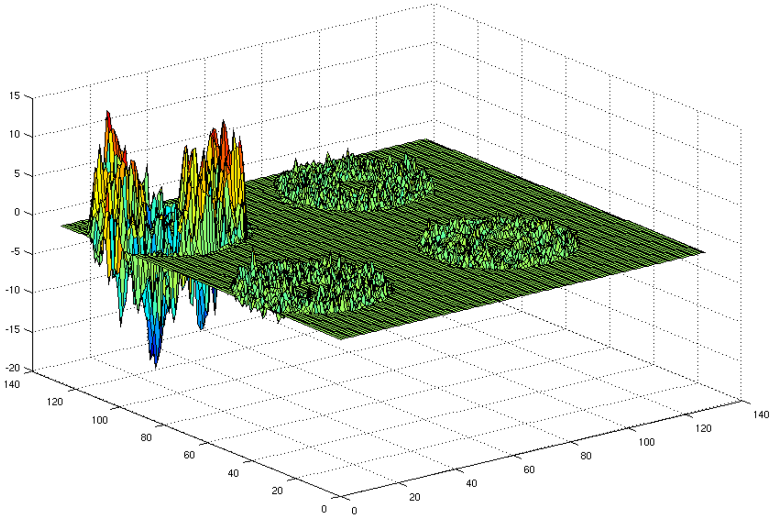

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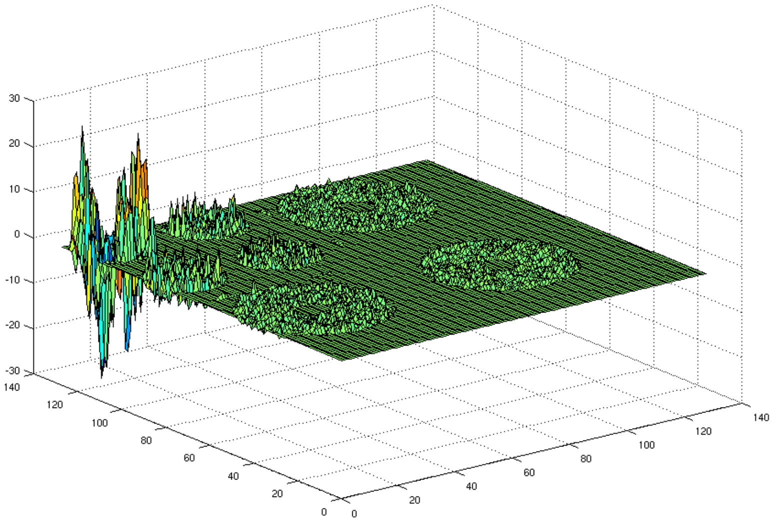


Preconditioned conjugate gradient method

Solve

$$\underbrace{\left(W A^* C_\eta^{-1} A W^{-1} + \alpha C_c^{-1} \right)}_{M} c = \underbrace{W A^* C_\eta^{-1} s}_{b}$$

or

$$Mc = b$$

using conjugate gradient (CG) method

Computational cost of CG $\longrightarrow$ cost of applying M

$\searrow$ "condition number" of M

Preconditioned conjugate gradient method

Reduce the condition number of M by preconditioning:

$$N^{-1}Mc = N^{-1}b$$

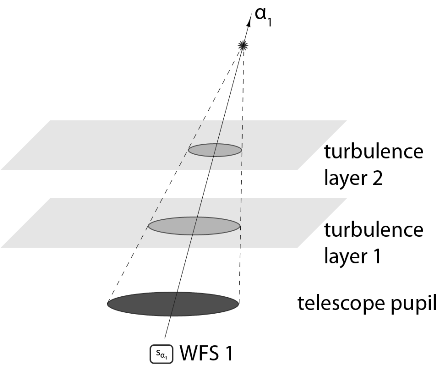
where N is such that

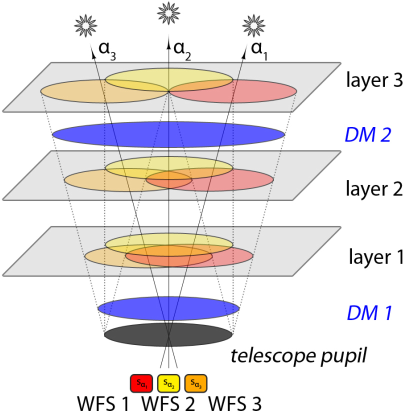
- $Nc = b$ is cheap to solve and
- $N \approx M$

Examples of preconditioners:

- Jacobi: $N = \mathrm{diag}(M) = \mathrm{diag}(W\,A^*\,C_\eta^{-1}A\,W^{-1}) + \alpha\,C_c^{-1}$
- Multigrid: $N = M$ on coarser scale (fewer bilinear elements, wavelet scales)
  - N^{-1} ... exact solution or
  - N^{-1} ... a few steps of an iterative method

