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Dynamic Evolutionary Modelling of Globular Star Clusters

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Outline

Introduction

- Types of dynamical models – evolutionary models

Monte Carlo models

- The method – advantages and limitations
- Results and problems

N-body models

- The method – advantages
- Results and limitations
- Prospects

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1. Introduction: Types of models

1. Static models:
   - Plummer's model (Plummer 1911)
   - King's model (King 1966, Peterson & King 1975)
   - Anisotropic models (King; Michie 1963)
   - Multi-mass models (Gunn & Griffin 1979; Meylan & Mayor et al; Pryor et al)
   - Non-parametric models (Gebhardt & Fischer 1995)
   - Schwarzschild's method (van de Ven et al 2006)
   - Jeans' equations (Leonard et al 1992)

2. Dynamic Evolutionary Models
   - Gas/fluid models (Angeletti & Giannone 1980 [M3])
   - Fokker-Planck models (Cohn and co-workers 1997 [M15], 1992 [N6624], 2003 [47 Tuc]; Drukier 1993, 1995 [N6397]; Phinney 1993 [M15])
   - Monte Carlo model (Giersz & H 2003,2008-9,2011 [ω Cen, M4, N6397, 47 Tuc])
   - *N*-body model (Zonoozi et al 2011 [Pal 14])

2. Monte Carlo models: method

Follows orbits of stars (*not* the stars)

- time step governed by relaxation time – very long
- simulations very fast – few days for 47 Tuc

Includes

- stellar evolution
- (most) binary interactions
- Galactic tide

Active codes

- Rasio group (Northwestern, USA)
- Mirek Giersz (CAMK, Poland)

Recent improvements in Giersz's code

- On-the-fly binary interactions
- Escape procedure uses correct time scale

Monte Carlo models: results and problems

2003: ω Cen (pilot project)

- Mass segregation (unpublished)

2008: M4

- A post-collapse cluster, despite its King profile
- Used in collaboration for planning and interpretation of observing programmes

2009: N6397

- Core exhibits gravothermal oscillations

2011: 47 Tuc

- Evolution so far mainly driven by mass-loss from stellar evolution
- Far from core collapse (>20Gyr), despite high concentration

Main problem: finding good initial conditions

- Time-consuming non-automatic trial and error
- Automatic method under development

2. N-body models: method and results

Example code: NBODY6 (Aarseth*)

- All point-mass dynamics
- Stellar evolution from fitting formula (Hurley et al)
- Realistic tide, cluster orbit and Galactic potential
- Rotation
- Collisions (sticky particles)
- GPU enabled

Results (restricted to modelling entire evolution of a specific globular cluster): Zonoozi et al (2011): Pal 14

- Likely flattened IMF
- Binaries dynamically unimportant
- Initial mass ~50 000$M_\odot$, initial half-mass radius ~20pc
- Time taken (with binaries) 1 month

