The Mass-Radius Relation of Hot Stellar Systems

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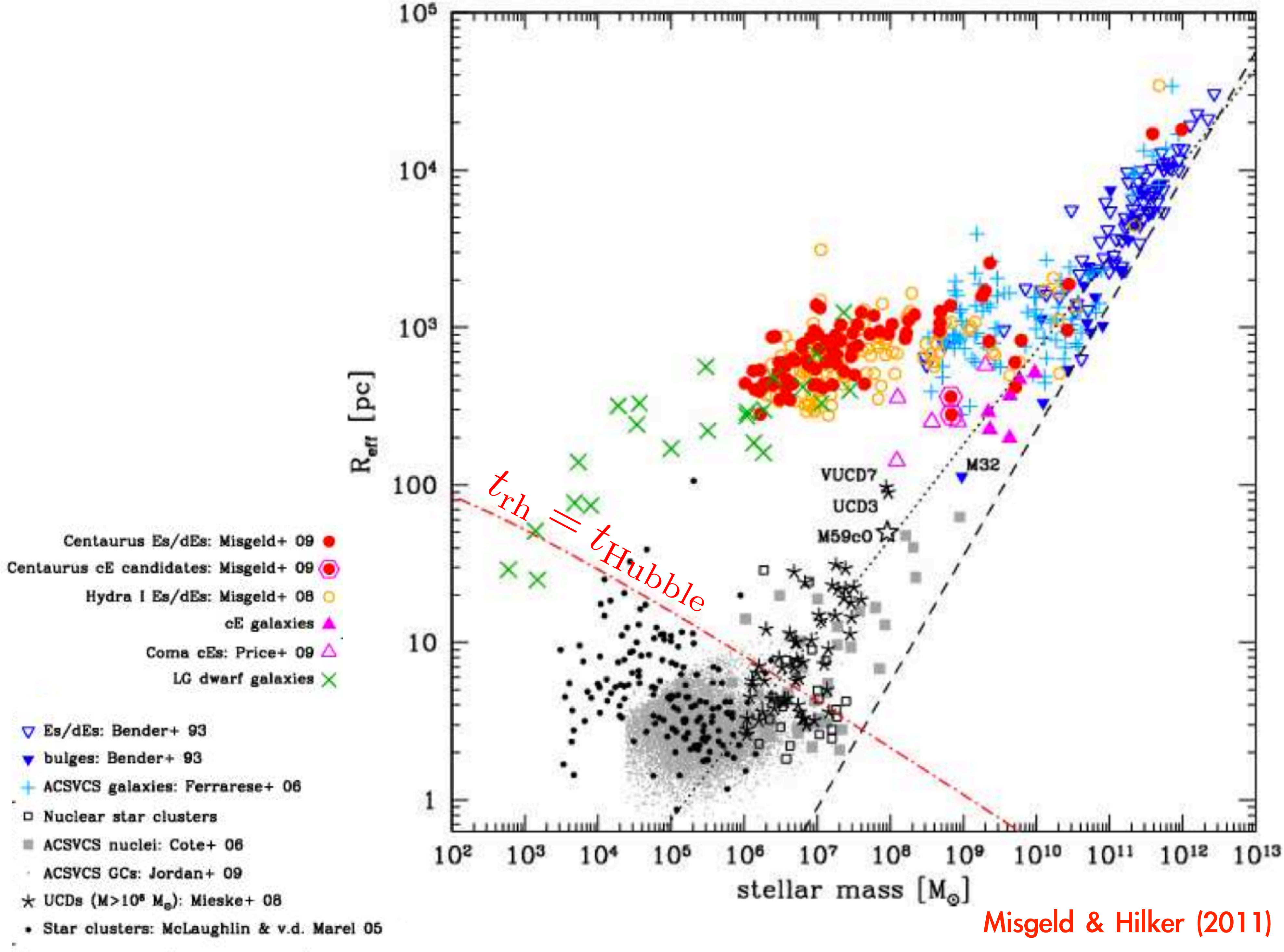
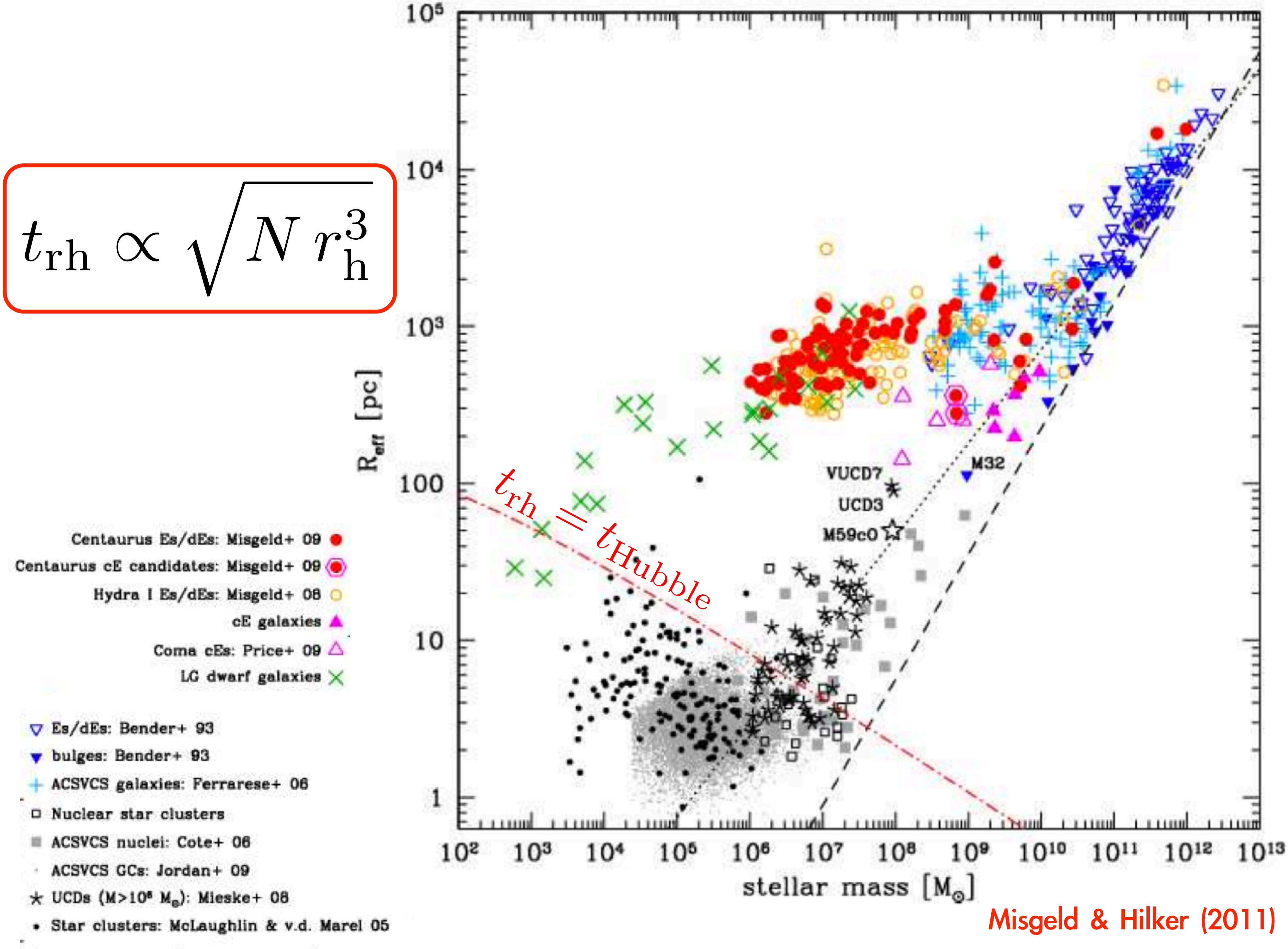
Poul Alexander (Cambridge)

ESO, April, 2011



Misgeld & Hilker (2011)

