Tides, Rotation or Anisotropy?
New Self-consistent Nonspherical Models for Globular Clusters

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Beyond the traditional paradigm of GCs: Theoretical Motivations

Spherical King models provide a successful zeroth-order interpretation of GCs

But more realistic equilibrium models should include ...

- External tidal field:
  Energy truncation in King models is imposed heuristically to mimic the role of tides, but the triaxial tidal field should be imposed self-consistently
- Internal rotation:
  Present-day GCs are only slowly rotating. But in the past?
  Solid-body or differential?
- Anisotropy in the velocity space:
  Quasi-relaxed systems are expected to be approximately isotropic, but:
  - less-relaxed objects may keep memory of their formation process.
  - evolution in a (variable) tidal field may produce non-trivial kinematical signatures.

Deviations from spherical symmetry are induced!
Physical origin of the observed flattening? van den Bergh AJ 2008

Beyond the traditional paradigm of GCs: Observational Motivations

- New measurements of shapes and sizes of 116 GGCs are available

  Chen & Chen ApJ 2010

- Existence of the “extra-tidal light” is frequently reported

  e.g McLaughlin & van der Marel ApJ 2005, Jordi & Grebel A&A 2010

- Proper motions of thousands of stars have been measured in selected GCs

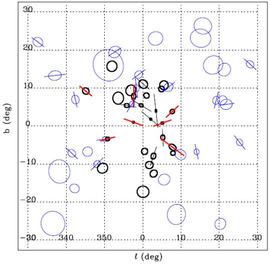
  ω Cen e.g. Anderson & van der Marel 2010

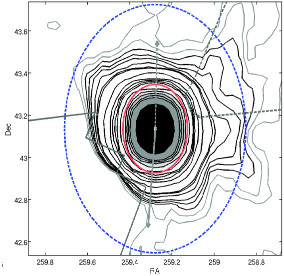
  47 Tuc e.g. McLaughlin et al ApJS 2006

- New kinematical measurements (velocity dispersion profile, rotation curve) are available

  e.g. Sollima et al MNRAS 2009, Lane et al 2009, 2010

Renewed modeling efforts are needed





Triaxial Tidal Models

Tidal models: Definitions

Bertin & Varri ApJ 2008

- Distribution function

$$f_K(E) = \begin{cases} A\left[\exp\left(-aE\right) - \exp\left(-aE_0\right)\right] & \text{if } E \leq E_0 \\ 0 & \text{if } E > E_0 \end{cases}$$

King AJ 1966

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Phi_c$$

TIDAL MODELS: Definitions

Bertin & Varri ApJ 2008

- Distribution function

$$f_K(H) = \begin{cases} A\left[\exp\left(-aH\right) - \exp\left(-aH_0\right)\right] & \text{if } H \leq H_0 \\ 0 & \text{if } H > H_0 \end{cases}$$

Weinberg ASPC 1993, Heggie & Ramamani MNRAS 1995

$$H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Phi_T + \Phi_c$$

$$\psi(\mathbf{r}) = a\{H_0 - [\Phi_c(\mathbf{r}) + \Phi_T(\mathbf{r})]\}$$

"Tidal approximation"

$$\Phi_T(\mathbf{r}) = \frac{1}{2}\Omega^2\left(z^2 - \nu x^2\right) \qquad \nu \equiv 4 - \frac{\kappa^2}{\Omega^2}$$

Concentration $\leftrightarrow$ $W_0 \equiv \psi(\mathbf{0})$

Tidal strength $\leftrightarrow$ $\epsilon \equiv \dfrac{\Omega^2}{4\pi G\rho_0}$

- Two domains separated by the boundary surface of the configuration, defined by $\psi(\mathbf{r}) = 0$, which is unknown *a priori*.

$$\hat{\nabla}^2\psi = -9\left[\frac{\hat{\rho}(\psi)}{\hat{\rho}(W_0)} + \epsilon(1-\nu)\right] \quad \text{for} \quad \psi > 0 \quad \text{(Poisson)}$$

$$\hat{\nabla}^2\psi = -9\epsilon(1-\nu) \quad \text{for} \quad \psi < 0 \quad \text{(Laplace)}$$

