Reducing the Fractal Iterative Method (FRiM)

to the cost of half an iteration

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Reminder on the Fractal Iterative Method (FRiM)

- minimum-variance reconstruction algorithm for large Adaptive Optics systems

Thiébaut & Tallon JOSA A, 2010

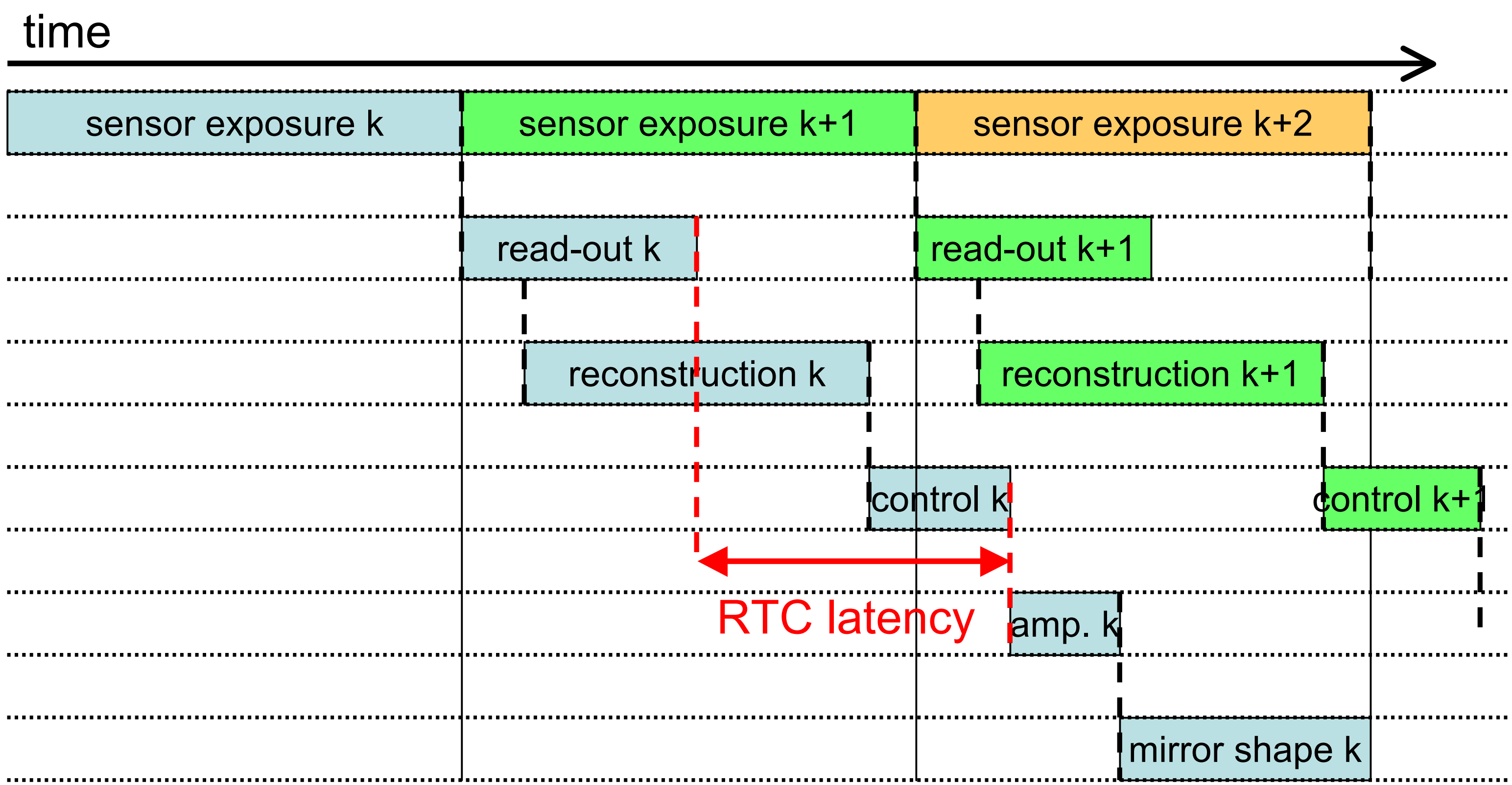
- performance assessed on Octopus, ESO end-to-end AO simulator, since 2008

- preconditioned conjugate gradients (PCG), as most of the iterative AO reconstructors