$$\boxed{t_{\mathrm{rh}} \propto \sqrt{N\, r_{\mathrm{h}}^{3}}}$$



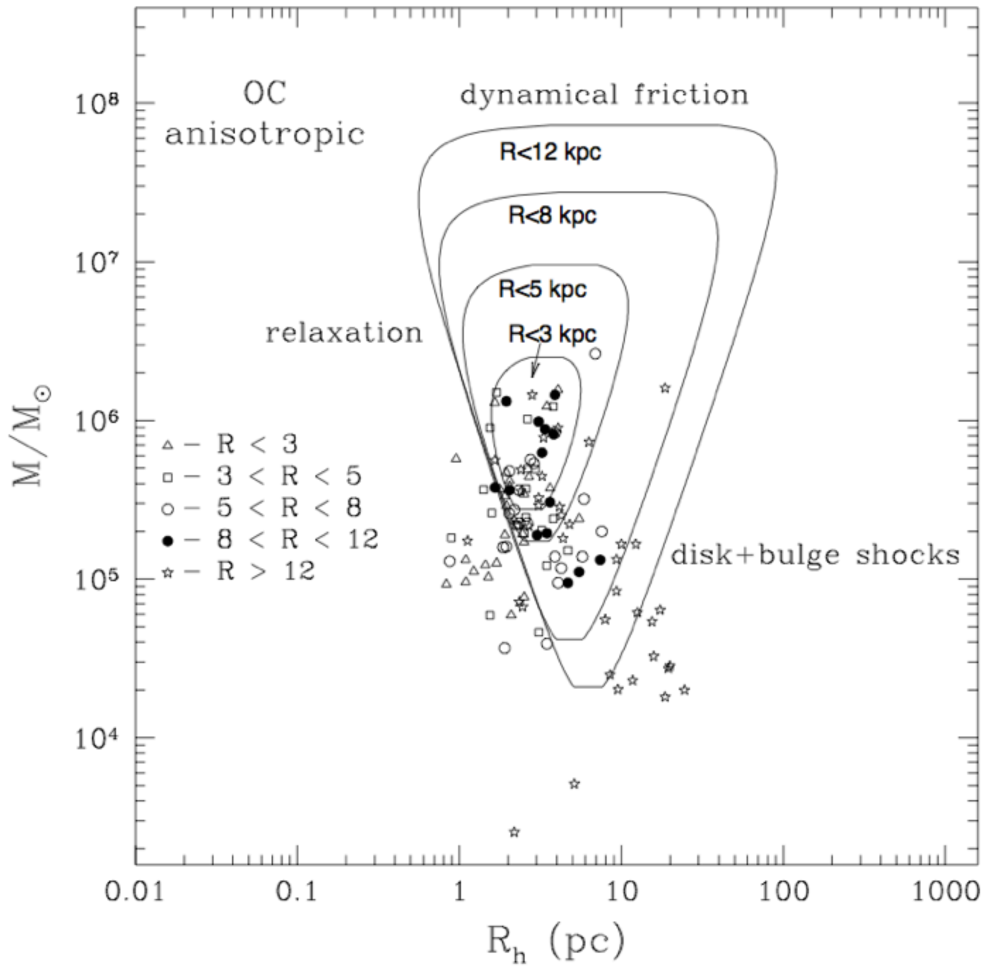
Misgeld & Hilker (2011)

One interpretation: preferential survival

According to the calculations reported in § 3.1,

$$t_{ev} \sim 30 t_{rh} ,$$

where t_{rh} is the half-mass relaxation time (eq. [5]).



Gnedin & Ostriker (1997) also: Fall & Rees (1977)

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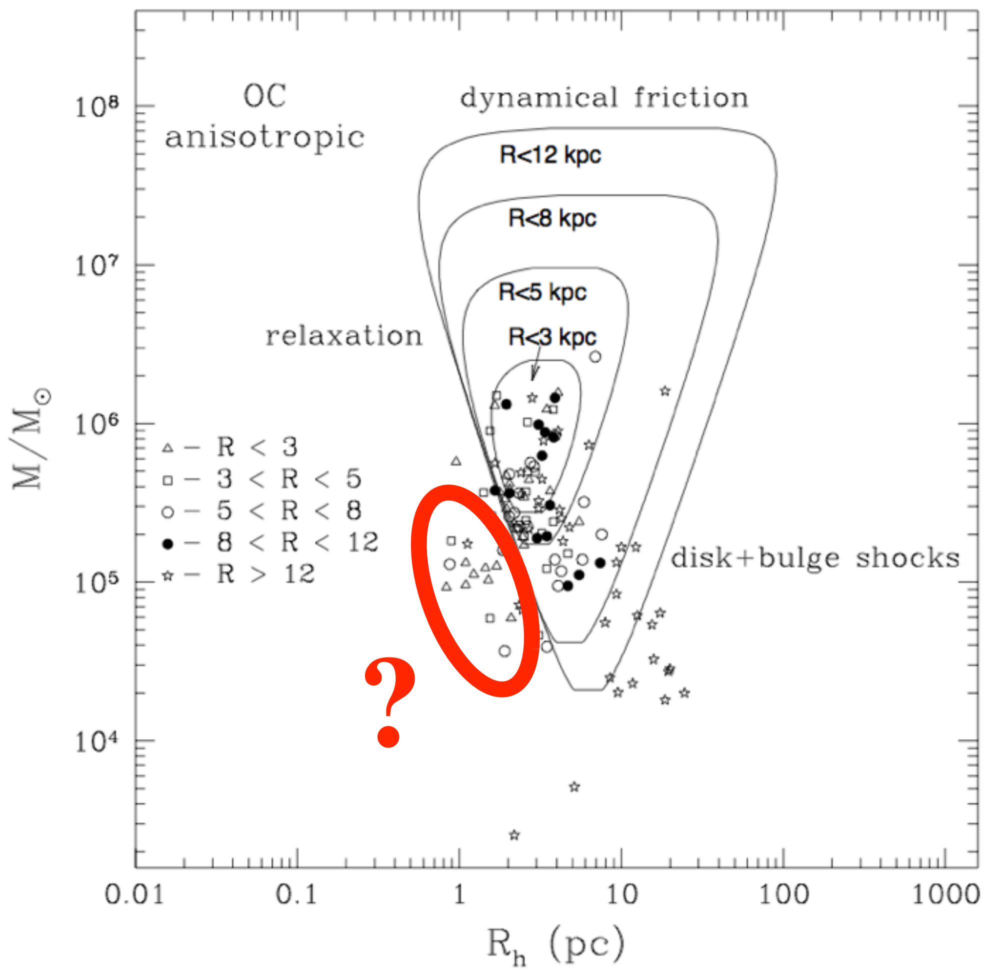

Gnedin & Ostriker (1997) also: Fall & Rees (1977)

Modelling of Star Clusters anno 2011

(the "Kitchen Sink" approach)

- Realistic number of stars
  - $N \sim 10^5$ in direct N-body codes (Aarseth, Baumgardt, Hurley, Mackey, Makino, Portegies Zwart)
  - $N \sim 10^6$ in Monte Carlo codes (Fregeau, Giersz, Heggie, Rasio)
- Realistic stellar mass spectrum
  - $0.1 < m/M_\odot < 100$
  - Salpeter, Kroupa, etc.
- Primordial binary population (Hurley, Ivanova, Kroupa)
- Stellar evolution prescription (Giersz, Hurley, Portegies Zwart, Tout)
- Special treatment of binary interaction (Church, Hurley)
- Stellar collisions (Lombardi, Gaburov)
- External tidal fields
  - Circular orbits
  - Eccentric orbits
  - Disc, bulge + halo components (Dehnen, Heggie, Vesperini)
  - Triaxial potentials (Peñarrubia, Walker & Gilmore 2010)
  - Chaotic time-dependent tides (Kruijssen, Renaud)

Modelling of Star Clusters anno 2011

(the "Kitchen Sink" approach)

everything but the kitchen sink (*uncountable*)

1. (*idiomatic*) Almost everything, whether needed or not.

   *She must have brought **everything but the kitchen sink** along on the trip, and how she lifted her suitcase, I do not know.*

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Modelling of Star Clusters anno 1965

(the "one-step-at-a-time" approach)

- *N*-body simulations
  - *N* ~ 10-100 (Aarseth, Von Hörner)
  - all stars the same mass
  - no tidal field
- Theory: evolve the distribution function (Hénon)
  - Velocity distribution is isotropic
  - All stars have the same mass
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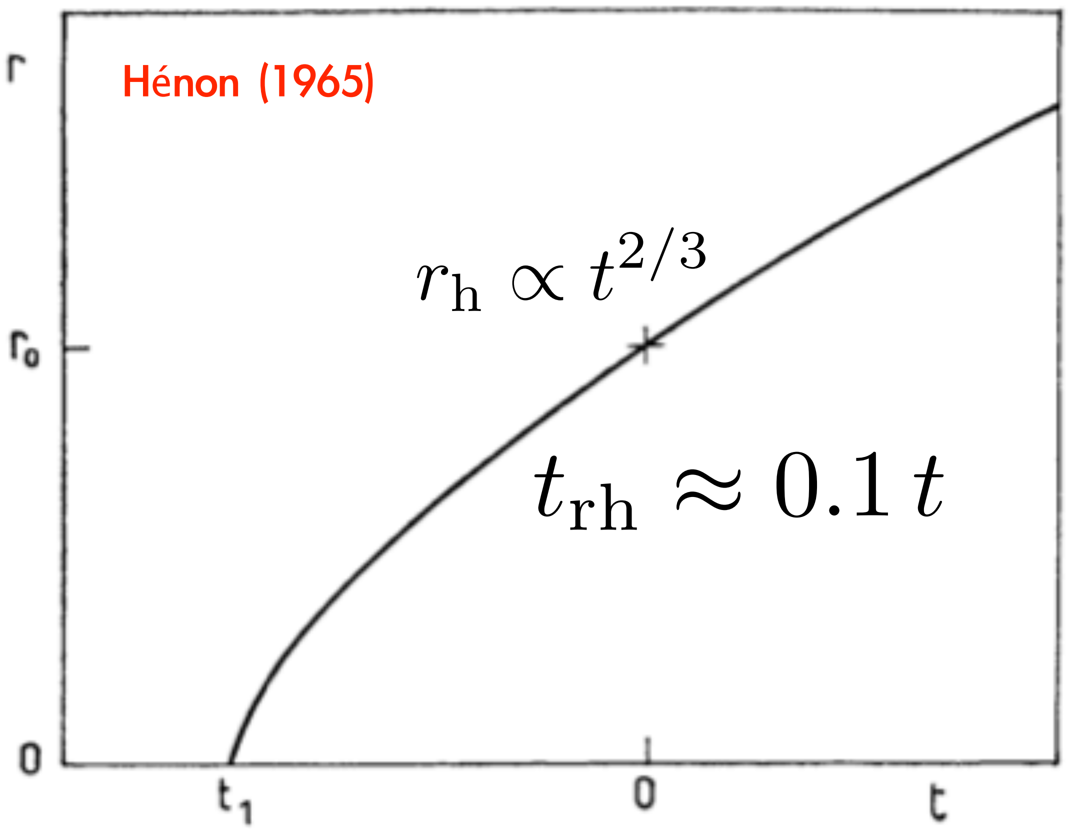

FIG. 3. — Modèle homologique isolé : évolution des dimensions.

