Principles of interferometry

Interferometry Primer

Evolution of solar–mass stars

ESO Garching,
March 2-5, 2010

Chris Haniff

Astrophysics Group, Cavendish Laboratory,
University of Cambridge, UK

1 March 2010

This is a “primer” – please ask questions

What is the difference between a CCD and an interferometer?

Outline

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

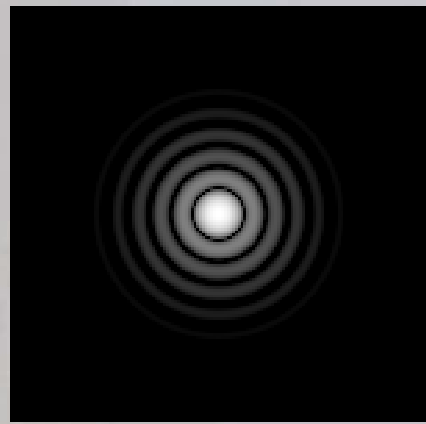
Outline

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

How do we characterize what a source looks like ?

How do we normally think about images?

- Consider a perfect telescope in space observing an unresolved point source:
  - This produces an Airy pattern with a characteristic width: $\theta = 1.22\lambda/D$.



- What does a more complicated object look like?
  - Each point in the source produces a displaced Airy pattern.
  - The superposition of these limits the detail visible in the final image.

So even in perfect conditions, a telescope image is NOT a perfect representation of what's in the sky.

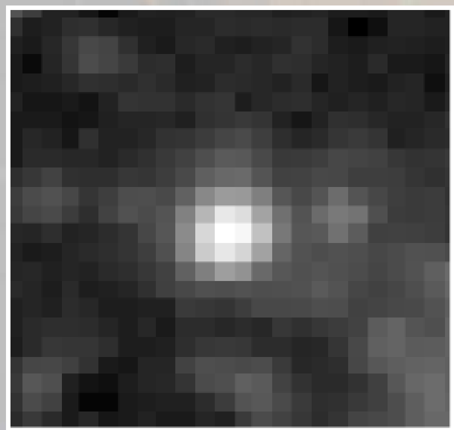
How do we describe this process mathematically?

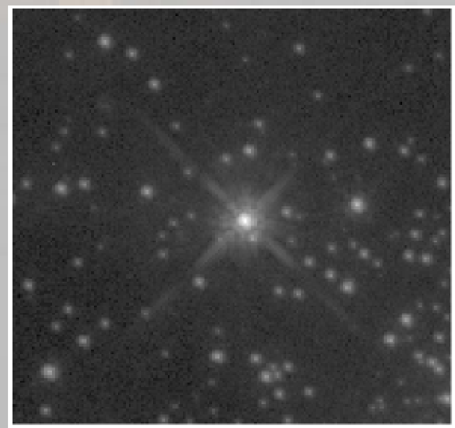
- The fundamental relationship for isoplanatic imaging is:

$$I(l, m) = \iint P(l-l', m-m')\, O(l', m')\, dl'\, dm' \,,$$

i.e. the observed brightness distribution is the true source brightness distribution convolved with a point-spread function, P(l, m).

Note that here l and m are angular coordinates on the sky, measured in radians.





This week you need to forget this and think differently

- Take the Fourier transform of the convolution on the previous slide to get:

$$I(u, v) = T(u, v) \times O(u, v) ,$$

where *italic* functions refer to the Fourier transforms of their roman counterparts, and u and v are now spatial frequencies measured in radians^{-1}.

- The essential properties of the target are encapsulated in its Fourier spectrum, $O(u, v)$.
- The essential properties of the imaging system are encapsulated in a complex multiplicative transfer function, $T(u, v)$.
- The transfer function is just the Fourier transform of the PSF.

Measuring I(l,m) ≡ measuring *I(u,v)*.

How shall we understand the "Transfer function"?