Elliptical PDE in a free boundary problem

TIDAL MODELS: Perturbation method

Bertin & Varri ApJ 2008

- Tidal effect = (small) perturbation acting on the configuration described by the spherical King models: $\epsilon \ll 1$

$$\psi(\hat{\mathbf{r}};\epsilon) = \sum_{k=0}^{\infty} \frac{1}{k!} \psi_k(\hat{\mathbf{r}})\, \epsilon^k$$

- Expansion of the general term of the series $\psi_k(\hat{\mathbf{r}})$ in spherical harmonics $\rightarrow$ one-dimensional (radial) Cauchy problems.

- This perturbation problem is singular!
  The convergence radius of the asymptotic series vanishes $\hat{r} \rightarrow \hat{r}_{tr}$, i.e. the validity of the expansion breaks down when $\psi_0 = \mathcal{O}(\epsilon)$.
  - Introduction of an intermediate region (boundary layer)
  - Asymptotic matching *à la* Van Dyke for $(\psi^{(\mathrm{int})}, \psi^{(\mathrm{lay})})$ and $(\psi^{(\mathrm{lay})}, \psi^{(\mathrm{ext})})$

    Van Dyke, Perturbation Methods in Fluid Mechanics, 1975

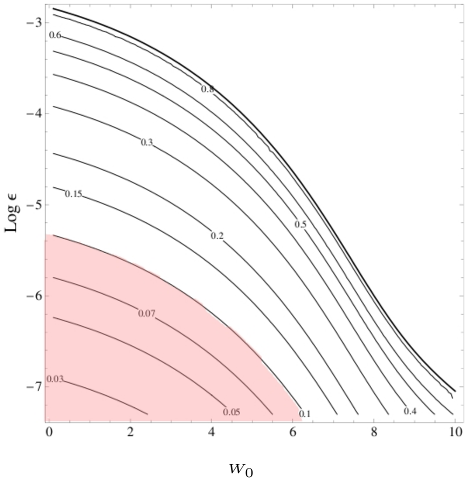
- Inspiration: rigidly rotating polytropes   Chandrasekhar MNRAS 1933, ... Smith Ap&SS 1975

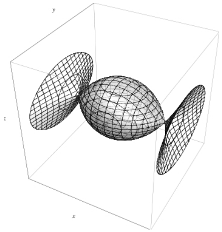
- Full explicit solution to two orders in ϵ.

- *By induction*, the k-th order solution $\psi^{(k)}$ contains only the $l = 0, 2, .., 2k$ harmonics with even m.