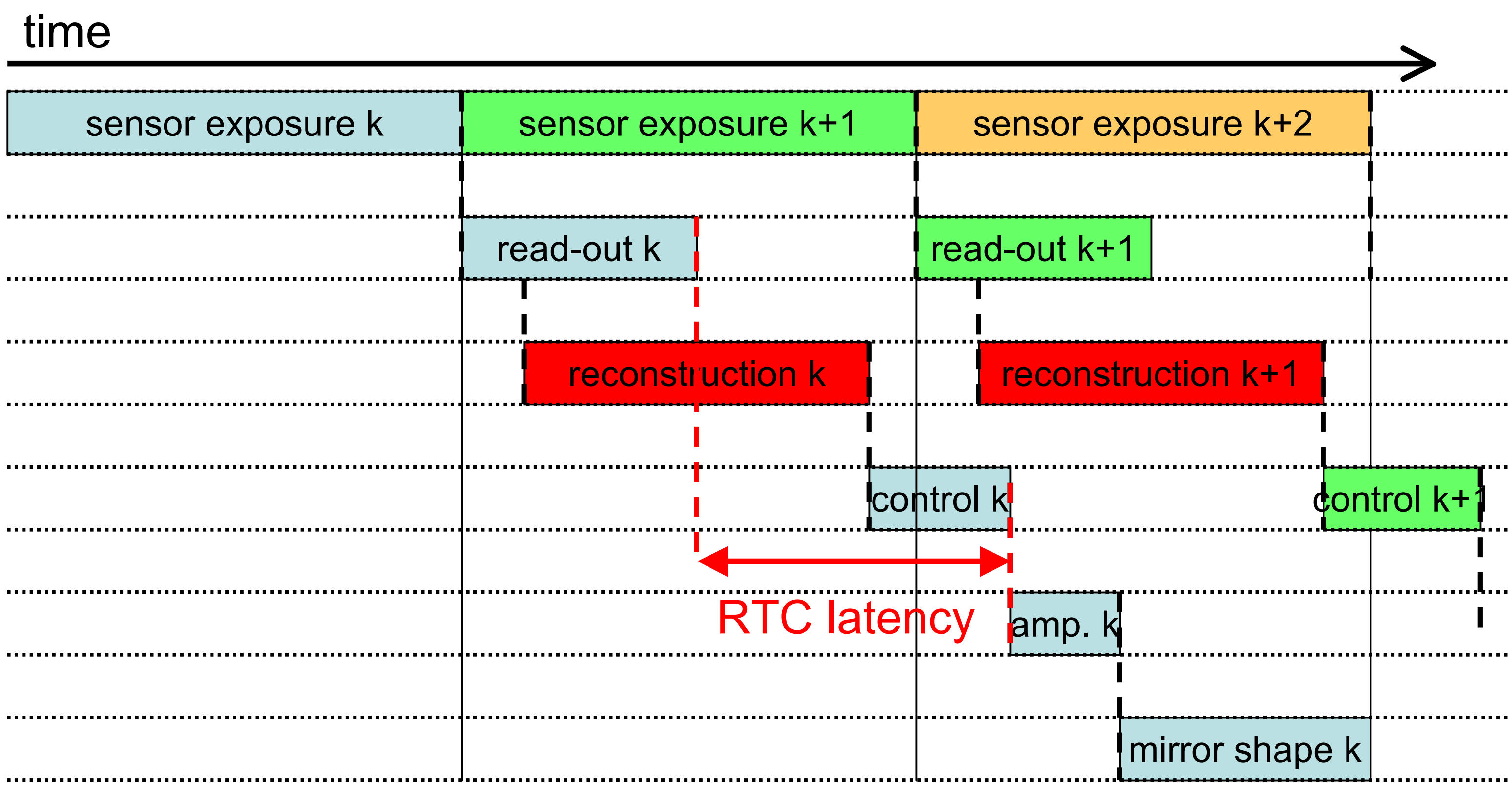
- iterative method PROS
  - no full matrix multiplication
  - neither matrix inversion, nor matrix storage in FRiM
  - sparse/fast operators in FRiM
  - easier to update the model

- iterative method CONS?
  - **latency increases with the number of PCG iterations**

RTC latency requirements in AO

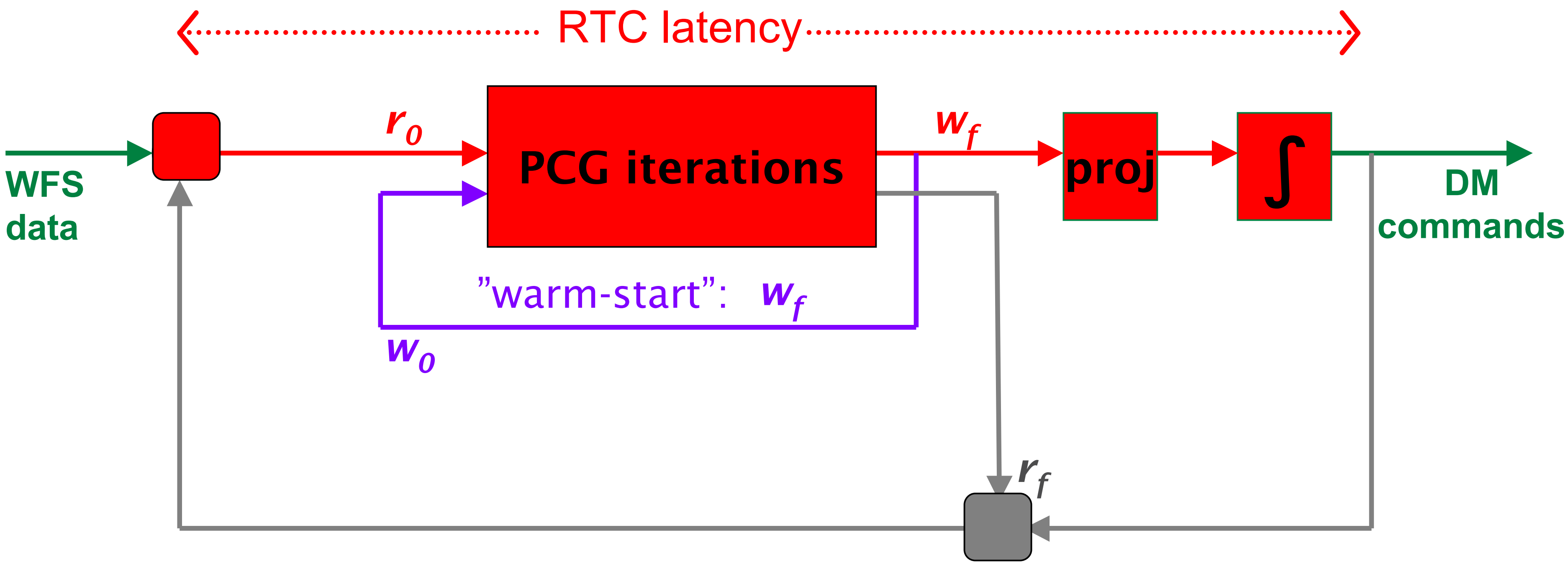


RTC latency requirements in AO



- most time spent in the reconstruction, and particularly in the PCG

FRiM iterations *vs* RTC latency requirements



- projection on DM (proj) and pseudo open-loop control with integrator (∫) (Gilles, 2003)
- r_0 and r_f, PCG residuals (initial and final) for iterations
- w_0 and w_f, starting guess and final estimate
- **"warm-start":** $w_0 \leftarrow w_f$ (because slow evolution of the turbulent wavefront)