*http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm

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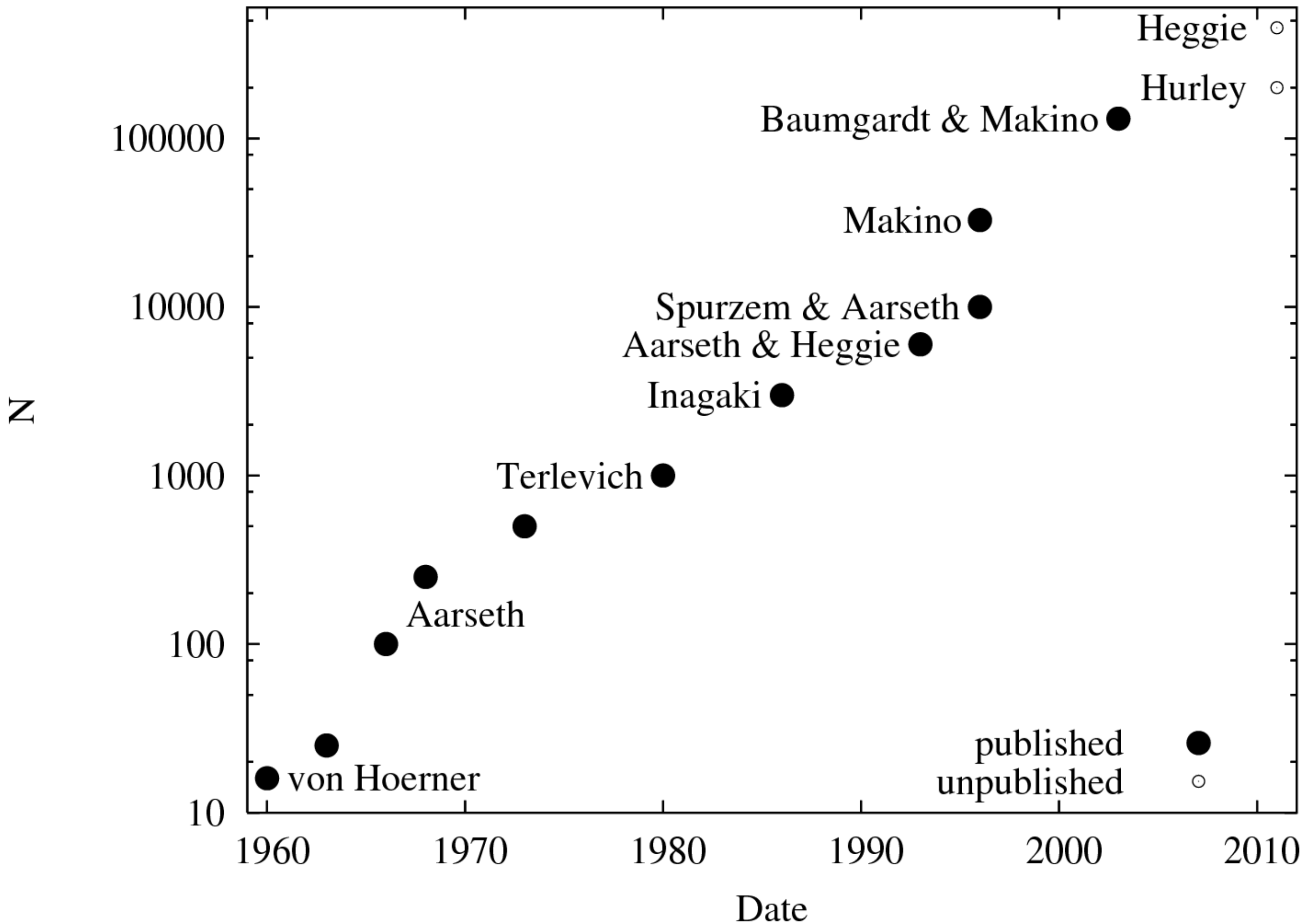
N-body models: the challenge



Largest N-body models extending past core collapse

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N-body models: why they take so long

- Each force takes N calculations
- Each of N stars needs updating at least every crossing time
- About N crossing times per relaxation time ($t_{rh} \propto N^{1/2} R_h^{3/2}$)

$\Rightarrow$ computing time $W \propto N^3$ per relaxation time

$$\propto N^{5/2} R_h^{-3/2}$$

- Binaries very time-consuming (factor ~10 for few % binary fraction); dependence on N, R_h complicated by distribution of hardness

Examples

- Pal 14: $M \sim 10^4 M_\odot$, $R_h \sim 34$pc; $W \sim 1$ month (with binaries; Zonoozi et al)
- M4: $M \sim 10^5 M_\odot$, $R_h \sim 3$pc; $W \sim 10^3$ years (with binaries; from scaling)