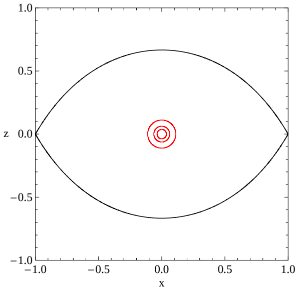
Two tidal sub-critical regimes

Extension parameter: $\delta_e \equiv \dfrac{x_e}{r_J}$





Weak deformation: $\delta_e << 1$

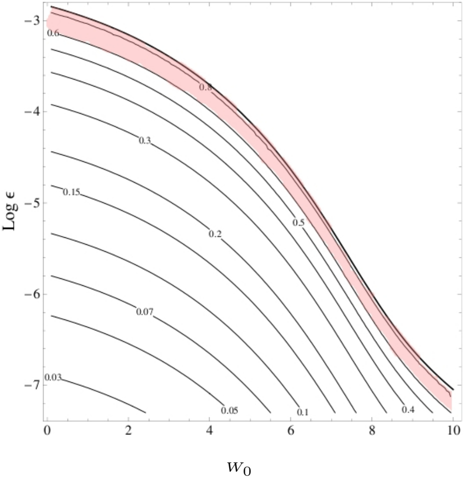


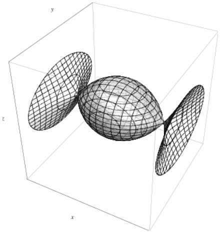
Tidal Models: Parameter space

Varri & Bertin ApJ 2009

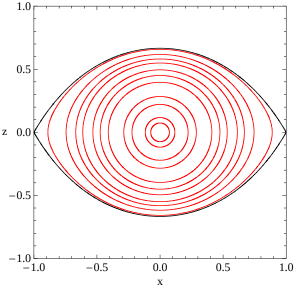
Two tidal sub-critical regimes

Extension parameter: $\delta_e \equiv \frac{x_e}{r_J}$





Strong deformation: $\delta_e \approx 1$



Tidal Models: Properties

Varri & Bertin ApJ 2009

Deformation shaped by the tidal potential:

- compression along $\hat{z}$
- elongation along $\hat{x}$

$$e = [1 - (\hat{c}/\hat{a})^2]^{1/2}$$
$$\eta = [1 - (\hat{b}/\hat{a})^2]^{1/2}$$
$$\hat{a} \geq \hat{b} \geq \hat{c}$$

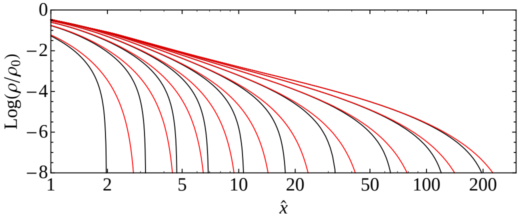
$$e_0, \eta_0 = \mathcal{O}(\epsilon^{1/2})$$

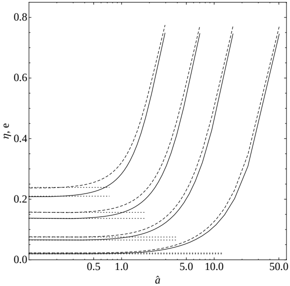
Non-trivial!

Quadrupole moments calculated analytically!

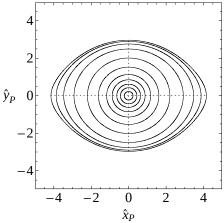
$$Q_{ij}^{(2)} = Q_{ij,1}\epsilon + Q_{ij,2}\frac{\epsilon^2}{2} = \int_V (3x_i x_j - r^2\delta_{ij})\rho(\mathbf{r})d^3r$$

$W_0 = 1, ..., 10 \qquad \epsilon = \epsilon_{cr}(W_0) \qquad \nu = 3$





No isophotal twist!



Axisymmetric models with:

i) solid-body rotation

ii) differential rotation

- If total angular momentum is non-vanishing, in the Maxwell-Boltzmann distribution function:

$$E \rightarrow H = E - \omega J_z$$

where ω represents the (solid-body) angular velocity of the system.

Landau & Lifchitz Stat Phys 1967

- Distribution function:

$$f_K(H) = \begin{cases} A\left[\exp\left(-aH\right) - \exp\left(-aH_0\right)\right] & \text{if } H \leq H_0 \\ 0 & \text{if } H > H_0 \end{cases}$$

$$H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \Phi_{cen} + \Phi_C \qquad \Phi_{cen}(\mathbf{r}) = -\frac{1}{2}\omega^2(x^2 + y^2)$$

$$\psi(\mathbf{r}) = a\{H_0 - [\Phi_c(\mathbf{r}) + \Phi_{cen}(\mathbf{r})]\}$$

Concentration $\leftrightarrow$ $W_0 \equiv \psi(\mathbf{0})$ Rotation strength $\leftrightarrow$ $\chi \equiv \dfrac{\omega^2}{4\pi G\rho_0}$

- Formally, the same singular perturbation problem - reduced to 2D.