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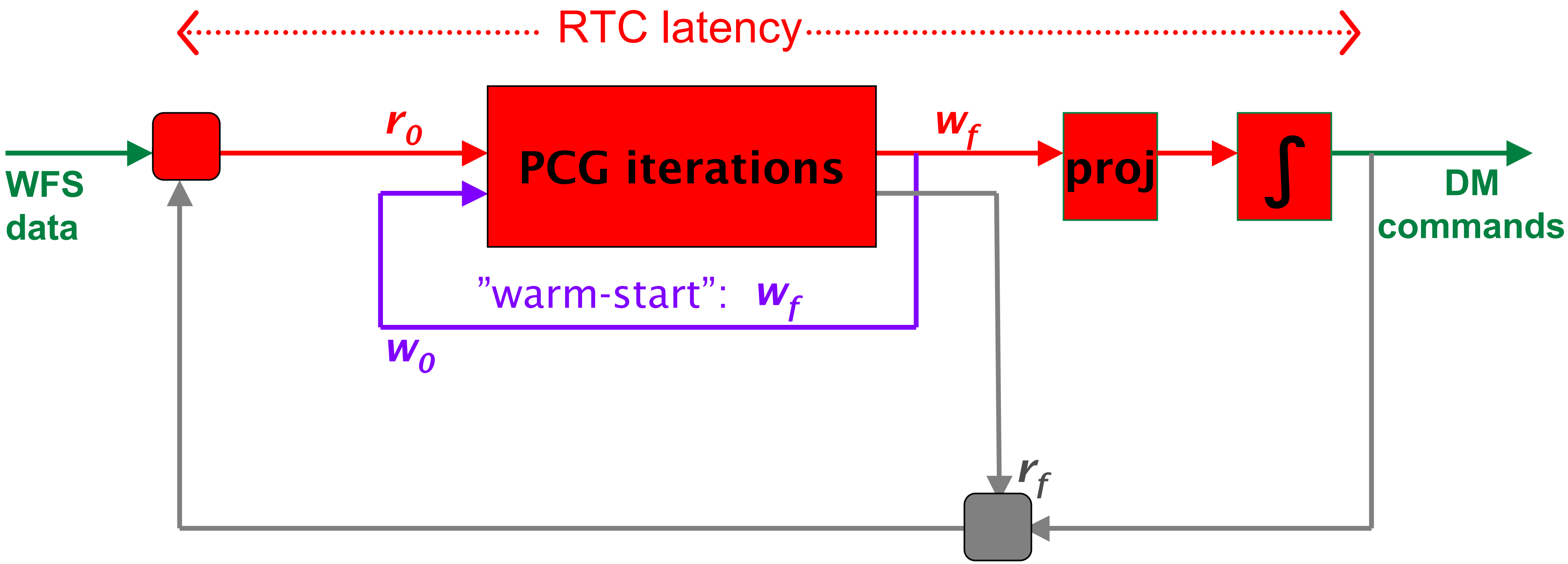

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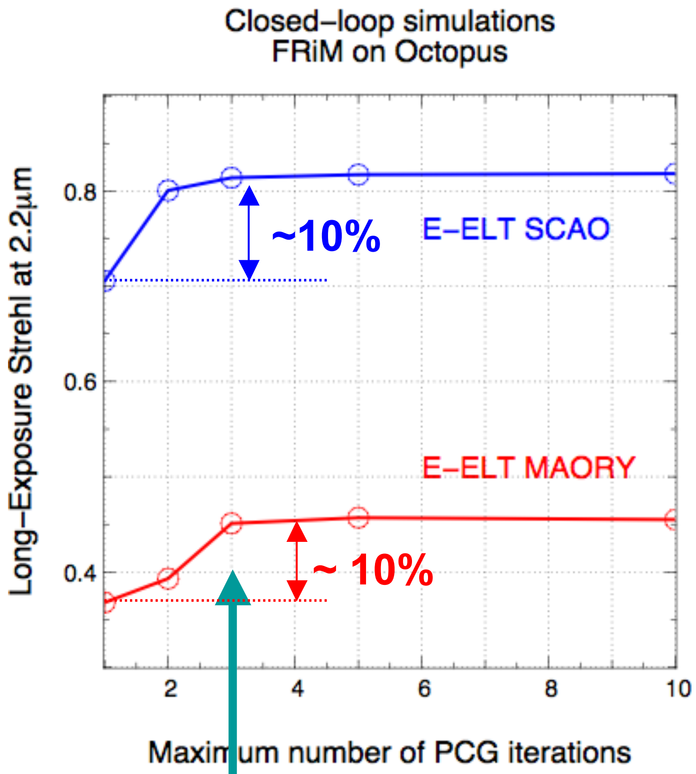
- **“warm-start”:** $w_0 \leftarrow w_f$ (because slow evolution of the turbulent wavefront

- iterations need to be done sequentially
- more iterations means more latency until commands are applicable
- several iterations required to obtain best performance in this classical scheme