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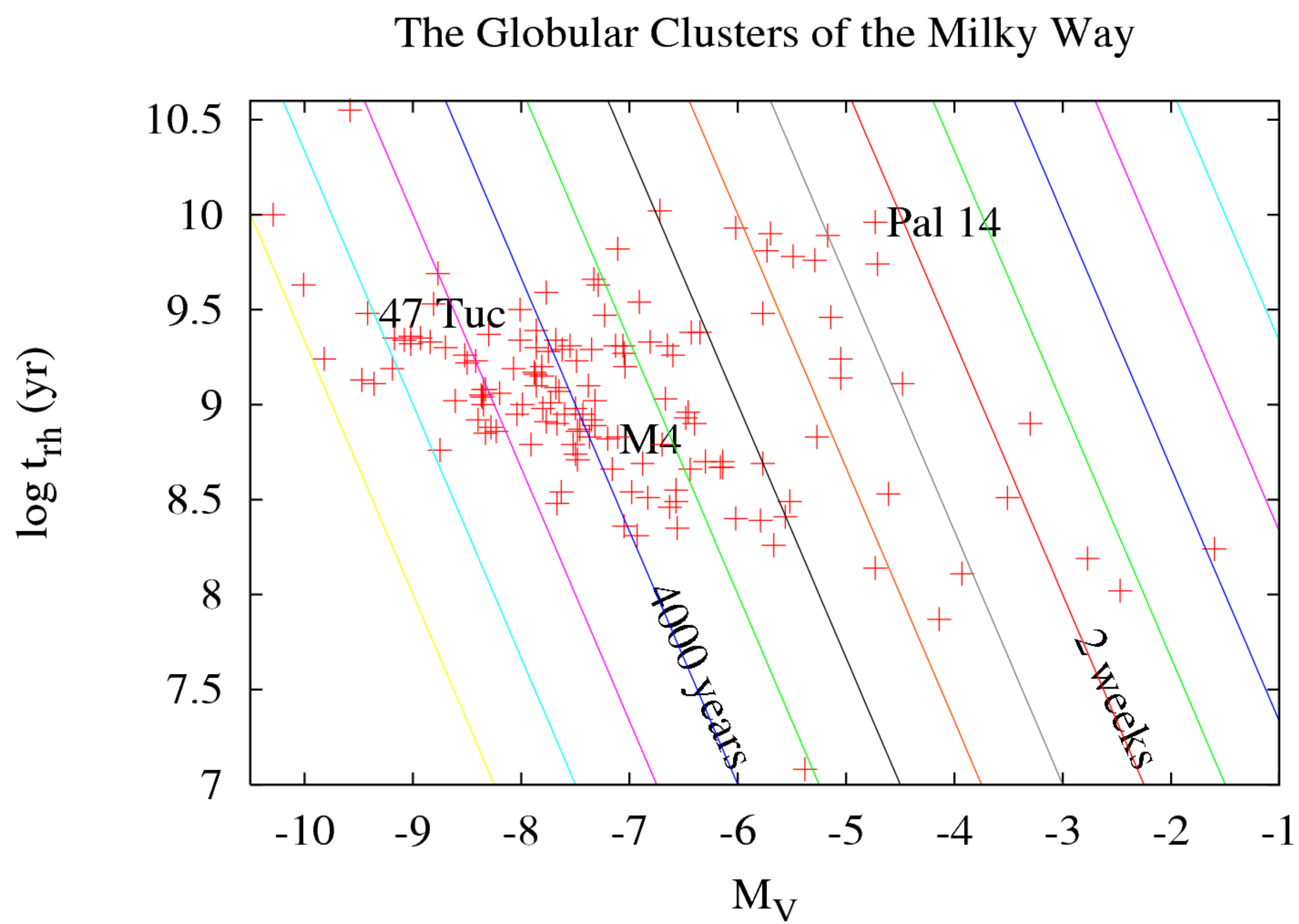
N-body models:
the globular clusters of the Milky Way



Data: Harris

N-body models: M4

Initial conditions:
- H & Giersz 2008 (Monte Carlo model)
- N = 484 710
- 7% primordial binary fraction
- $R_h = 0.58$pc