See Kormendy & Anand Ap&SS 1971, Vandervoort ApJ 1980 for other DFs

RIGIDLY ROTATING MODELS: Properties

Varri & Bertin AIPC 2010

Deformation shaped by the centrifugal potential:
- "elongation" on $(\hat{x}, \hat{y})$

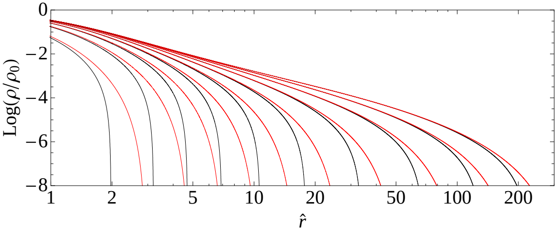
$$e = [1 - (\hat{b}/\hat{a})^2]^{1/2}$$
$$\hat{a} \geq \hat{b}$$

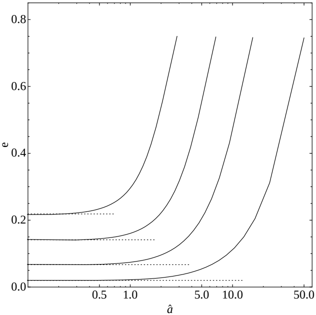
$$e_0 = \mathcal{O}(\chi^{1/2})$$
Non-trivial!

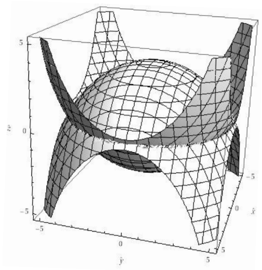
Quadrupole moments calculated analytically!

$$\hat{Q}_{zz}/\hat{Q}_{xx} = -2$$
$$\hat{Q}_{yy}/\hat{Q}_{xx} = 1$$
for every χ and W_0

$W_0 = 1, ..., 10$ $\quad\quad \epsilon = \chi_{cr}(W_0)$







Differentially rotating models: Definitions

Varri & Bertin in prep.

- New family in which internal rotation is rigid in the center but vanishes in the outer parts of the system, where the truncation on the energy E is effective.

$$I(E, J_z) \equiv E - \frac{\omega J_z}{1 + b J_z^{2c}}$$

$$I \sim H = E - \omega J_z \qquad \text{for low } |J_z|$$

$$I \sim E \qquad \text{for high } |J_z|$$

- Distribution function:

$$f_W(I) = \begin{cases} A \exp(-aE_0) \left\{ \exp\left[-a\left(I - E_0\right)\right] - 1 + a(I - E_0) \right\} & \text{if } E \leq E_0 \\ 0 & \text{if } E > E_0 \end{cases}$$

Continuous truncation in phase space

Wilson AJ 1975, Hunter AJ 1977

- Solution of the Poisson equation with iteration method:

$$\hat{\nabla}^2 \psi^{(n)} = -\frac{9}{\hat{\rho}_0} \hat{\rho}\left(\hat{r}, \theta, \psi^{(n-1)}\right)$$

$\psi(\hat{\mathbf{r}}) = a[E_0 - \Phi_c(\hat{\mathbf{r}})]$ sets the boundary

Prendergast & Tomer AJ 1970

Differentially rotating models: General properties Varri & Bertin in prep.

- Dimensionless parameters:

  Concentration: $W_0 \equiv \psi(\mathbf{0})$

  Central solid-body rotation: $\bar{\omega}^2 \equiv \frac{\omega^2}{4\pi G\rho_0}$

  Differential character: $\bar{b} \equiv \left(\frac{9b^{1/c}a^{-2}}{4\pi G\rho_0}\right)^c, c$

- For any W_0 there exists a $\bar{\omega}_{max}$, above which models cannot be constructed because the procedure does not converge
- Non-monotonic polar eccentricity profile
- In the center, isotropy and solid-body rotation:
- In the outer parts, transition to tangential anisotropy and no rotation
- Rapidly rotating models exhibit a toroidal core iff (NSC!)