- In general the transfer function is obtained from the auto-correlation of the complex aperture function:

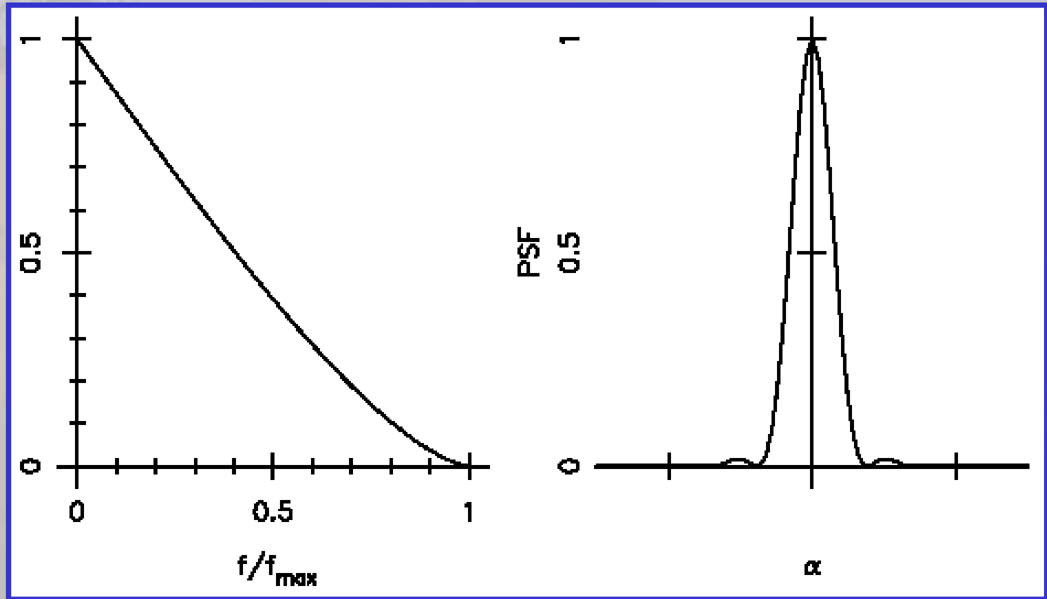
$$T(u, v) = \iint A^*(x, y)\, A(x+u, y+v)\, dx\, dy\ .$$

Here x and y denote co-ordinates in the aperture. In the absence of aberrations A(x, y) is equal to 1 where the aperture is transmitting and 0 otherwise.

- Some key features of this formalism worth noting are:
  - For each spatial frequency, *u*, there is a physical baseline, B, in the aperture, of length λu.
  - Different shaped apertures measure different Fourier components of the source.
  - Different shaped apertures give different PSFs:
    - Important, e.g., for planet detection.

We can use a circular aperture as an example

- For a circularly symmetric aperture, the transfer function can be written as a function of a single co-ordinate: $T(f)$, with $f^2 = u^2+v^2$.