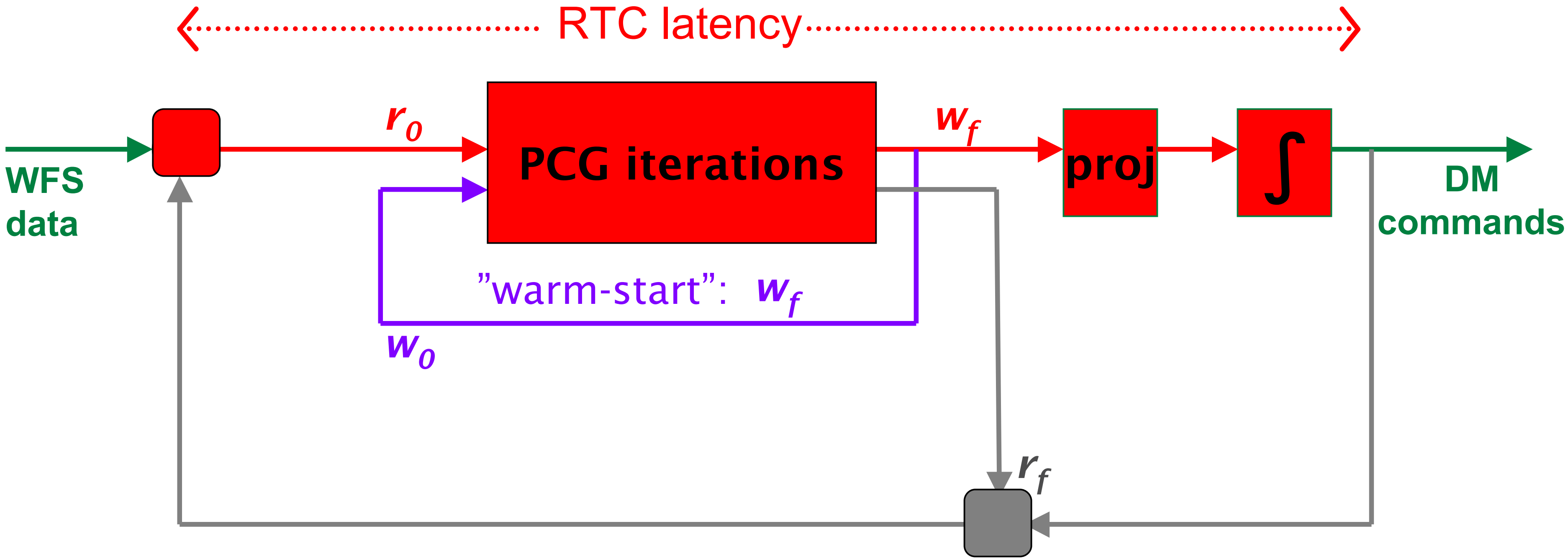
Examples of FRiM performance *vs* #iterations

- **E-ELT**
  - 42-m telescope (central obs.:0.28)
  - 500 Hz loop frequency
  - Cn2 profile : 9 layers, r_0= 12.9 cm
- **single-conjugate AO:**
  - DMs: 0 km (85 x 85)
  - 1 NGS, 84x84 subap.
  - 10^5 photons/frame/subap.
  - τ_0 = 2.8ms
- **multi-conjugate AO (MAORY):**
  - DMs: 0 km (85 x 85), 4 km (47 x 47), 12.7 km (53 x 53)
  - 6 LGS, 84 x 84 subap.
    - on a Ø 2 arcmin circle
    - 500 ph/subap.
    - RON 3e-
  - 2 NGS for tip/tilt, 1 NGS for 2 x 2 subap.
    - on a Ø 2.7 arcmin circle
    - 500 ph/subap., H band
    - RON 5e-



3 PCG iterations required for the best performance

back to the classical scheme



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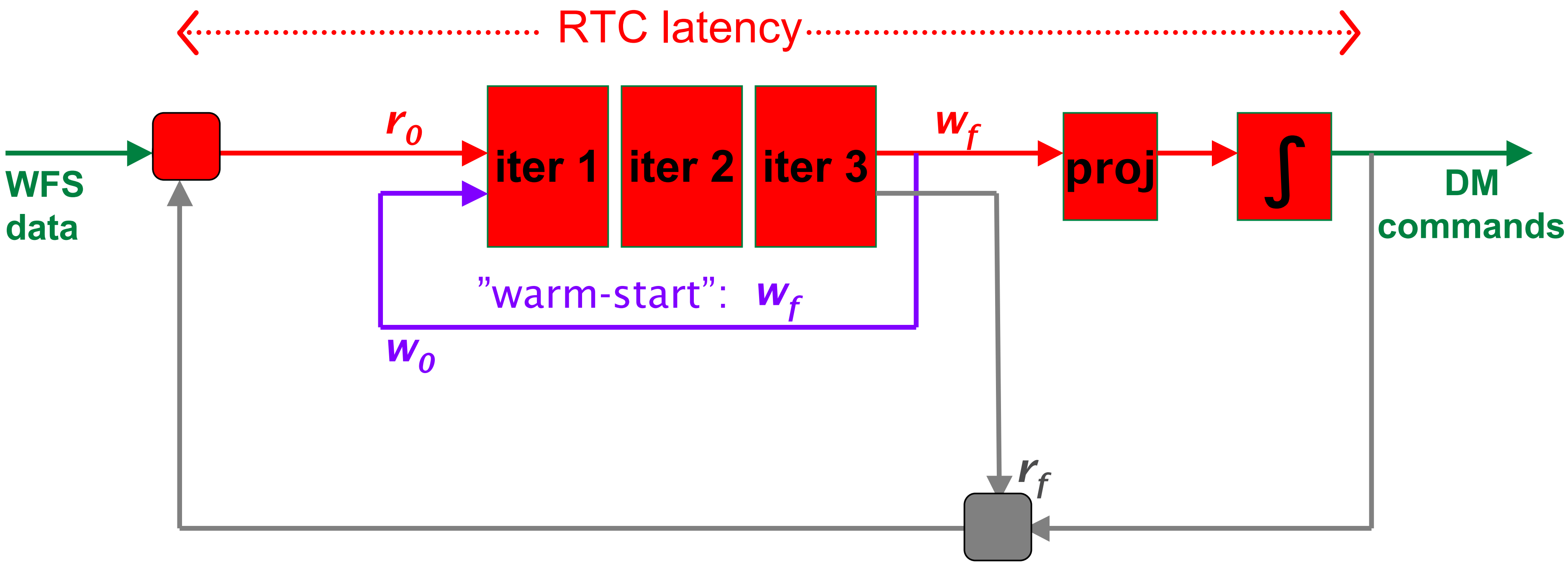