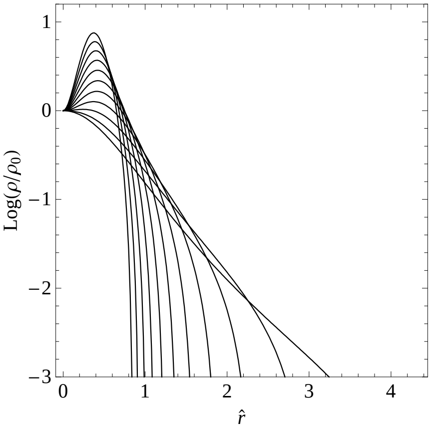
$$\frac{< \hat{v}_\phi >^2}{\hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial\psi}{\partial\hat{r}} > 0 \qquad \hat{r} \to 0$$

$$\bar{\omega}^2 > \frac{1}{3} + \frac{C_2}{18}\sqrt{\frac{5}{2}} \qquad C_2 = \frac{18}{5e^{W_0}\gamma(5/2, W_0)}\int_0^{\hat{r}_e} d\hat{r}'\,\hat{\rho}_2(\hat{r}')/\hat{r}'$$

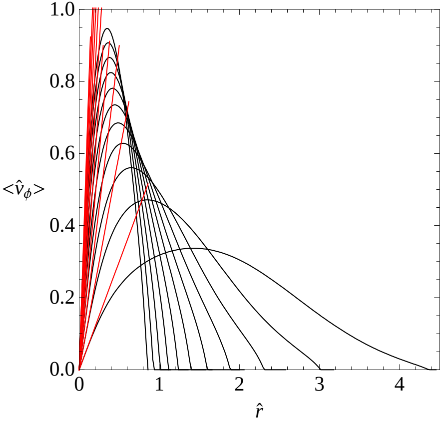
For toroidal cores in rapidly differentially rotating polytropes, see Stoeckely ApJ 1965, Geroyannis ApJ 1990

Differentially rotating models: Examples

Varri & Bertin in prep.



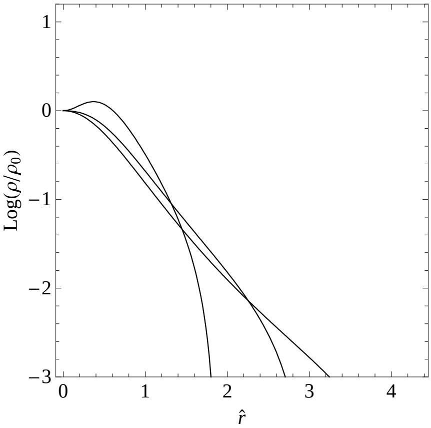
$W_0 = 2 \quad \bar{\omega}/\bar{\omega}_{max} \in [0.1, 1]$



$< \hat{v}_\phi > (\hat{r}, \theta) = 3\bar{\omega} \sin\theta \hat{r} + \mathcal{O}(\hat{r}^3)$

Differentially rotating models: Examples

Varri & Bertin in prep.



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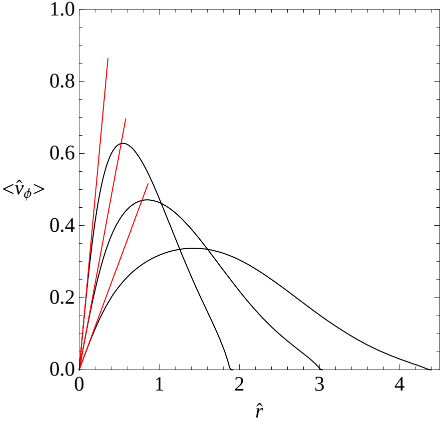

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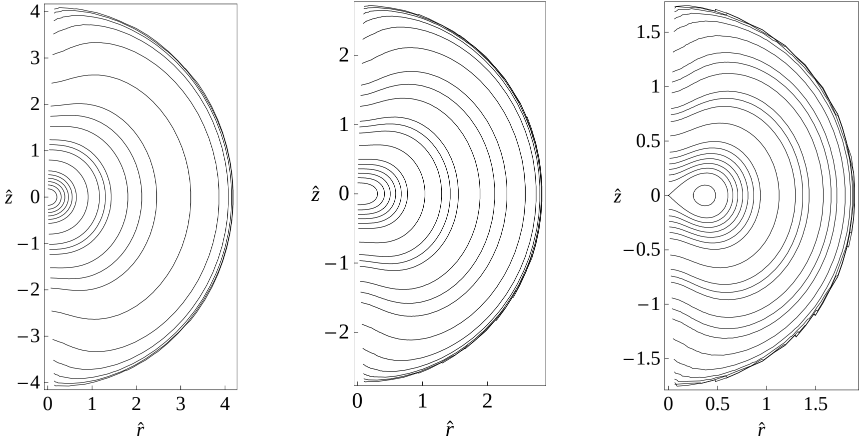
Differentially rotating models: Examples