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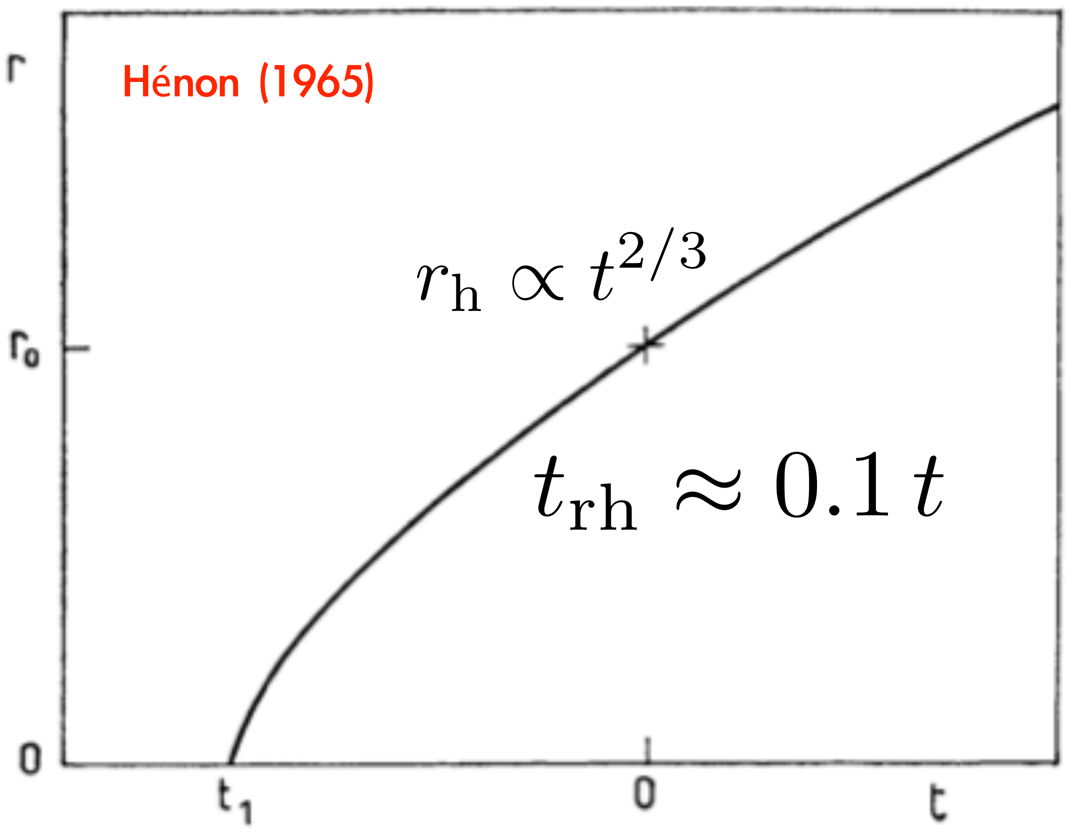

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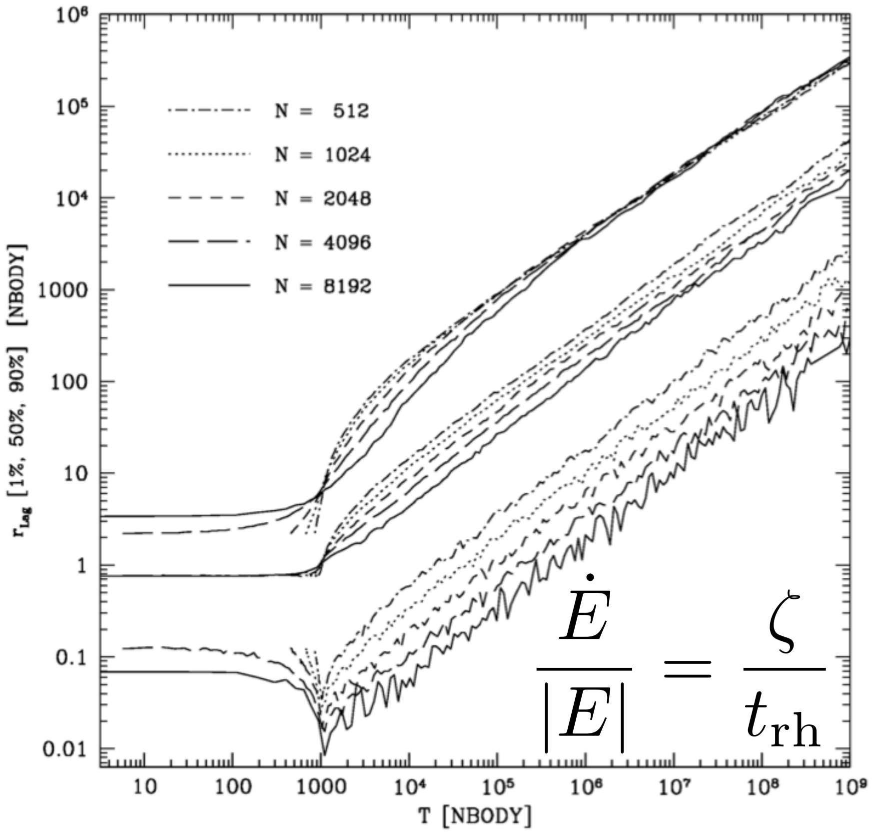
ADVERTISEMENT: The 2 seminal papers on star cluster evolution by Michel Hénon (1961, 1965) are now translated into English by Florent Renaud and are on arXiv:1103.3498/9, for FREE

Modelling of Star Clusters “Retro” Style

ret•ro [1] |ˈretrō|
adjective
imitative of a style, fashion, or design from the recent past : *retro 60s fashions*.

- Direct *N*-body integrations
- Isolated star clusters (i.e. no tides)
- No primordial binaries
- All stars the same mass
- Plummer density distribution



$$\frac{\dot{E}}{|E|} = \frac{\zeta}{t_{\rm rh}}$$

Figure 2. Evolution of the Lagrangian radii for clusters with different initial particle numbers. Times are shifted such that all clusters go into core collapse at the same time. Low-*N* clusters expand quicker after core collapse, and because of this faster expansion their relaxation times reach those of high-*N* clusters. After 10^5 *N*-body time-units, all clusters do therefore expand with the same rate.

Baumgardt et al. (2002)

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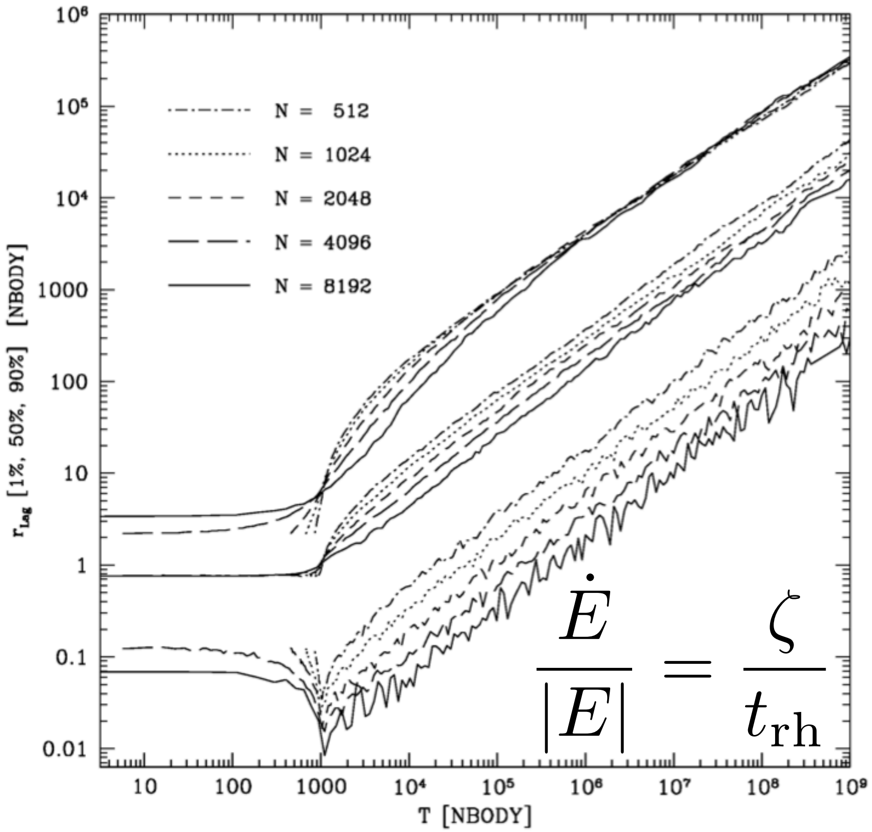
iDial iPhone Retro



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Hénon

The detailed mechanism for the transfer of energy from the source to the rest of the system must now be specified. We take the view that *the rate of flow of the energy is controlled by the system as a whole, not by the singularity*. To justify this assumption,

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Hénon (1975)

i.e. rate of energy production, $\dot{E}$, is self-regulative!

Hénon **Eddington**

Like in stars!

