Bias correction for high precision science

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Outline

The problem

Existing bias-removal methods

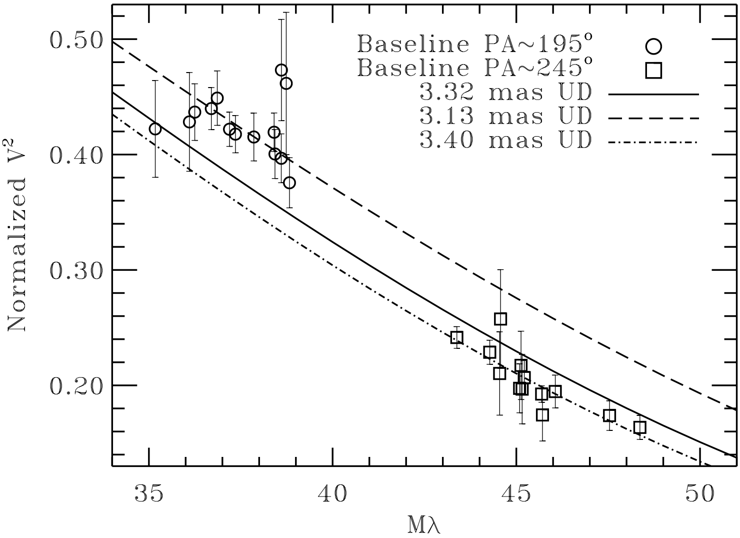
New method

Examples

Acknowledgements

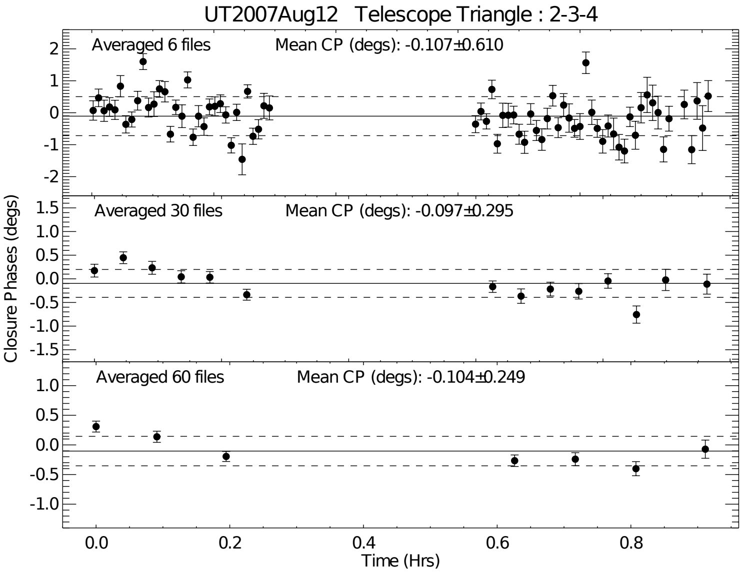
- Hrobjarthur Thorsteinsson
- James Gordon

High-precision measurements yield new science



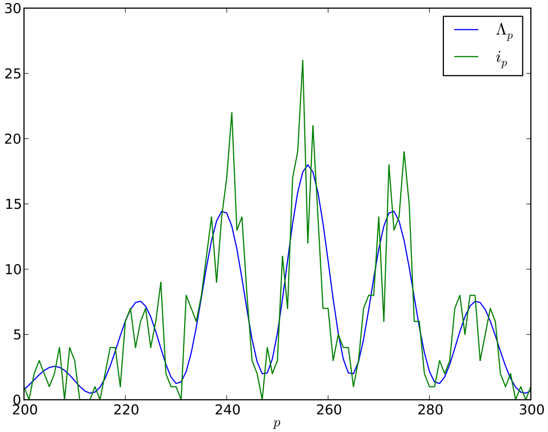
Van Belle *et al.* 2001

Closure phases are a high-precision observable



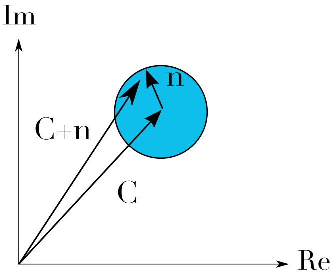
Zhao *et al.* 2007

Detection noise affects all interferometry data



- Mean photon rate $\{\Lambda_p, \quad p = 1 \ldots N_{\mathrm{pix}}\}$
- Detected flux $\{i_p, \quad p = 1 \ldots N_{\mathrm{pix}}\}$

Zero-mean noise can lead to a non-zero bias



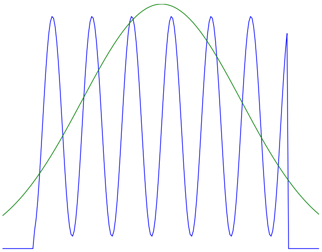
$$C = \sum_p \exp(2\pi i u x_p)\Lambda_p \qquad c = \sum_p \exp(2\pi i u x_p) i_p = C + n$$

$$\begin{aligned}\left\langle |C+n|^2 \right\rangle &= \left\langle |C|^2 \right\rangle + 2\left\langle \Re\left[C n^*\right]\right\rangle + \left\langle |n|^2 \right\rangle \\ &= \left\langle |C|^2 \right\rangle + \left\langle |n|^2 \right\rangle\end{aligned}$$

