CuReD – fast wavefront reconstruction towards the real world

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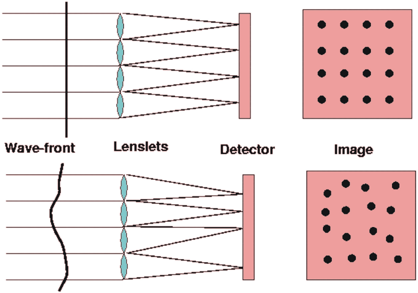
Shack-Hartmann wavefront sensor



(source: Tokovinin)

SH WFS model:

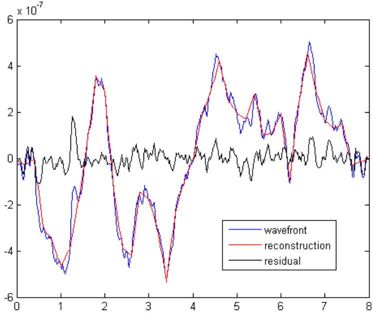
$$s_x[i] = \frac{1}{|\Omega_i|} \int_{\Omega_i} \frac{\partial \phi}{\partial x},$$

$$s_y[i] = \frac{1}{|\Omega_i|} \int_{\Omega_i} \frac{\partial \phi}{\partial y}.$$

measurements are averaged gradients of the wavefront

Goal: Reconstruct the wavefront from the measurements of the Shack-Hartmann wavefront senor

The Cumulative Reconstructor idea



measurements in 1D:

$$s_i = \int_{a_i}^{a_{i+1}} \frac{\partial \phi}{\partial x} dx$$

and with

$$\phi(a_{i+1}) - \phi(a_i) = \int_{a_i}^{a_{i+1}} \frac{\partial \phi}{\partial x} dx$$

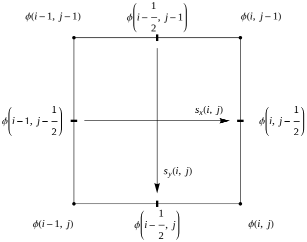
gives the iteration rule

$$\phi(a_{i+1}) = \phi(a_i) + s_i$$

fast and accurate reconstruction of the wavefront in 1D

Question: Can this be adapted to the 2D?

Extension to 2D



modified Hudgin geometry

- bilinear influence functions gives measurements

$$s_x(i,j) = \phi\left(i, j-\frac{1}{2}\right) - \phi\left(i-1, j-\frac{1}{2}\right)$$

(s_y respectively)

- reconstruct chains on the midpoints

$$\phi^x(i, j-\frac{1}{2}) = \phi^x\left(i-1, j-\frac{1}{2}\right) + s_x(i,j)$$

- connect the mean values of the x chains to the mean chain of the orthogonal y chains

$$\underset{m}{\text{mean}}\ \phi^x\left(m, j-\frac{1}{2}\right) = \frac{1}{N}\sum_{m=1}^{N} \phi^y\left(m-\frac{1}{2}, j-\frac{1}{2}\right)$$

- bilinear interpolation onto the corner points

adaptation of the algorithm to general geometries possible

The Cumulative Reconstructor - Graphical Representation

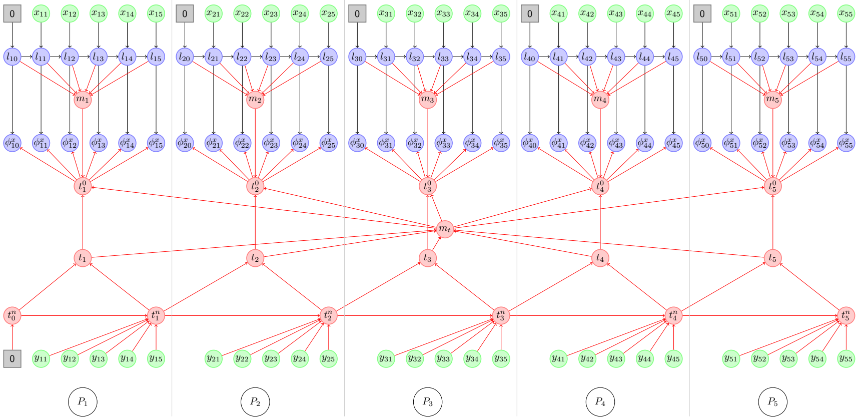


Figure: Graphical representation of the CuRe for a 5x5 subaperture domain

not included: second part of computation for modified Hudgin geometry, transition to Fried geometry