Eddington's chief additions was to show that radiation pressure was necessary to prevent collapse of the sphere. He developed his model despite knowingly lacking firm foundations for understanding opacity and energy generation in the stellar interior. However, his results allowed for calculation of temperature, density and pressure at all points inside a star, and

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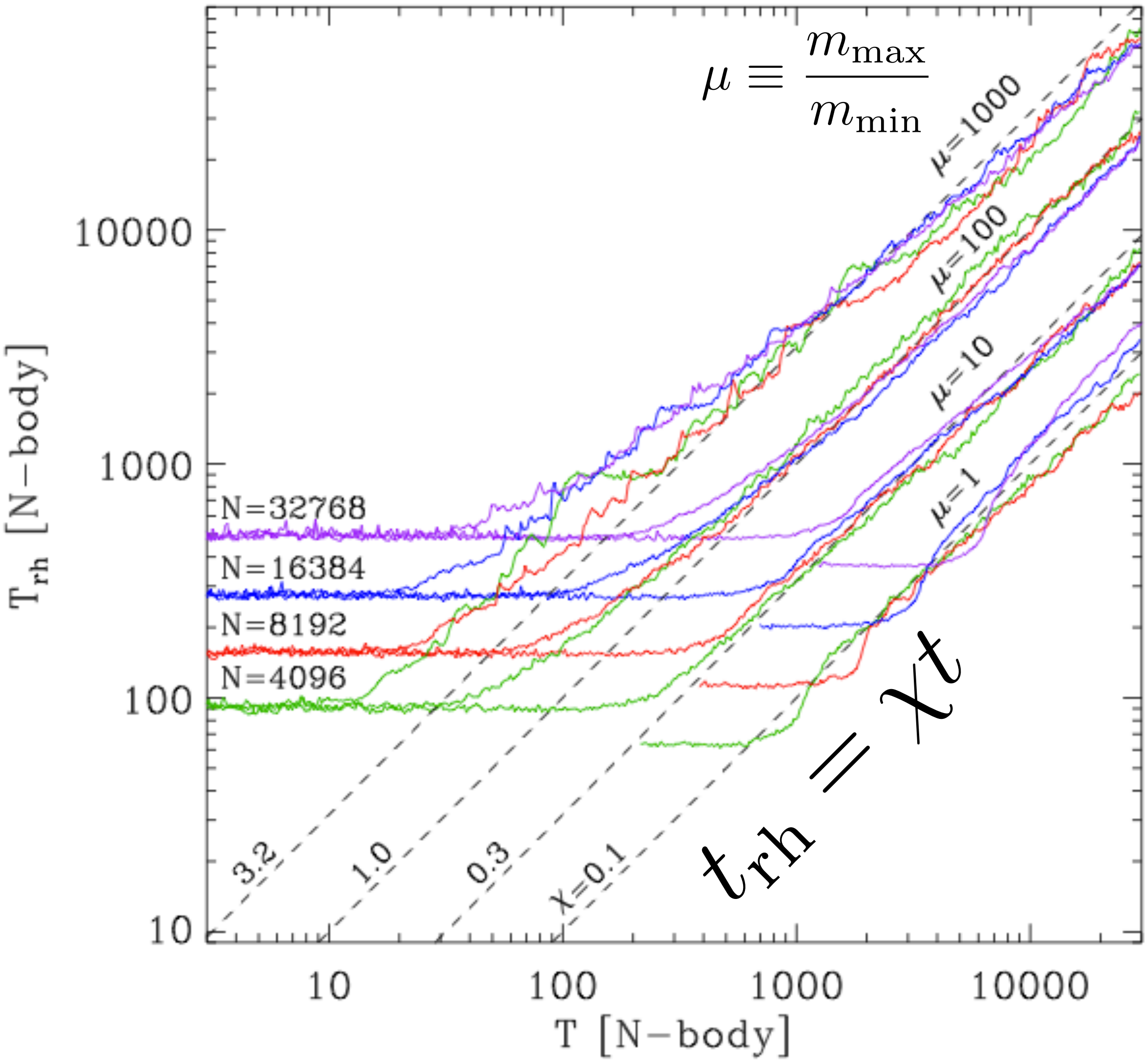
Adding a stellar mass spectrum



Gieles, Baumgardt, Heggie & Lamers (2010)

See also:
Inagaki & Saslaw (1985)
Lee & Goodman (1995)

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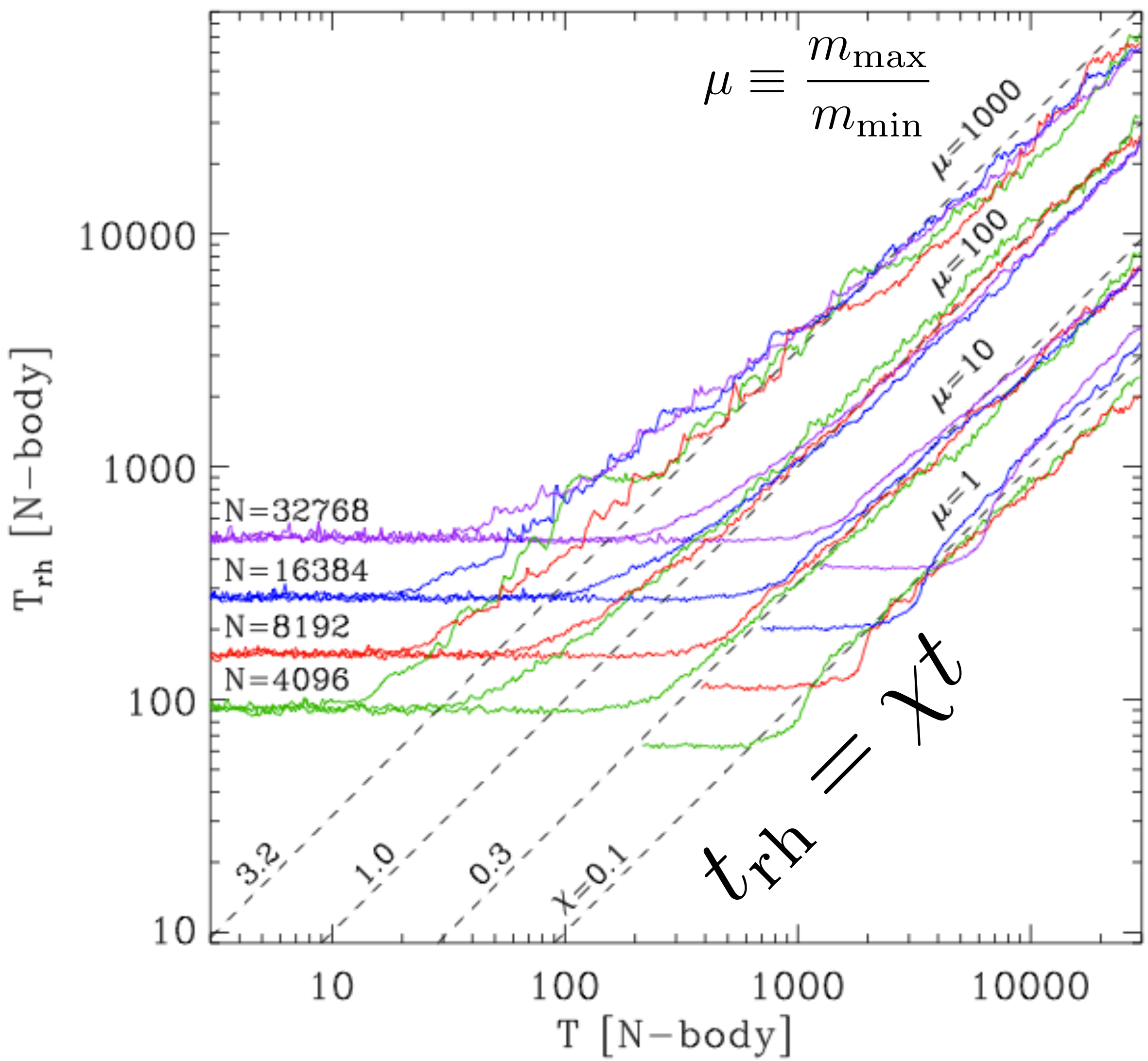

~ embedded clusters

~ open clusters

~ globular clusters

= virtual clusters
(perhaps neutron star clusters)

Gieles, Baumgardt, Heggie & Lamers (2010)

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Adding stellar evolution

- Changing stellar mass function : $\mu(t)$
- Decreasing total mass : $M(t) \propto t^{-\delta}$
- Increasing energy : $\dfrac{\mathrm{d}E}{E} = x\dfrac{\mathrm{d}M}{M}$

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$x = 3$ for homologous mass loss + adiabatic expansion $\left(E \propto -\dfrac{M^2}{r_{\mathrm{h}}}\,;\, r_{\mathrm{h}} \propto \dfrac{1}{M}\right)$
$x > 3$ for mass loss from the centre