Software: NBODY6
Hardware: 8 cores, 3 GPU (GeForce GTX 285)
Starting date: 30 October 2010
Finish date (estimated):
- 2165 (extrapolating rate of progress so far)
- 2070 (extrapolating current rate of progress)
- 2016 (assuming scaling $\propto N^{5/2} R_h^{-3/2} f_b$, and using time-dependence from the Monte Carlo model)
- 2011 July (by scaling from a scaled-down model with same relaxation time)

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N-body models: scaling down

The idea: model a cluster with N stars by a model with N*<<N stars
The principle: get the time scale of the major evolutionary effects correct

Stellar evolution: set by stellar evolution models
Two-body relaxation: time scale $t_r \propto N^{1/2}R^{3/2}$
Interaction of binaries: interface between hard and soft binaries at binary separation $R/N \propto N^{-4/3}$
Internal evolution of binaries: need to scale stellar radii $R_* \propto N^{-4/3}$

But then other processes do not scale properly:

Collision time scale:
$1/(n\sigma v) \propto R^3/(N^{3/2}R_*) \propto \mathbf{N^{-7/6}}$
Escape time scale:
$N^{-1/4}t_r \propto \mathbf{N^{-1/4}}$
Sampling effects of upper mass function

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Example: models of 47 Tuc

Single-component King model

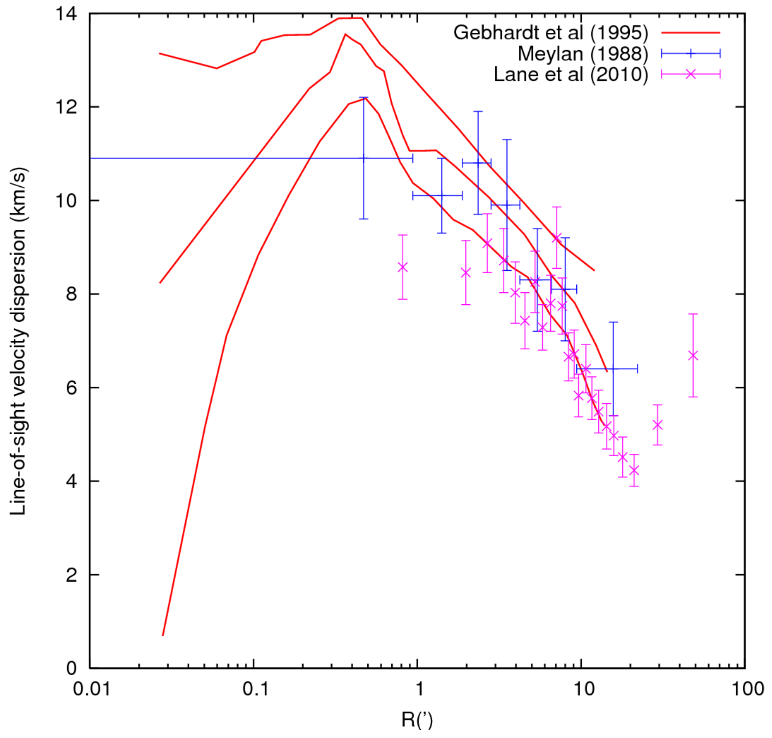
- Freire et al 2001: pulsar accelerations consistent with central line-of-sight velocity dispersion 11.6 ±1.4 km/s; > 11.6 (Freire et al 2003)
- McLaughlin et al 2006: 11.6±0.8 (proper motions)

Multi-component King model

- Meylan 1989: 10.1-10.4 (surface brightness and velocity dispersion profiles)

Comments:

- Pulsar data consistent with data of Gebhardt et al (1995) and Meylan (1998), but not Lane et al (2010)