MCAO Features

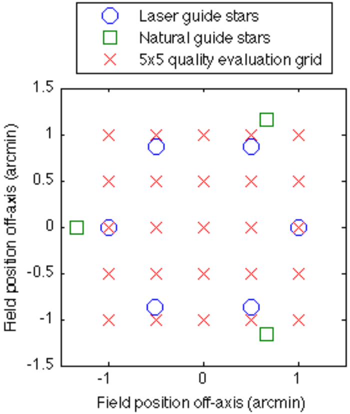




1. Laser guide stars:
   - spot elongation
   - tip/tilt indetermination
   - cone effect
2. Reconstructing more layers than mirrors (fitting step)
3. Pseudo-open loop control (POLC)

Configuration:

- Telescope aperture diameter: 42 m
- 6 laser guide stars (LGS)
  - 84×84 subapertures
- 3 natural guide stars (NGS)
  - 1 sensor with 2×2 subapertures
  - 2 sensors with 1×1 subapertures
- 3 DMs
  - at 0, 4000, 12,700 m
  - 9,296 active actuators



Simulated data:

- OCTOPUS – official simulation tool of ESO
- 9 atmospheric layers
- quality evaluated in 25 directions

Speed estimates

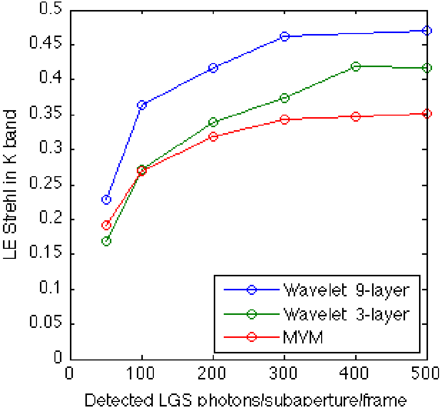
Number of floating point operations

	MVM	wavelets 3-layer CG	wavelets 3-layer Jacobi PCG	wavelets 9-layer CG
FLOP	562,779,840 100%	25,413,497 4.5% (20 it.)	7,335,062 1.3% (5 it.)	130,272,942 23.1% (20/4 it.)

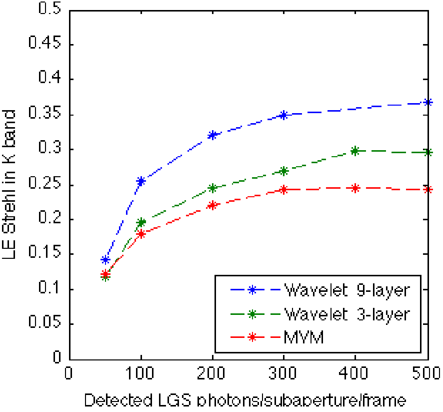
wavelets 3-layers: discretization grid coincides with bilinear actuators of the mirrors

Results in OCTOPUS: Quality

Low flux: LGS @ 50-500 photons/subap/frame, elongated spots
NGS @ 500 photons/subap/frame



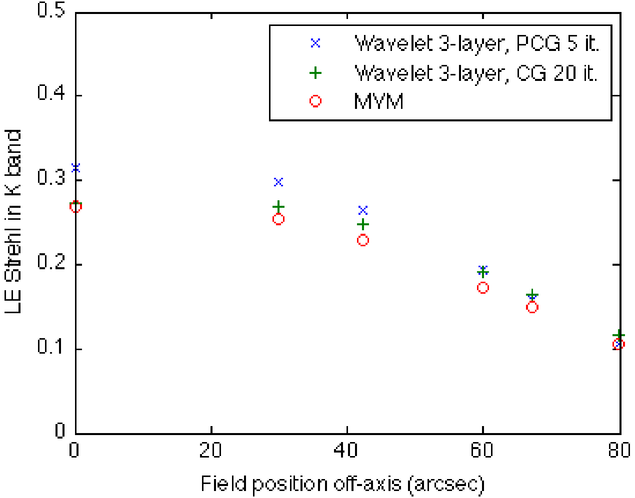
On-axis



Field average

Results in OCTOPUS: Preconditioning

Low flux: LGS @ 100 photons/subap/frame, elongated spots
NGS @ 500 photons/subap/frame



Summary and Outlook

Wavelet method

- CG-based
- efficient representation of atmosphere statistics
- wavelet basis $\longleftrightarrow$ bilinear basis: discrete wavelet transform

Wavelet method: numerical results

- high reconstruction quality
- promising speed estimates

Outlook

- RTC prototype
- Parallelization
- Multigrid preconditioner