Varri & Bertin in prep.

Moderate rotation | Rapid rotation



$\bar{\omega}/\bar{\omega}_{max} = 0.1$ $\bar{\omega}/\bar{\omega}_{max} = 0.2$ $\bar{\omega}/\bar{\omega}_{max} = 0.4$

$W_0 = 2$

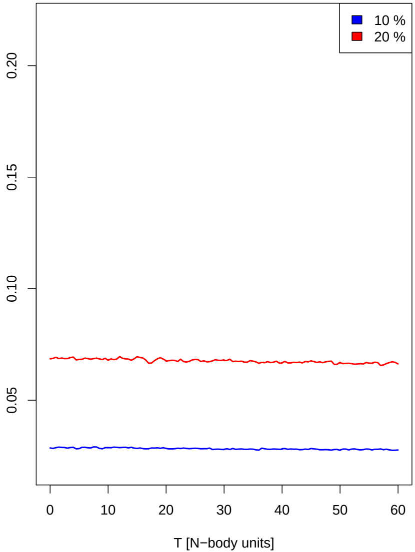
Sections in meridional plane of isodensity surfaces

Differentially rotating models: Dynamical stability

- Starlab Portegies Zwart et al MNRAS 2001
- $N = 65536$
- Isolated models
- Single mass, no stellar evolution

☑ Models with moderate rotation are stable

$t = K_{ord}/|W|$ $W_0 = 6$



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☑ Rapidly rotating models, even with the toroidal core, can be stable

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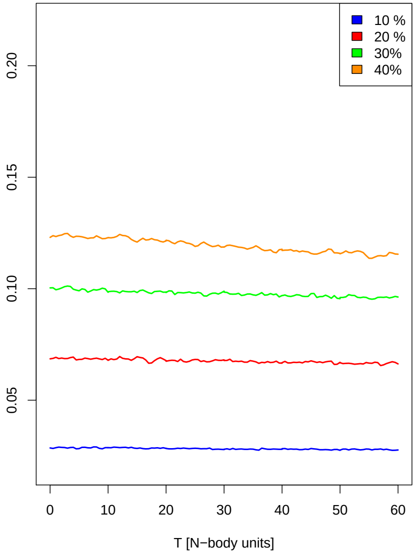

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☒ Extreme rotation regime is unstable

☑ Consistent with Ostriker & Peebles (1973) criterion! $t = 0.14 \pm 0.03$

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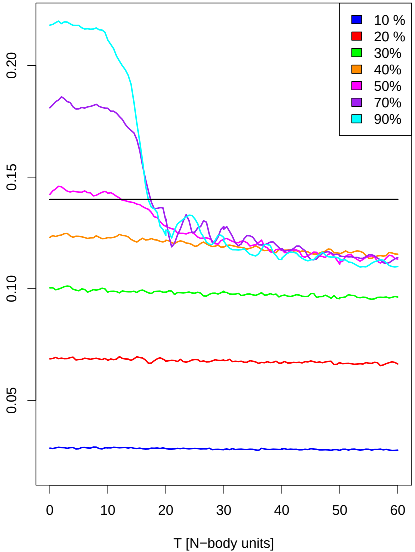