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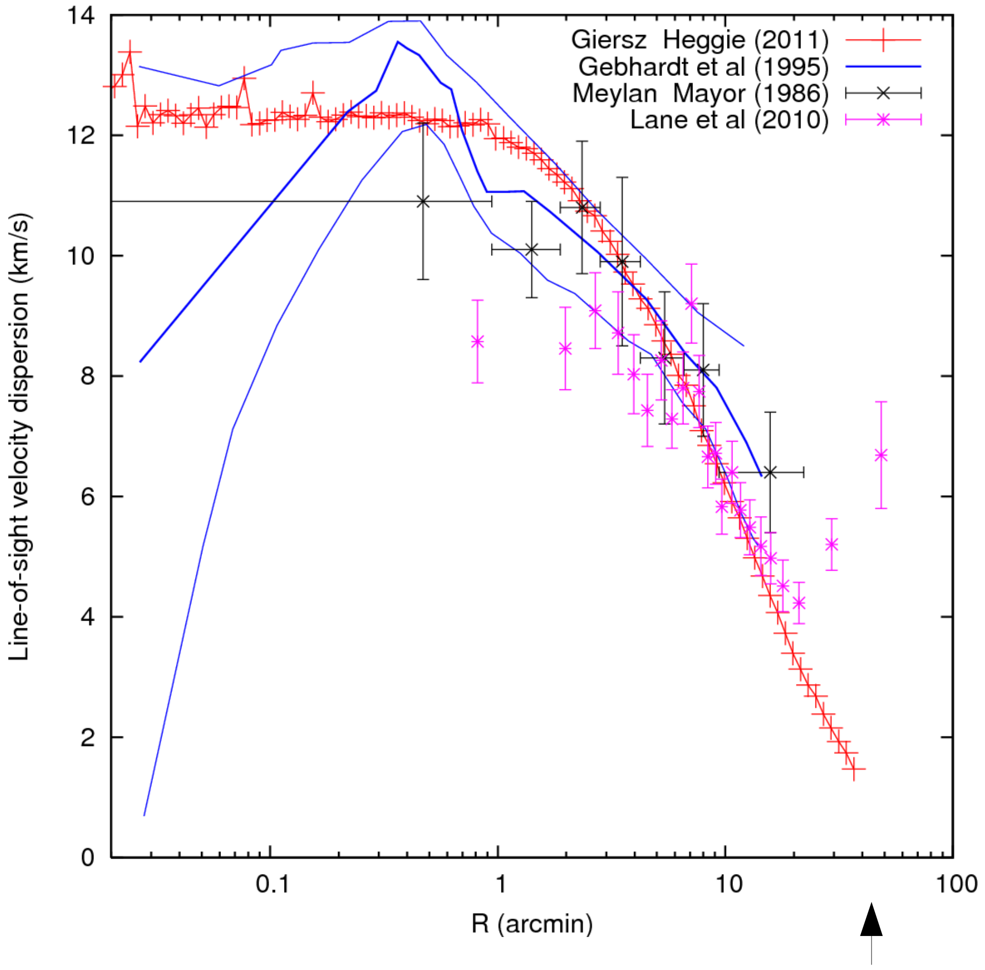
Example: models of 47 Tuc (continued)

Monte Carlo model (Giersz & H 2011)

- Consistent with pulsar acceleration
- Consistent (just) with Gebhardt et al (1995) *except* at large radii
- Tidal radius $\simeq 42'$
- Monte Carlo model treats tide as cut-off