- but 3 iterations of FRiM takes too long for the latency specifications

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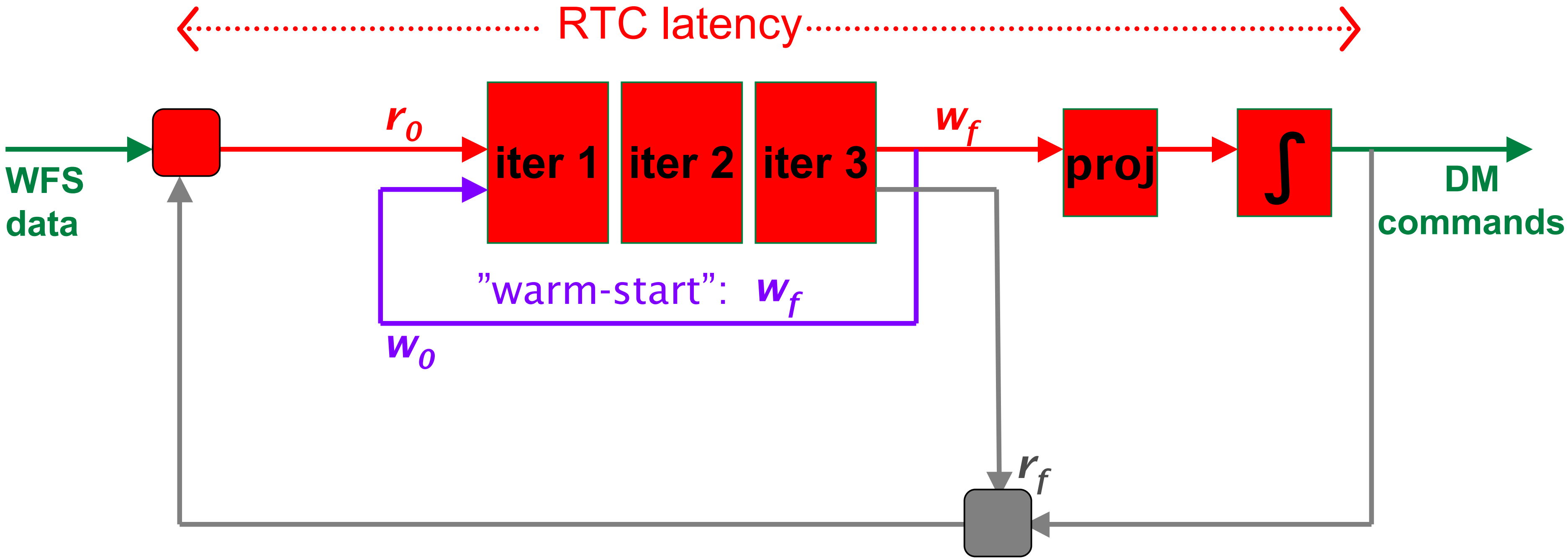

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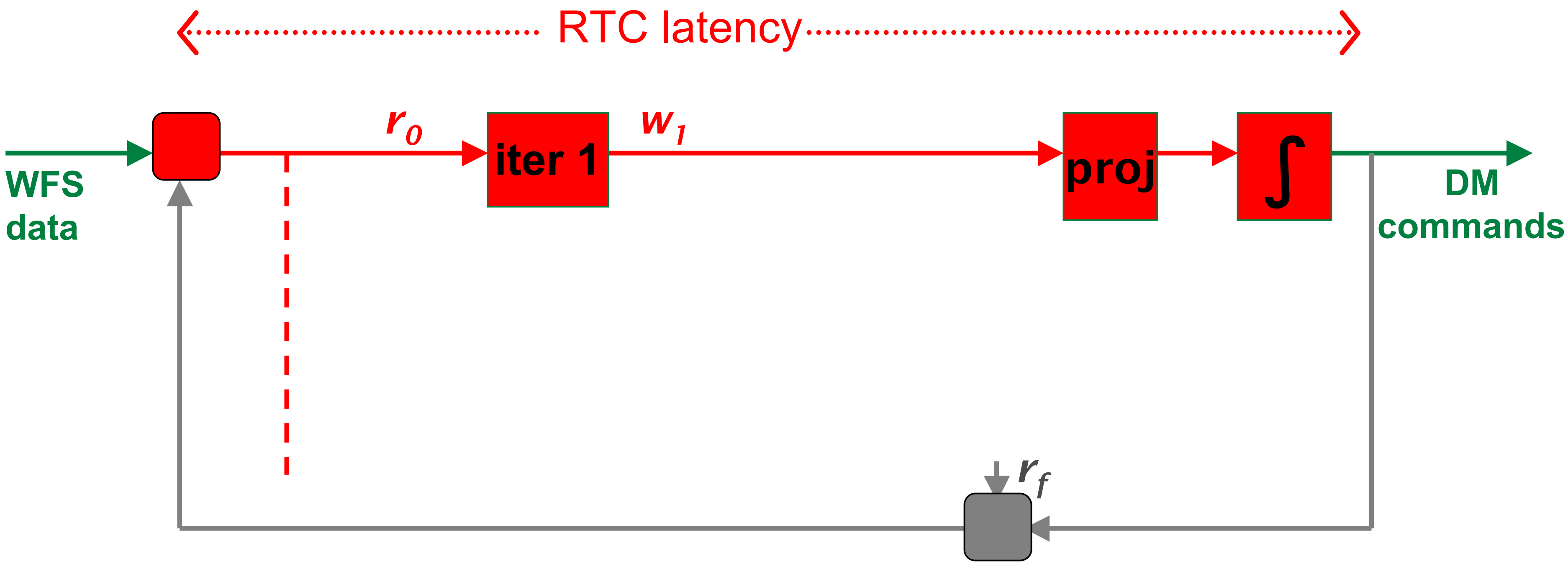
low-latency scheme for FRiM