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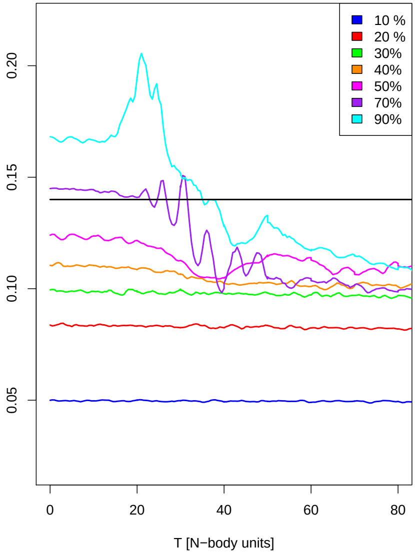
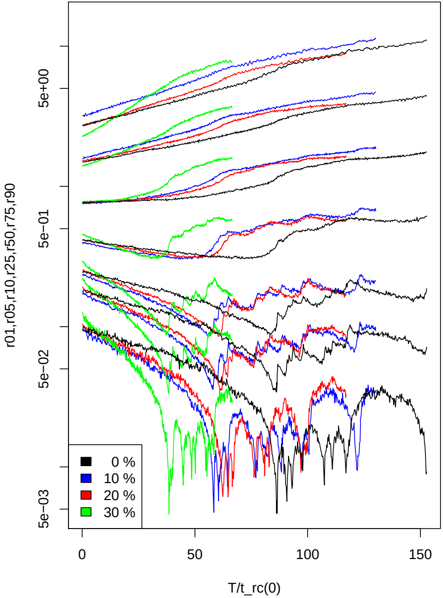


Differentially rotating models: Long term evolution



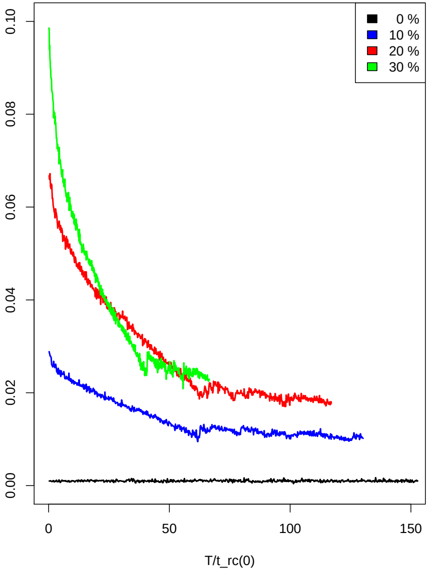
- Starlab
- $N = 16384$
- $W_0 = 6$
- Moderate/rapid rotation
- Isolated models
- Single mass, no stellar evolution
- No primordial binaries

Rotation accelerates the dynamical evolution

Hachisu PASJ 1979, Akyama & Sugimoto PASJ 1989, Lagoute & Longaretti A&A 1996, Einsel & Spurzem MNRAS 1999, Kim et al. MNRAS 2002, 2004

Differentially rotating models: Long term evolution

$t = K_{ord}/|W|$



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- $N = 16384$
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- Isolated models
- Single mass, no stellar evolution
- No primordial binaries

Rotation still present in the post-core collapse phase

Conclusions

- Triaxial tidal models have been constructed as an extension of spherical King models; intrinsic and projected properties have been given.

  Bertin & Varri ApJ, 685, 1005-1019 (2008), Varri & Bertin ApJ, 703, 1911-1922 (2009)

- Extension of spherical King models to the case of internal solid-body rotation has been performed.

  Varri & Bertin AIPC, 1242, 148-155 (2010)

- Promising family of differentially rotating models has been proposed.

  Varri & Bertin almost submitted

- Numerical study of dynamical stability and long term evolution in progress.

  Varri et al in preparation

Future work

- Rotating models: N-body simulations with tidal boundary, multimass.
- Tidal models: comparison with N-body simulations of star clusters with different degree of filling of the critical Hill surface.
- Comparison with observations: interpretation of observed flattening, "extra-tidal light", kinematics (rotation, anisotropy).

  Alice Zocchi Master's thesis: dynamical study of GCs in different relaxation conditions, King vs. $f^{(\nu)}$ models Bertin & Stiavelli Rep.Prog.Phys. 1993, Bertin & Trenti ApJ 2003