Domain decomposition - fighting the noise propagation

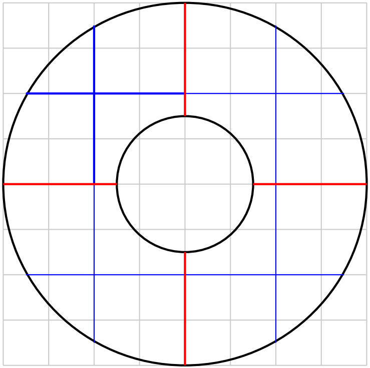


Figure: Decomposition of the domain for CuReD

decomposition of the domain to use the good properties of CuRe at small apertures

- divide the domain into subdomains Ω_i
- reconstruct the wavefront using CuRe on the subdomains
- connect these reconstructions at the boundaries $\partial\Omega_i$

1	2
3	4

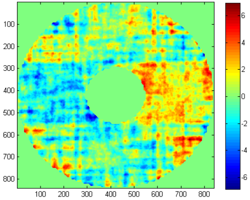
connect four parts at a time, hierarchically

$$d_{12} = \|\phi_2(i, \text{first})\| - \|\phi_1(i, \text{last})\|,$$

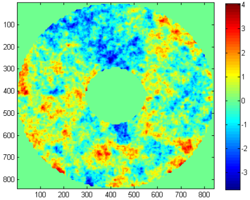
and d_{34}, d_{13}, d_{24} respectively.
shift according to these differences

Noise propagation CuReD

example residuals



decomposition level 0



decomposition level 4

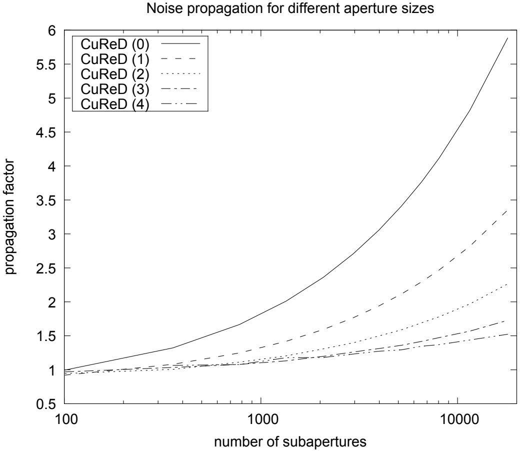
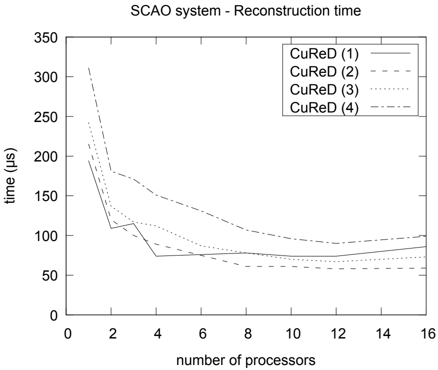


Figure: noise propagation for CuRe, CuReD and FrIM vs. number of subapertures

Computational Complexity

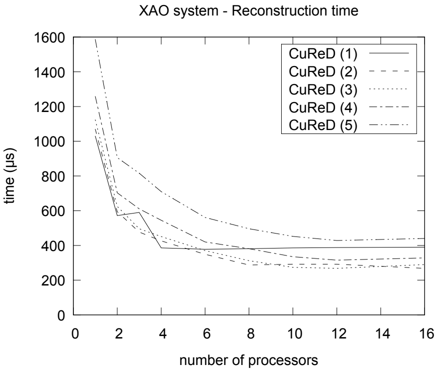
- theoretical computational complexity: $20n$

reconstruction of a 84x84 sensor (μs)



- MVM time: $12ms$

reconstruction of a 200x200 sensor (μs)



- MVM time: $402ms$

Octopus - simulation example of a telescope

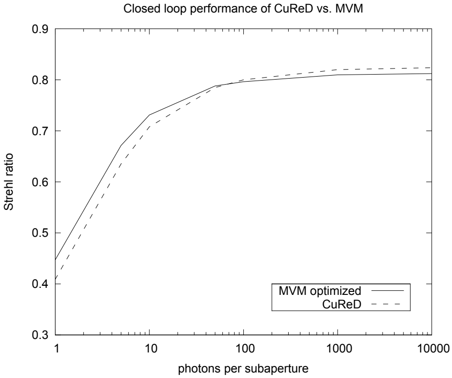


Figure: Comparison of MVM and CuReD vs. photon flux for a simulation of 1s

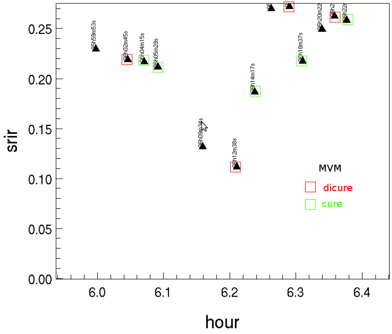
- E-ELT sized telescope (42 m)
- Simulation running at 1kHz
- Shack-Hartmann wavefront sensor with 84x84 subapertures
- annular aperture, 28% central obstruction
- bilinear mirror according to Fried geometry
- simple temporal integrator control