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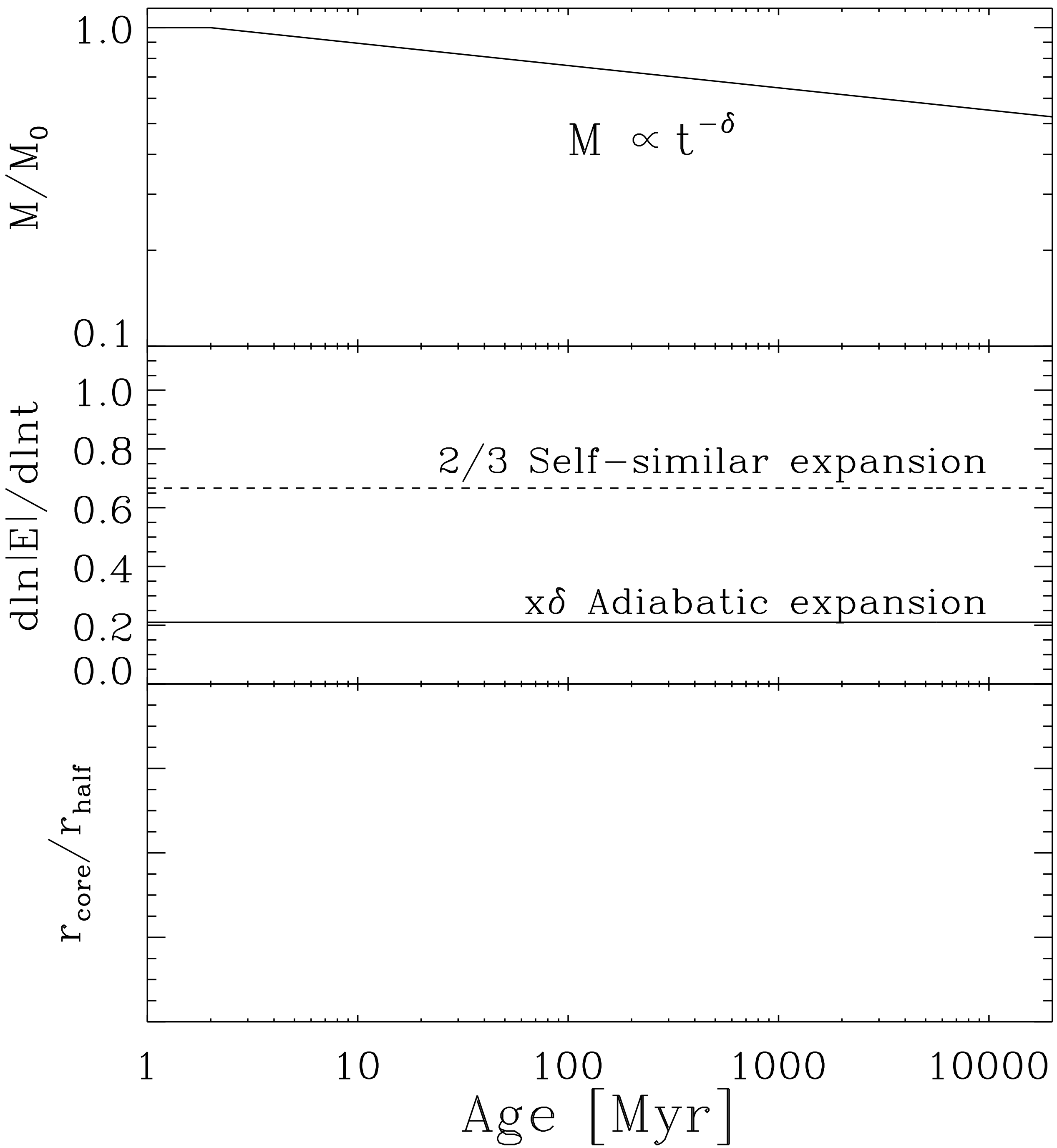
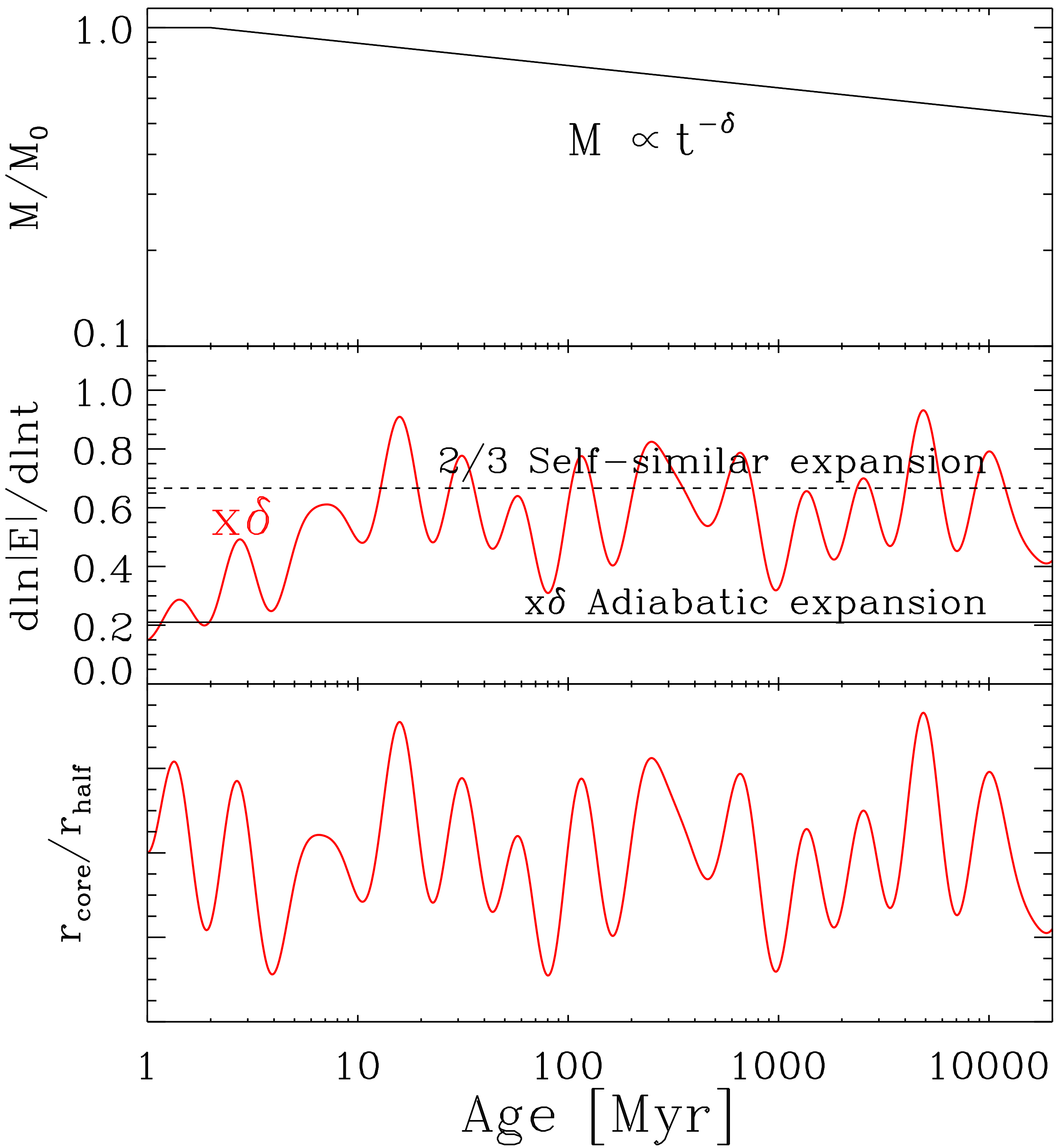


M/M$_0$
1.0
0.1
M $\propto t^{-\delta}$
dln|E|/dlnt
1.0
0.8
0.6
0.4
0.2
0.0
2/3 Self-similar expansion
xδ Adiabatic expansion
r$_{core}$/r$_{half}$
1
10
100
1000
10000
Age [Myr]