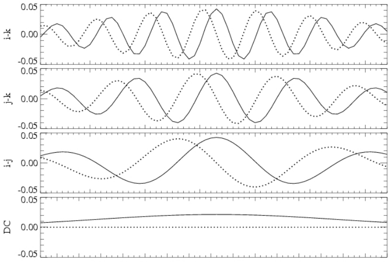
Photon-noise bias on the bispectrum is well-known

$$\left\langle C^{ij} C^{jk} C^{ki} \right\rangle = \left\langle c^{ij} c^{jk} c^{ki} \right\rangle - \left\langle \left| c^{ij} \right|^2 \right\rangle - \left\langle \left| c^{jk} \right|^2 \right\rangle - \left\langle \left| c^{ki} \right|^2 \right\rangle + 2 \left\langle N_{\mathrm{phot}} \right\rangle$$

Bias correction is only possible under restricted conditions



- "DFT conditions"
  - Integer number of fringe cycles in a scan
  - No "tapering" allowed
- No read noise allowed



We generalise the estimator for the complex amplitude

DFT estimator

$$c^{ij} = \sum_p \exp(2\pi i u^{ij} x_p) i_p$$

Generalised to

$$c^{ij} = \sum_p H_p^{ij} i_p$$

For example, a tapered DFT:

$$H_p^{ij} = W(x_p) \exp(2\pi i u^{ij} x_p)$$

Can represent *any* linear estimator, e.g. P2VM, ABCD

We allow any combination of read noise and photon noise

$$P_{\text{Poisson}}\left(N_p|\Lambda_p\right) = \sum_n^{\infty} \delta\left(N_p - n\right) \frac{\Lambda_p^{N_p}}{N_p!} \exp\left[-\Lambda_p\right].$$

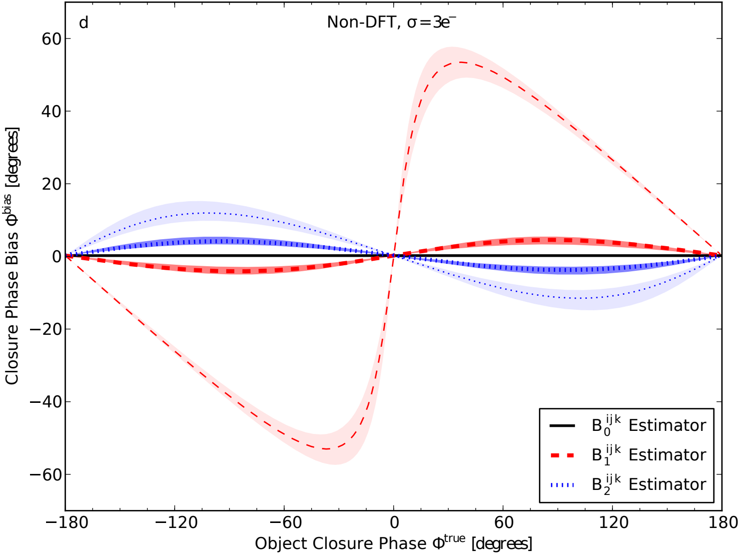
$$P_{\text{Gaussian}}\left(\epsilon_p|\sigma_p, N_p\right) = \frac{1}{\sqrt{2\pi\sigma_p^2}} \exp\left[-\frac{\left(\epsilon_p\right)^2}{2\sigma_p^2}\right].$$

$$P\left(i_p|\Lambda_p, \sigma_p\right) = \int_0^{i_p} P_{\text{Poisson}}\left(N_p|\Lambda_p\right) P_{\text{Gaussian}}\left(i_p - N_p|\sigma_p, N_p\right) dN_p.$$

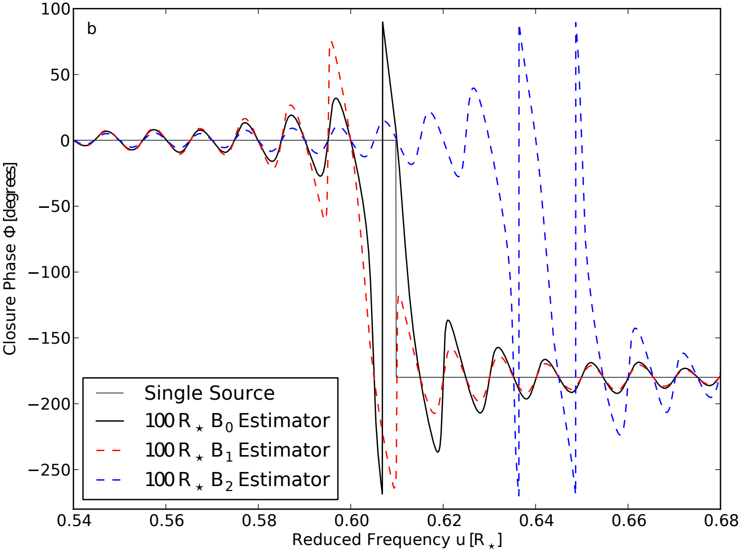
We derive a bias-free bispectrum estimator

$$
\begin{aligned}
B_0^{ijk} &= c^{ij} c^{jk} c^{ki} \\
&- c^{ij} \sum_p \left(i_p + \sigma_p^2 \right) H_p^{jk} H_p^{ki} \\
&- c^{jk} \sum_p \left(i_p + \sigma_p^2 \right) H_p^{ij} H_p^{ki} \\
&- c^{ki} \sum_p \left(i_p + \sigma_p^2 \right) H_p^{ij} H_p^{jk} \\
&+ \sum_p \left(2 i_p + 3 \sigma_p^2 \right) H_p^{ij} H_p^{jk} H_p^{ki} .
\end{aligned}
$$

The closure phase bias typically increases for non-zero closure phase



A practical example is closure-phase nulling



Summary

- Systematic errors can arise from detection noise
- We have presented a general bias-free bispectrum estimator
- A small step towards higher-precision science

Spare slides

The power spectrum variance contains coupled terms