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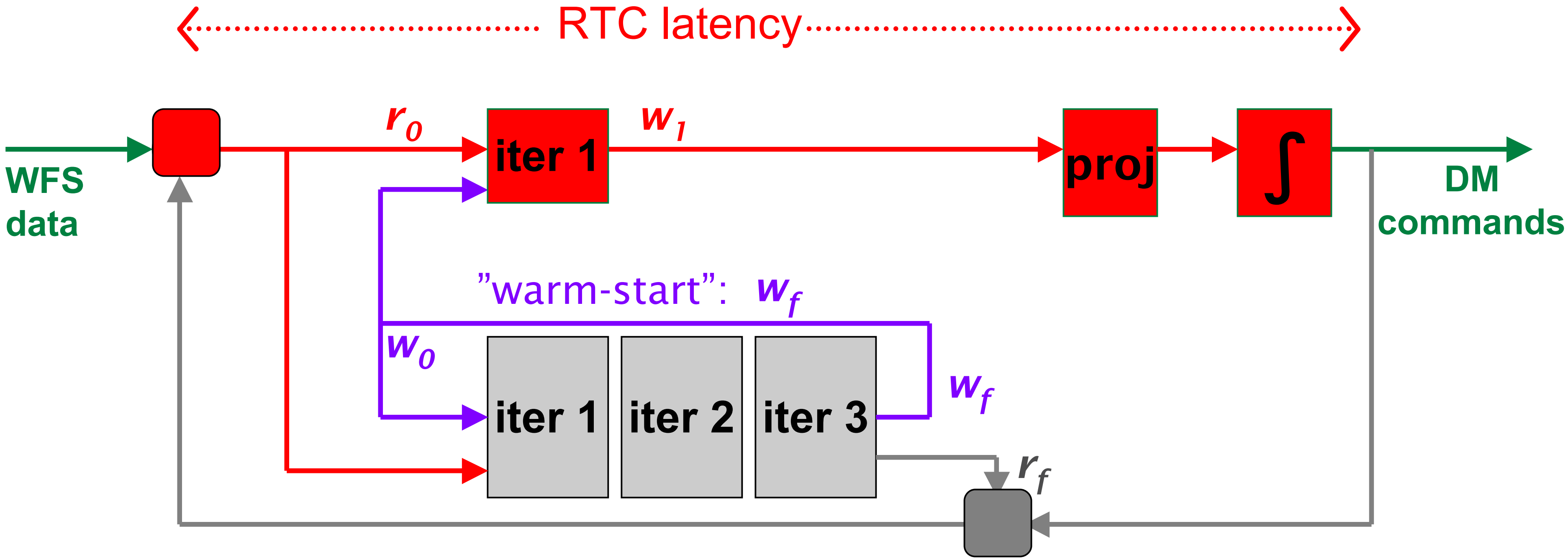

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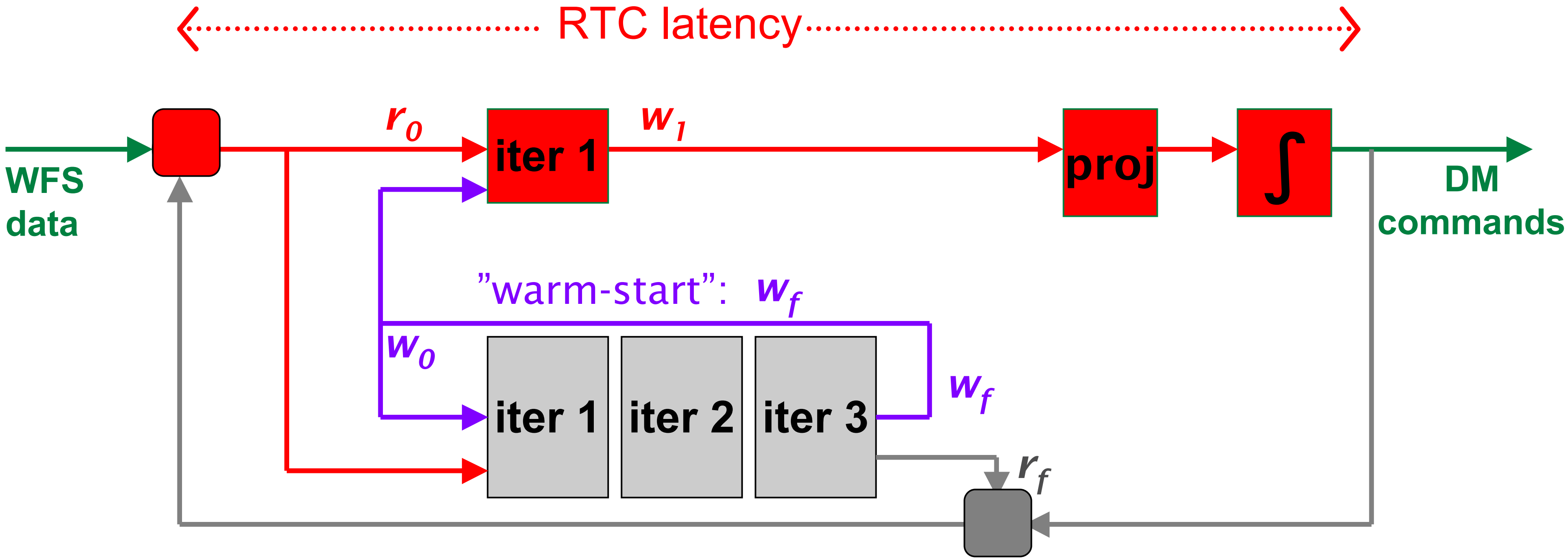