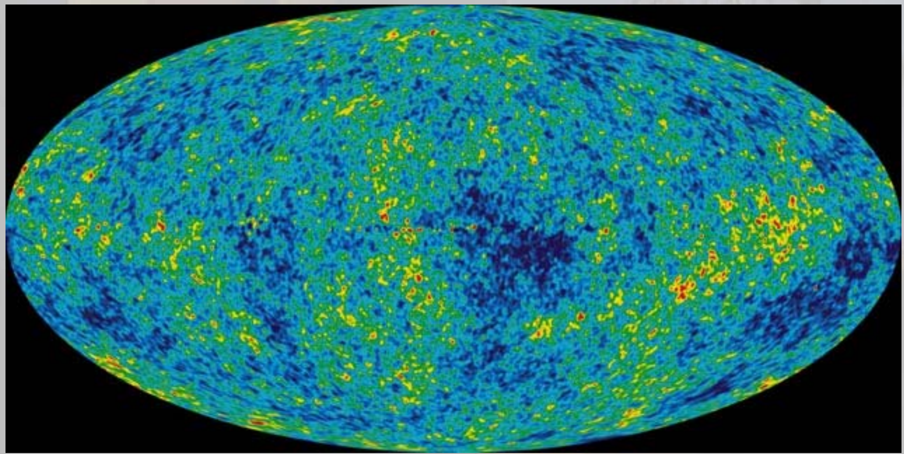


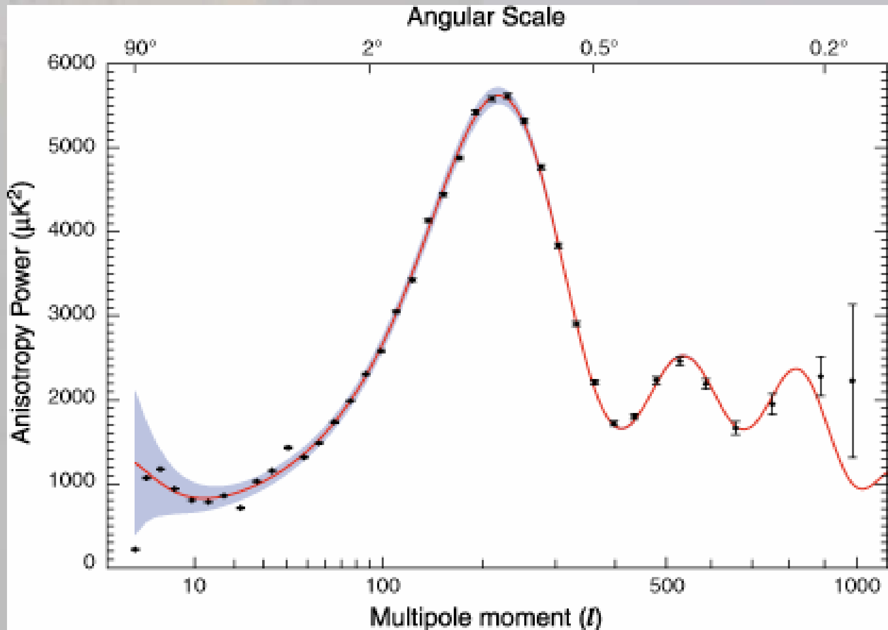
- The PSF is the familiar Airy pattern.
- The full-width at half-maximum of this is at approximately λ/D.
- $T(f)$ falls smoothly to zero at $f_{max} = D/\lambda$.

The behavior of $T(f)$ is why even perfect telescopes don't image sources perfectly.

So, what should you really learn from all this?

- Decomposition of an image into a series of spatially separated compact PSFs.
- The equivalence of this to a superposition of non-localized sinusoids, i.e. Fourier components.





What should you learn from all this?

- Decomposition of an image into a series of spatially separated compact PSFs.
- The equivalence of this to a superposition of non-localized sinusoids, i.e. Fourier components.
- The action of ANY incoherent imaging system as a filter for the Fourier spectrum of the source.
- The association of each Fourier component (or spatial frequency) with a distinct physical baseline in the aperture that samples the light.
- The form of the point-spread function as arising from the mix of different spatial frequencies measured by the imaging system.

If you can measure the Fourier components of the source, you should be able to do your science.

Recap & questions

Forget thinking about what a source looks like – start thinking about what its Fourier transform looks like.

If you can measure all the Fourier transform, that's the same as making an image.

Interferometers are devices to measure the Fourier content of your target.