$$\phi(t+1) = \phi(t) + g\Delta\phi$$

CuReD in the real world

- Test on the HOT bench:
  - using the MATLAB prototype
  - setup of subaperture map for CuReD
  - derivation of the mirror misalignment from system matrix
  - interpolation step from the Fried geometry to the actuator positions of the mirror
  - first system tests
- Test within DARC:
  - C prototype used
  - run with a 7x7 SH sensor
  - first tests with wavefront-to-actuator map
  - on-sky tests promising

- Las Palmas, Canary Islands (Spain), Roque de los Muchachos (2344m)
- 4.2m mirror diameter
- CuReD – tests performed by Durham University



Misalignment estimation using CuReD

Real life: in a real system the wavefront sensor and the mirror are not perfectly aligned

Problem: generic algorithms do not take the mirror misalignment into account
quality is not as good as possible

Goal: incorporate the misalignment into the CuReD interpolation step

Idea: use the CuReD to estimate the mirror misalignment

Assumption: we have a (measured) interaction matrix

Misalignment estimation algorithm

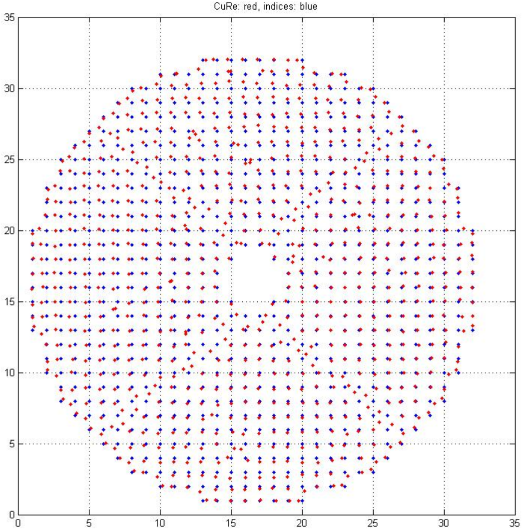
To estimate the misalignment of the mirror this algorithm is performed:

- obtain the measurements for pushing a single actuator from the interaction matrix
- use the CuReD algorithm to reconstruct the wavefront from these measurements
- calculate the (weighted) center of gravity for the reconstruction
- repeat these steps for all actuators
- select a sample of "credible" actuators
- obtain the misalignment parameters by minimization, e.g.

$$\min \sum_i \sqrt{(x_i^c - (a x_i + d_x))^2 + (y_i^c - (a y_i + d_y))^2} \tag{1}$$

  to estimate the shift (d_x, d_y) and the scale a from the calculated actuator positions x_i^c, y_i^c

Calculated actuator positions – map

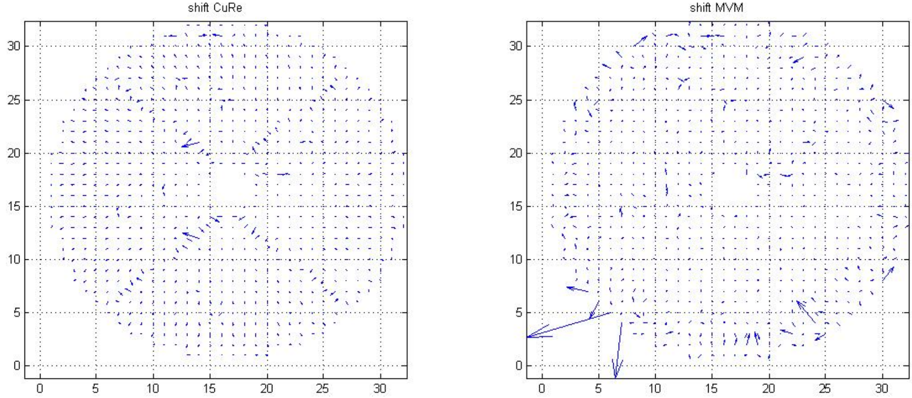


Map showing the expected vs. the with CuReD + CoG calculated actuator positions

blue: expected actuator positions
red: calculated actuator positions

"credible" actuator: without boundary and area at the spiders

Misalignment estimation: CuReD vs. MVM



Comparison of the misalignment estimation with CuReD and the reconstruction matrix (MVM)

References

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