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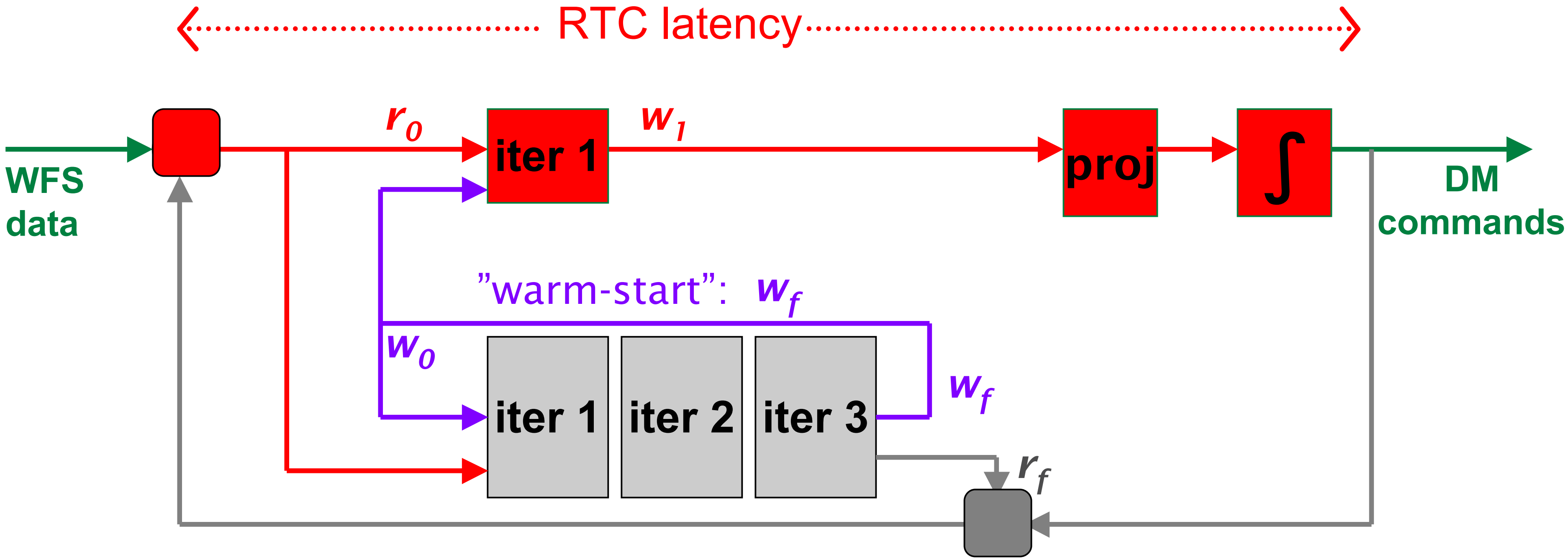


- consequences:
  - ✓ low-latency path is reduced to the cost of 1 iteration
  - ✓ 3 iterations in SCAO < 1 WFS exposure time
  - ✓ best performance is maintained thanks to an optimal "warm-start" (preliminary results by simulations)

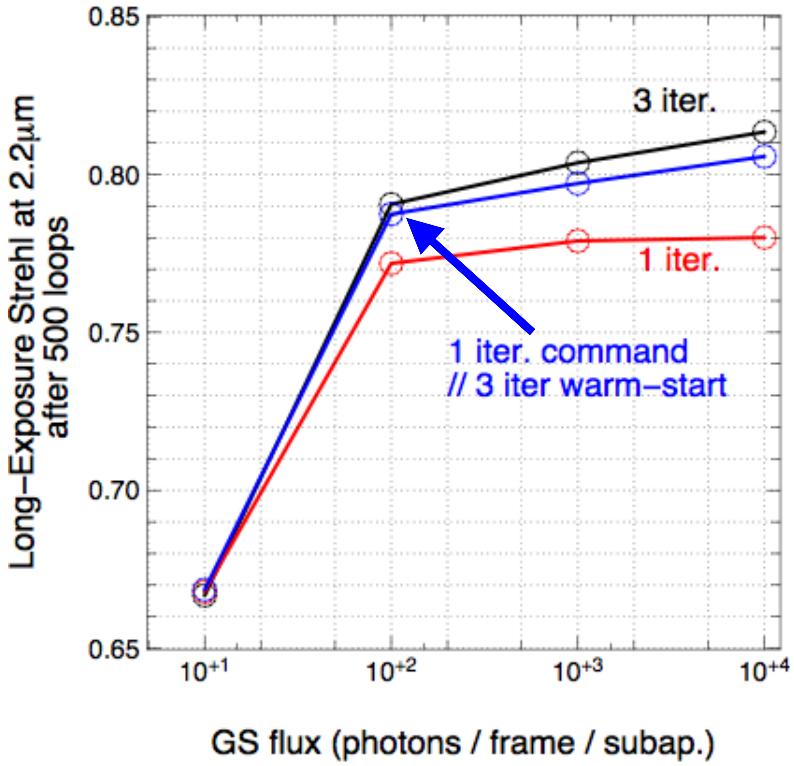
Results with low-latency scheme

- simulations E-ELT SCAO
- FRiM + Octopus
- 500Hz

1) vs measurement noise (flux)