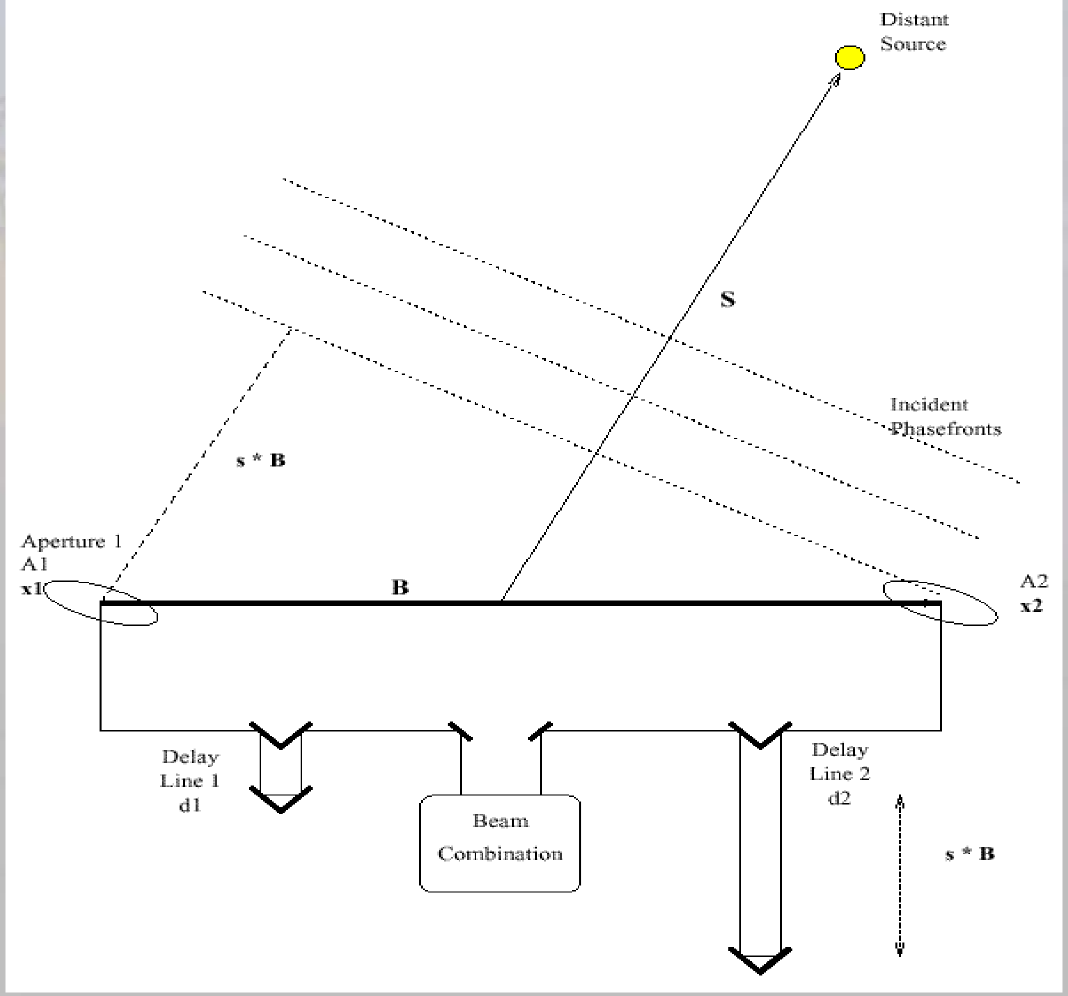
Outline

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

What/how do interferometers measure?