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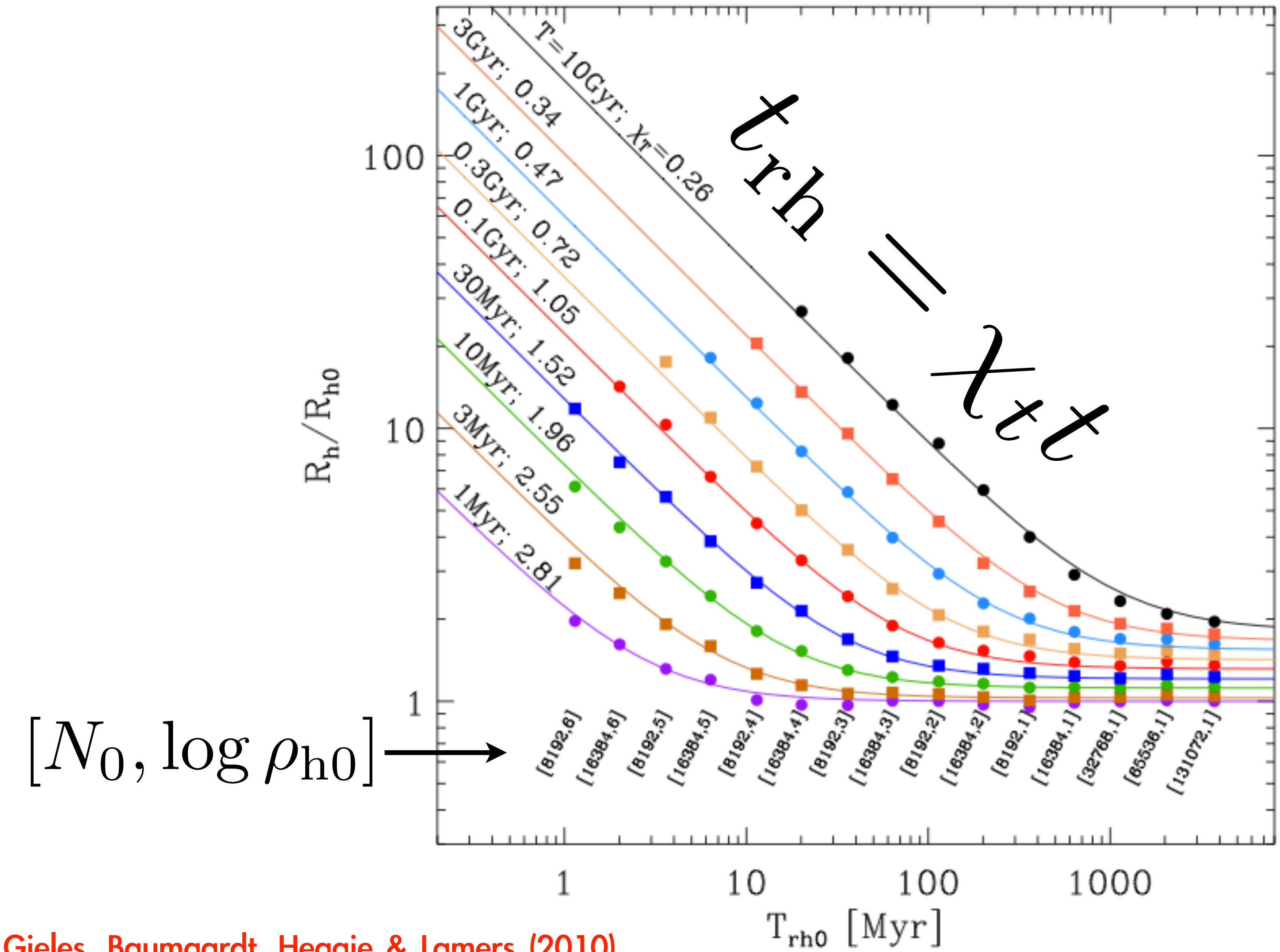
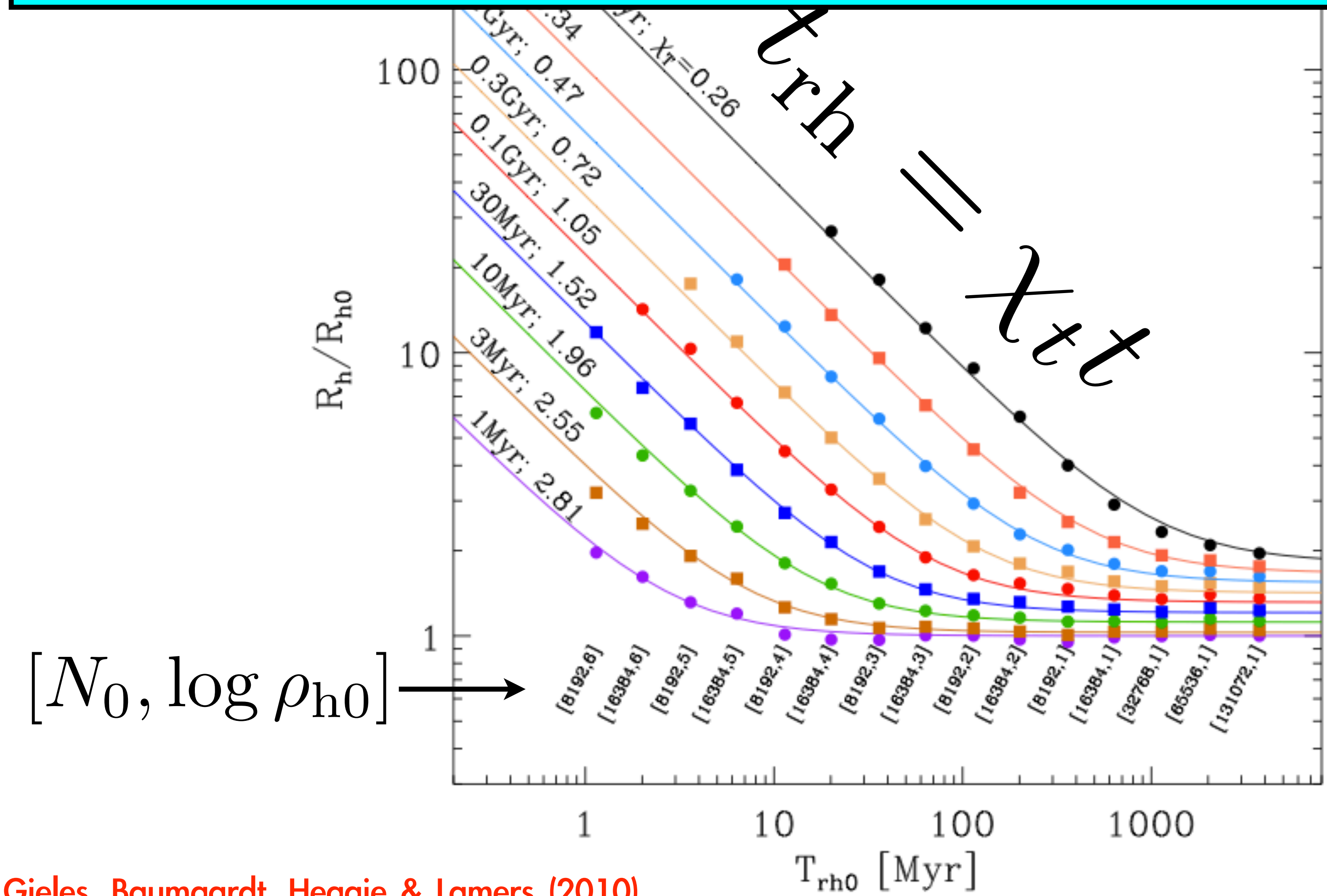
N-body models with stellar evolution w/o tides



Gieles, Baumgardt, Heggie & Lamers (2010)

There is a **balance** between $\dot{E}$ due to stellar evolution and the relaxation driven flux of energy



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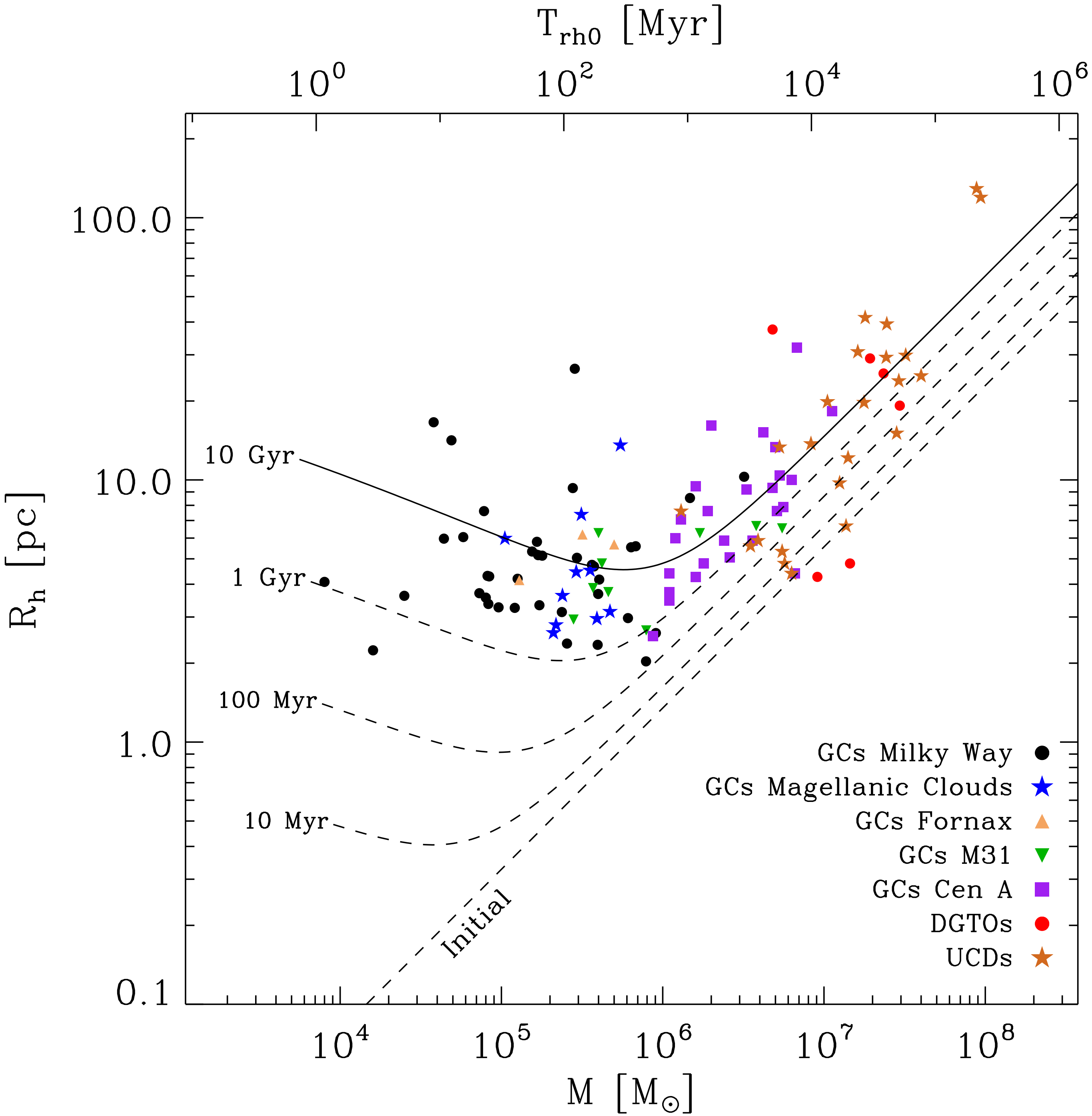