➡ no impact at low flux

➡ good “warm-start” = key point to afford 1 iteration at high SNR



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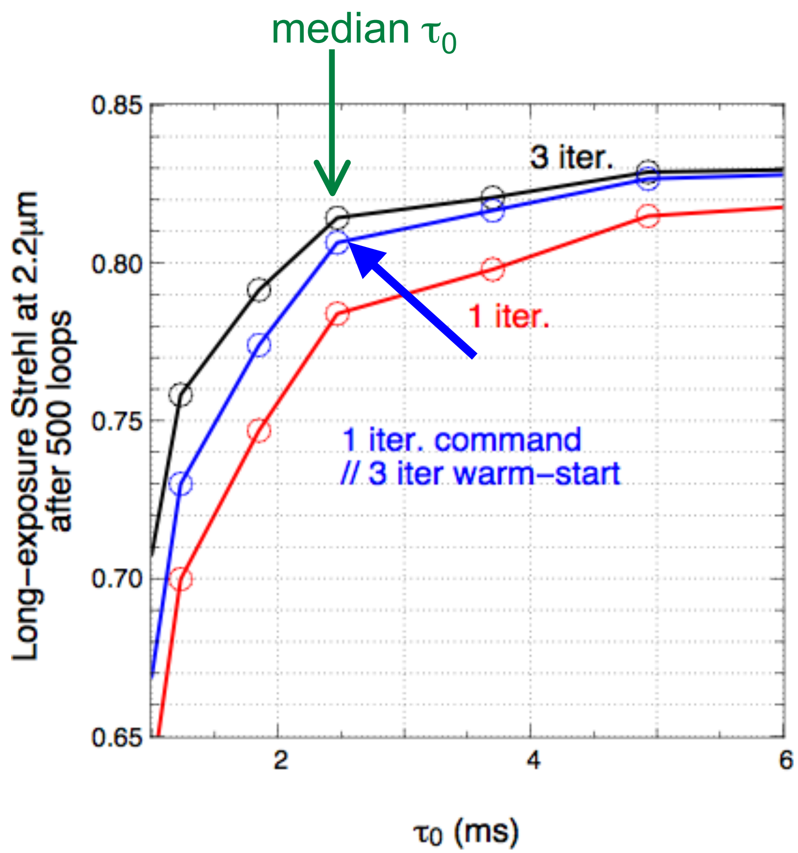
2) vs wind speed
(atmosphere coherence time τ_0)

high flux conditions

➡ again good "warm-start"= key point

➡ also helps for high wind speeds



reducing to the cost of half an iteration

- computations for the low-latency branch (only 1 iter.) can be revisited
- only the first iteration to be applied
  - PCG 1rst iteration = steepest descent
  - simplified computations (no need to update the residuals)

classical FRiM 1rst iter.	low-latency FRiM 1 iter.
$\boldsymbol{p} = \mathbf{M}_{\text{diag}} \cdot \boldsymbol{r}$	$\boldsymbol{p} = \mathbf{M}_{\text{diag}} \cdot \boldsymbol{r}$
$\rho = \boldsymbol{p}^{\mathrm{T}} \cdot \boldsymbol{r}$	$\rho = \boldsymbol{p}^{\mathrm{T}} \cdot \boldsymbol{r}$
$\boldsymbol{q} = \left(\mathbf{K}^{\mathrm{T}} \cdot \mathbf{S}^{\mathrm{T}} \cdot \mathbf{W}_e \cdot \mathbf{S} \cdot \mathbf{K} + \mathbf{I}\right) \cdot \boldsymbol{p}$	$\boldsymbol{q} = \mathbf{W}_e^{1/2} \cdot \mathbf{S} \cdot \mathbf{K} \cdot \boldsymbol{p}$
$\alpha = \rho / \boldsymbol{p}^{\mathrm{T}} \cdot \boldsymbol{q}$	$\alpha = \rho / (\boldsymbol{q}^{\mathrm{T}} \cdot \boldsymbol{q} + \boldsymbol{p}^{\mathrm{T}} \cdot \boldsymbol{p})$
$\boldsymbol{u} = \boldsymbol{u} + \alpha \cdot \boldsymbol{p}$	$\boldsymbol{u} = \boldsymbol{u} + \alpha \cdot \boldsymbol{p}$
$\boldsymbol{r} = \boldsymbol{r} - \alpha\, \boldsymbol{q}$	

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➡ half of the heavy computations of 1 iteration
➡ no longer an iterative reconstructor

Conclusions

- For iterative methods, RTC latency is constrained by the # of iterations
  - sequential iterations
  - although only 3 are required for best performance of FRiM

- We developed a new low-latency application of FRiM, based on:
  - 1 iteration for the commands computation
  - 3 iterations to improve the warm-start of the next reconstruction

- First results from simulations demonstrate the efficiency of the improved "warm-start"

- The computational cost of 1 iteration only is half the cost of a classical iteration

- With only 1 iteration, the reconstruction is no longer iterative.

This may be applied to any iterative method…