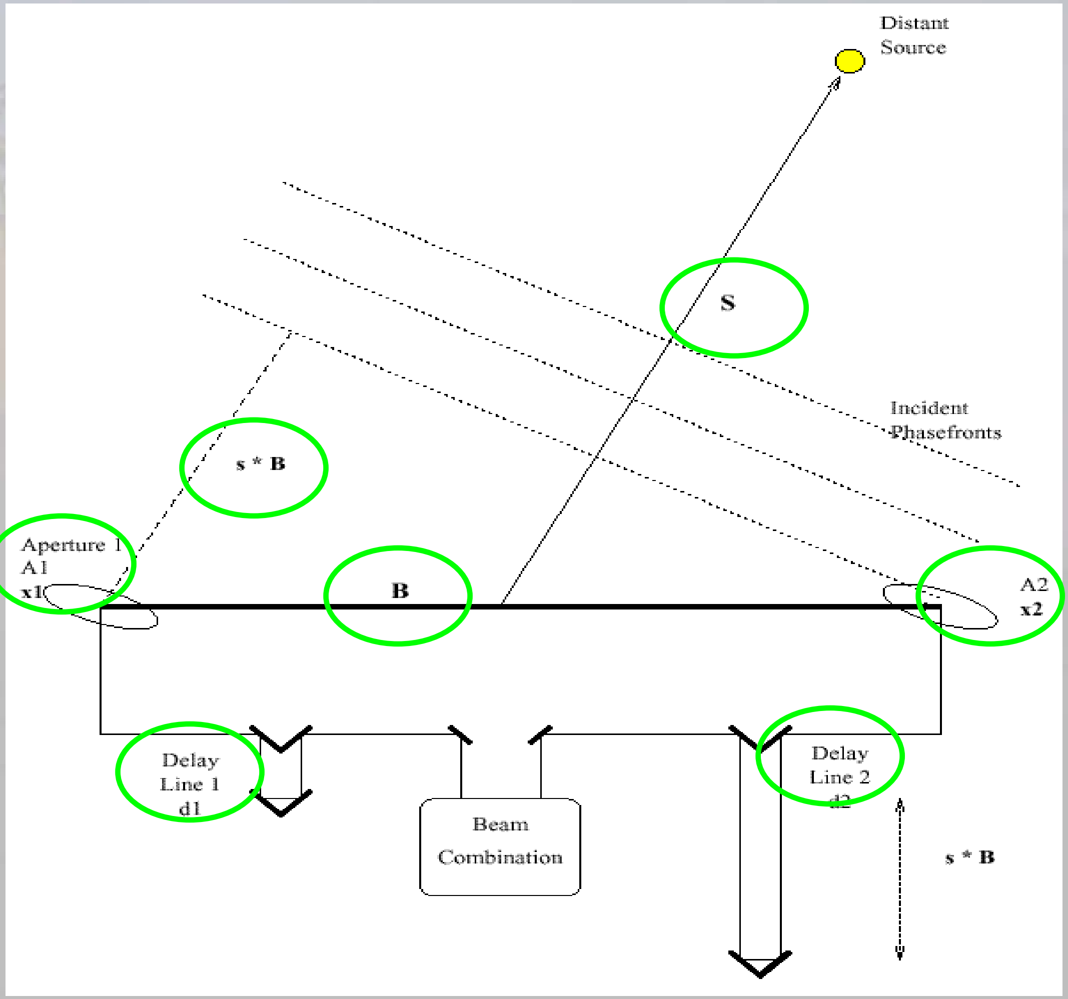
A 2-element interferometer – what happens?

- Sampling of the radiation (from a distant point source).
- Transport to a common location.
- Compensation for the geometric delay.
- Combination of the beams.
- Detection of the resulting output.