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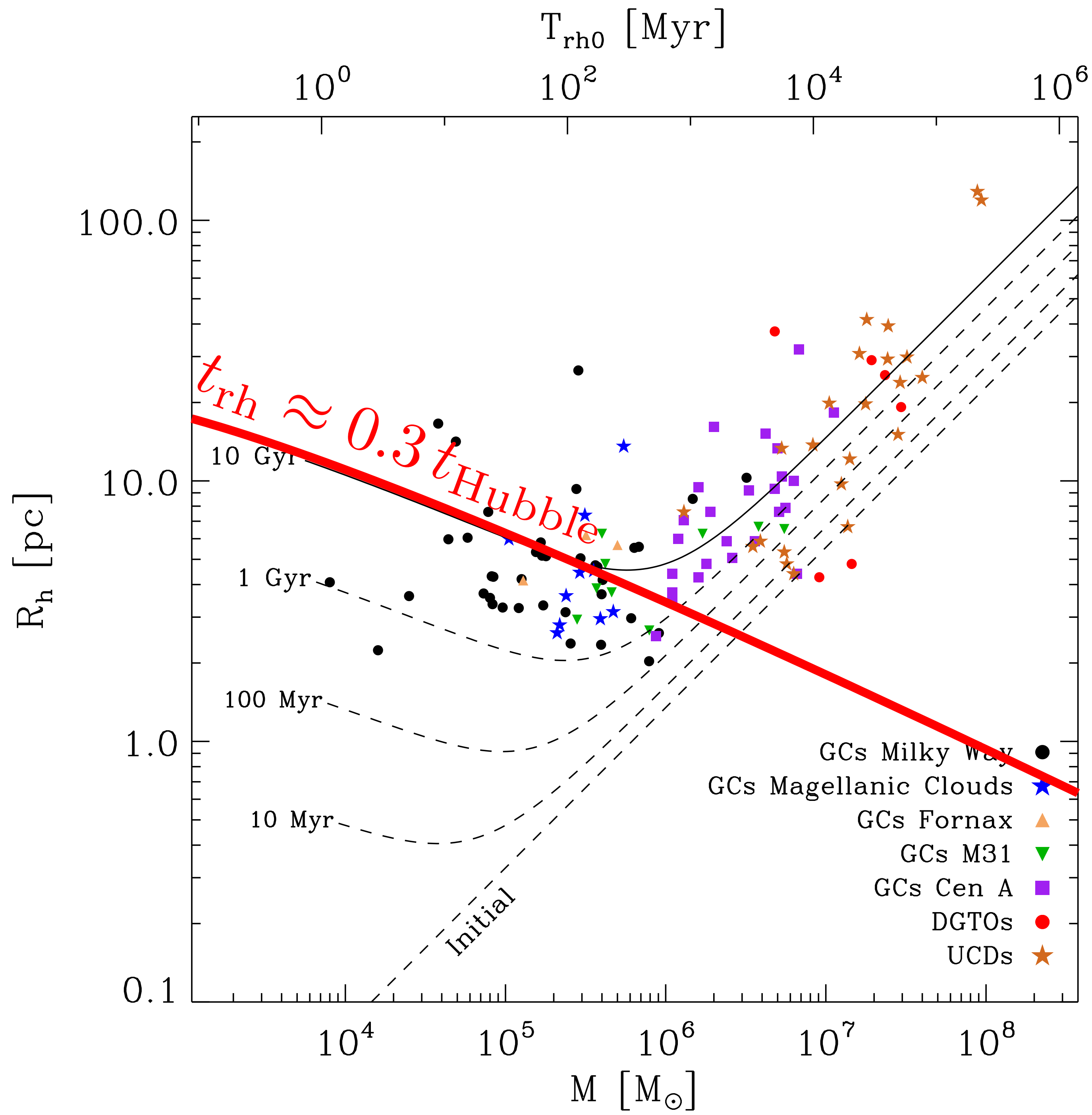

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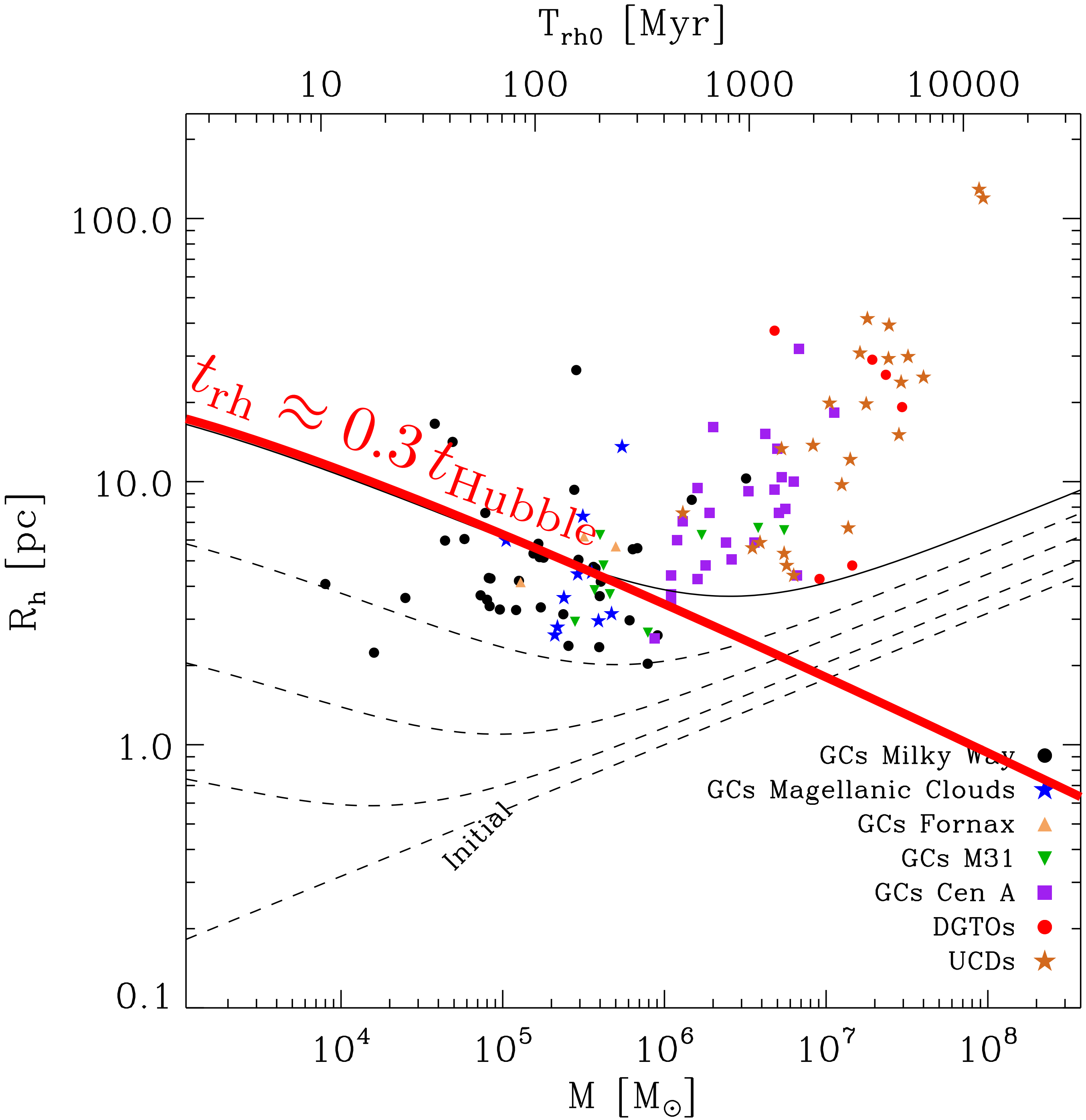
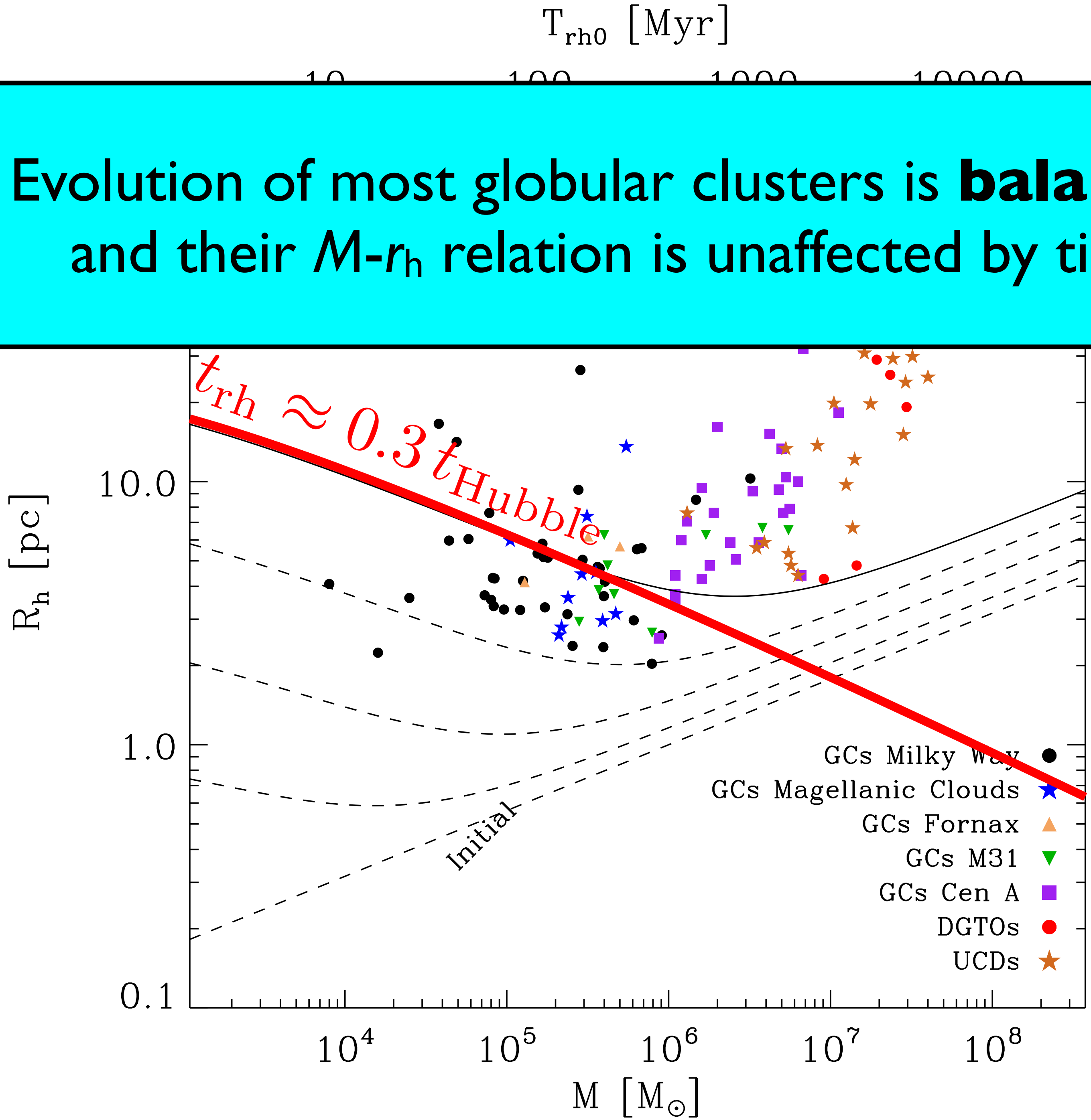