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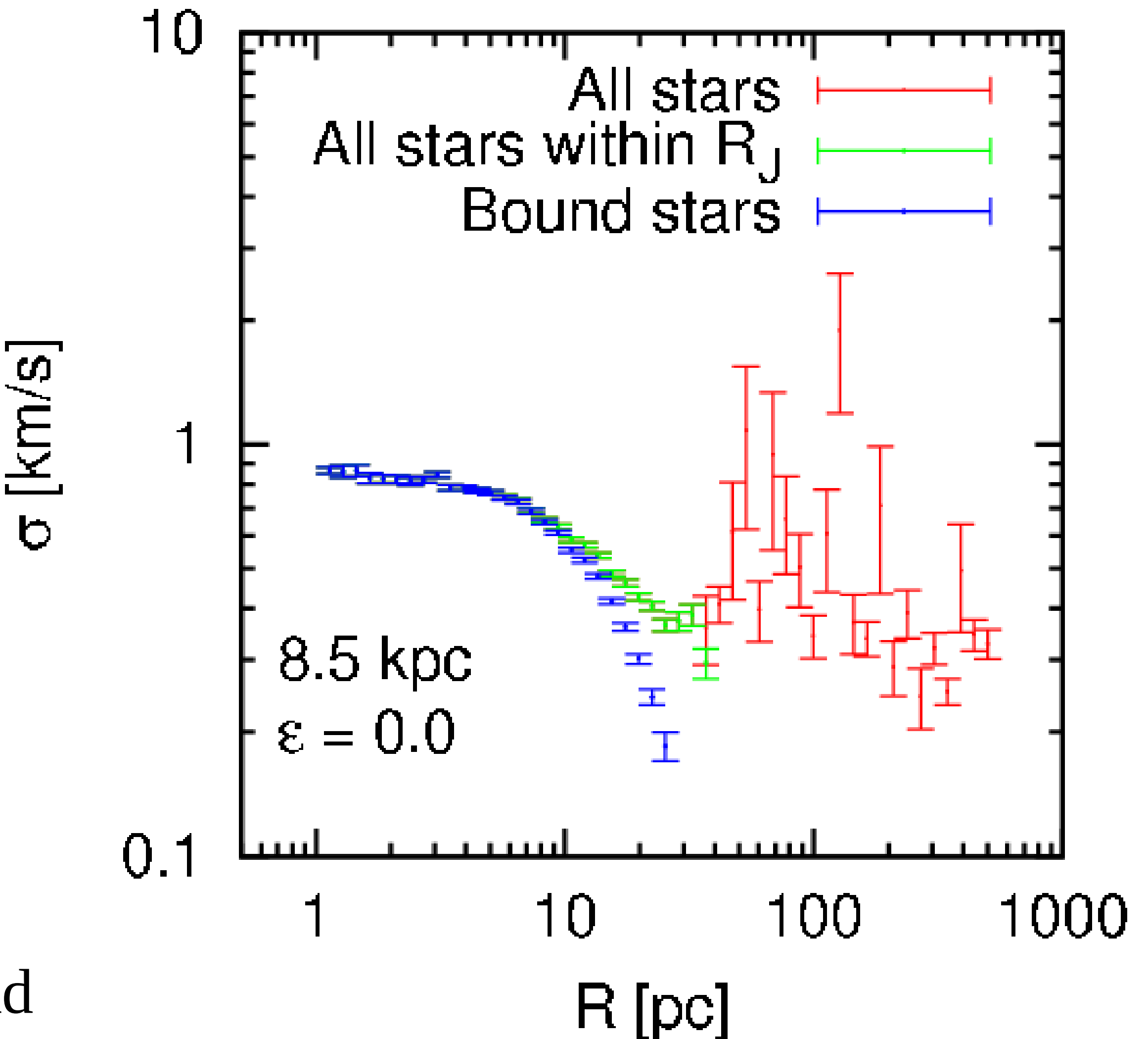
N-body models: tidal effects
An example using scaled models

- From Kuepper et al, 2010, MNRAS, 407, 2241
- Velocity dispersion profile
- Tidal radius ≃40pc
- At 25pc half the velocity dispersion contributed by stars inside the cluster but not bound to it.
- This is a population of "potential escapers", which can remain inside the cluster for ~ 10^8yr, some indefinitely
- The elevated velocity dispersion is not due to "tidal heating"
- A cluster may contain bound members with speeds above the escape speed



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N-body models: how to avoid scaling?

GPU server running M4 simulation with NBODY6 ↓

GPU cluster at NAOC (Beijing) ➔

- NBODY6++ under development

Image: P. Berczik

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Modelling individual globular clusters: summary

To model a cluster within a few days, you generally need either

- A Monte Carlo code, or
- A scaled-down N-body model, or
- A huge computer and new software

You need to run many models to find appropriate initial conditions

Monte Carlo models and scaled N-body models have complementary advantages

You need to ignore the second-generation problem