Gieles, Baumgardt, Heggie & Lamers (2010)

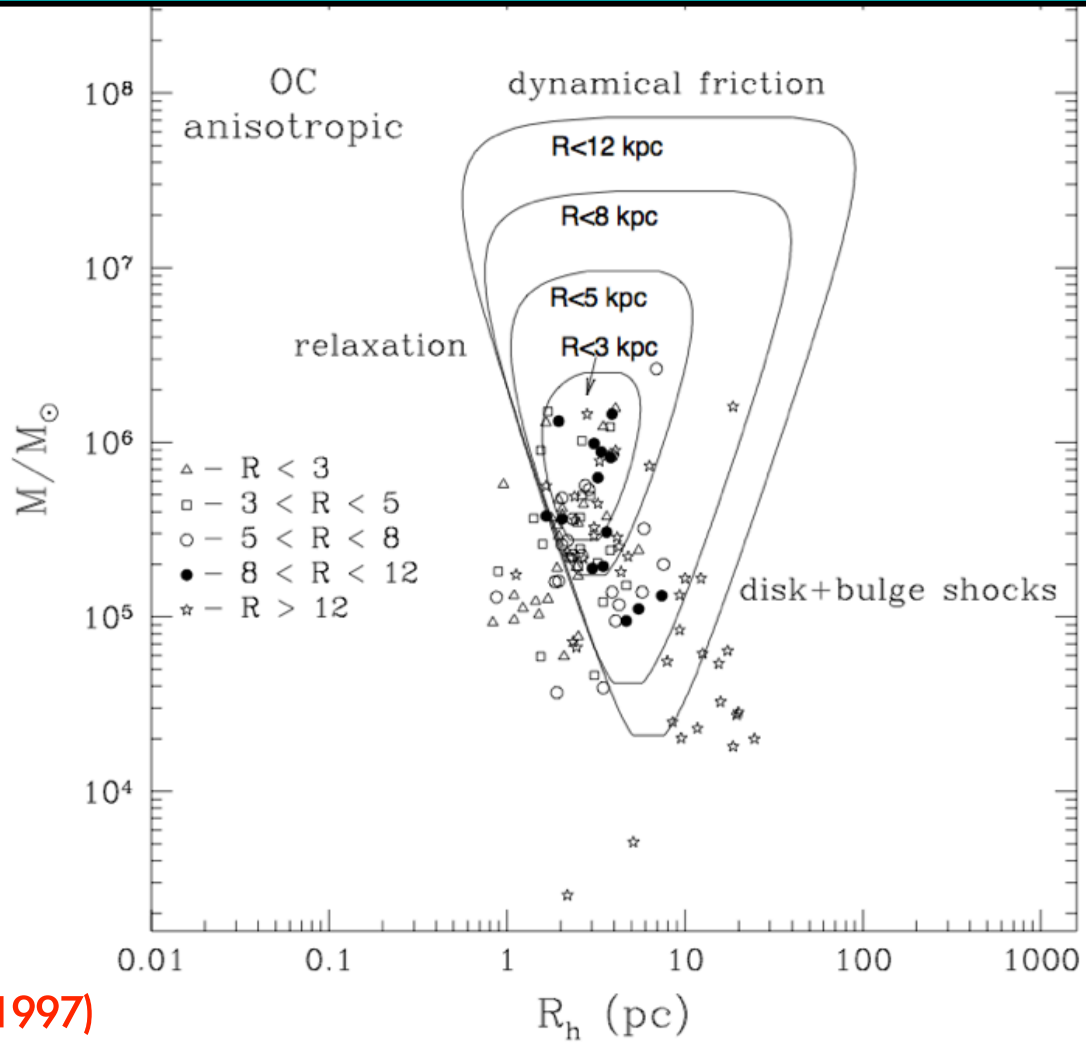
Evolution of most globular clusters is **balanced** and their M-r_h relation is unaffected by tides



Gieles, Baumgardt, Heggie & Lamers (2010)

Alignment with lines of constant t_{rh} because of **expansion**

not evaporation



Gnedin & Ostriker (1997)
also: Fall & Rees (1977)

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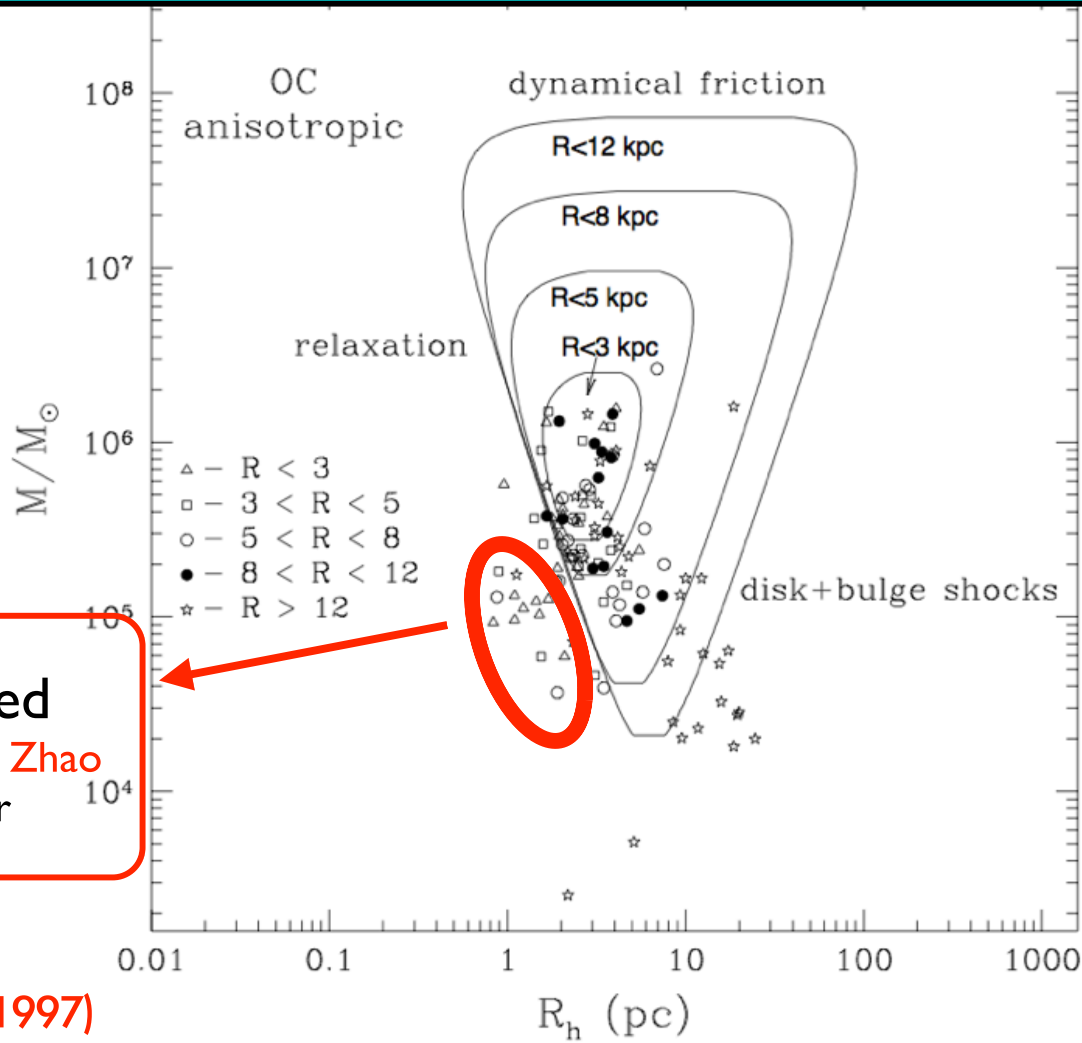

Tidally affected
see Gieles, Heggie & Zhao (2011) + poster

Gnedin & Ostriker (1997)
also: Fall & Rees (1977)

Summary

- Most globular clusters evolve with a central energy source that is in **balance** with the amount of energy that is conducted outwards by relaxation

- Stellar evolution is an important, perhaps dominant, energy source for 2-body relaxation

- Most Milky Way globular clusters are still expanding to their tidal boundary: alignment with lines of constant relaxation time:
  $r_{\rm h} \propto M^{-1/3}$ or $M \propto \rho_{\rm h}^{1/2}$