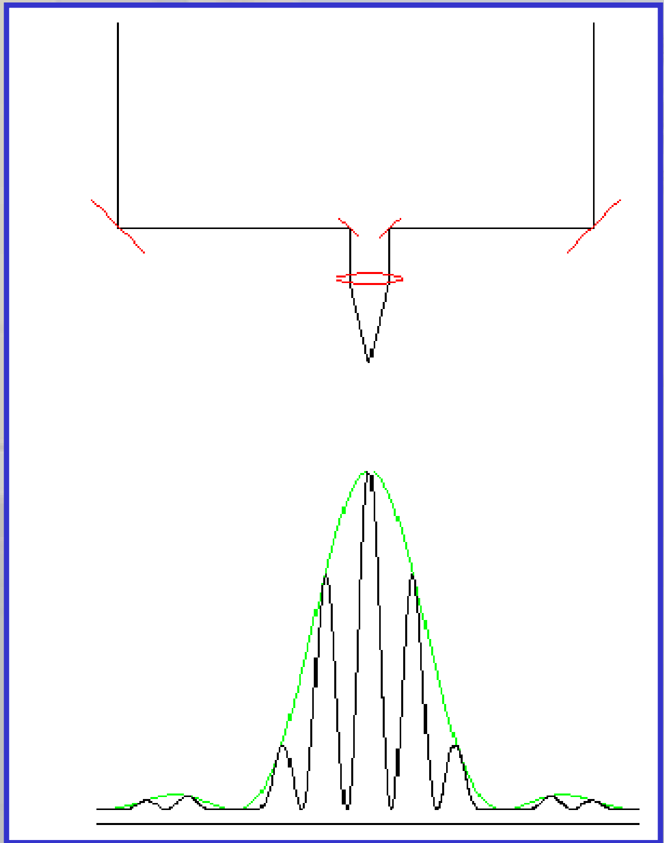


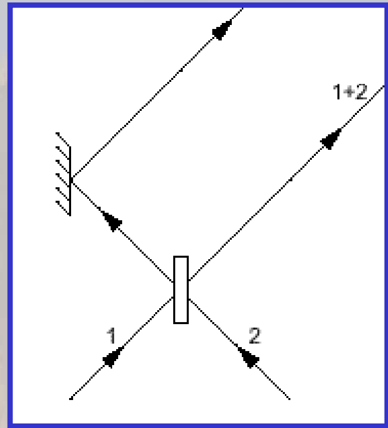
A 2-element interferometer – some jargon

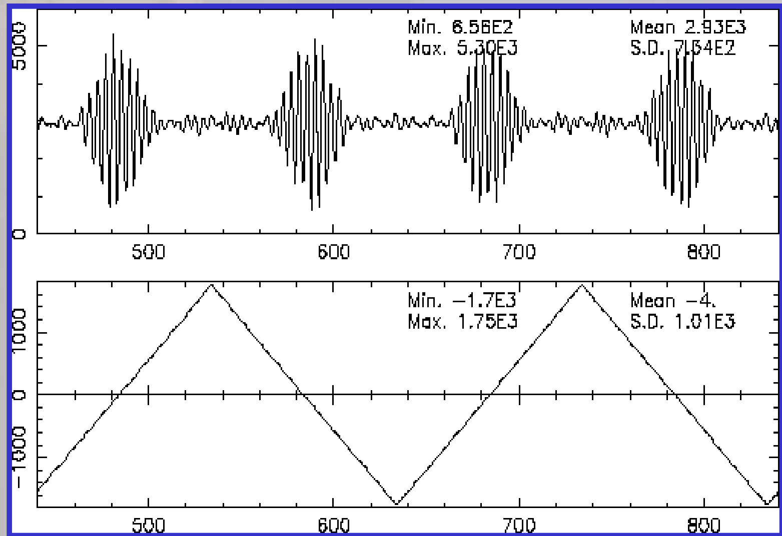
- Telescopes located at x_1 & x_2.
- Baseline $B = (x_1 - x_2)$:
  - Governs sensitivity to different angular scales.
- Pointing direction towards source is S.
- Geometric delay is $\hat{s}.B$, where $\hat{s} = S/|S|$.
- Optical paths along two arms are d_1 and d_2.



How do you "combine" the beams?







The output of a 2-element interferometer

- At combination the E fields from the two collectors can be described as:
  - ψ_1 = Aexp(ik[ŝ.B+d_1])exp(-iωt) and ψ_2 =Aexp(ik[d_2])exp(-iωt).
- So, <u>summing</u> these at the detector we get a resultant:

$$\Psi = \psi_1 + \psi_2 = A \left[\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2]) \right] \exp(-i\omega t) .$$

The output of a 2-element interferometer

- At combination the E fields from the two collectors can be described as:
  - $\psi_1 = A\exp(ik[\hat{s}.B+d_1])\exp(-i\omega t)$ and $\psi_2 = A\exp(ik[d_2])\exp(-i\omega t)$.
- So, summing these at the detector we get a resultant:

$$\Psi = \psi_1 + \psi_2 = A\left[\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])\right]\exp(-i\omega t) .$$

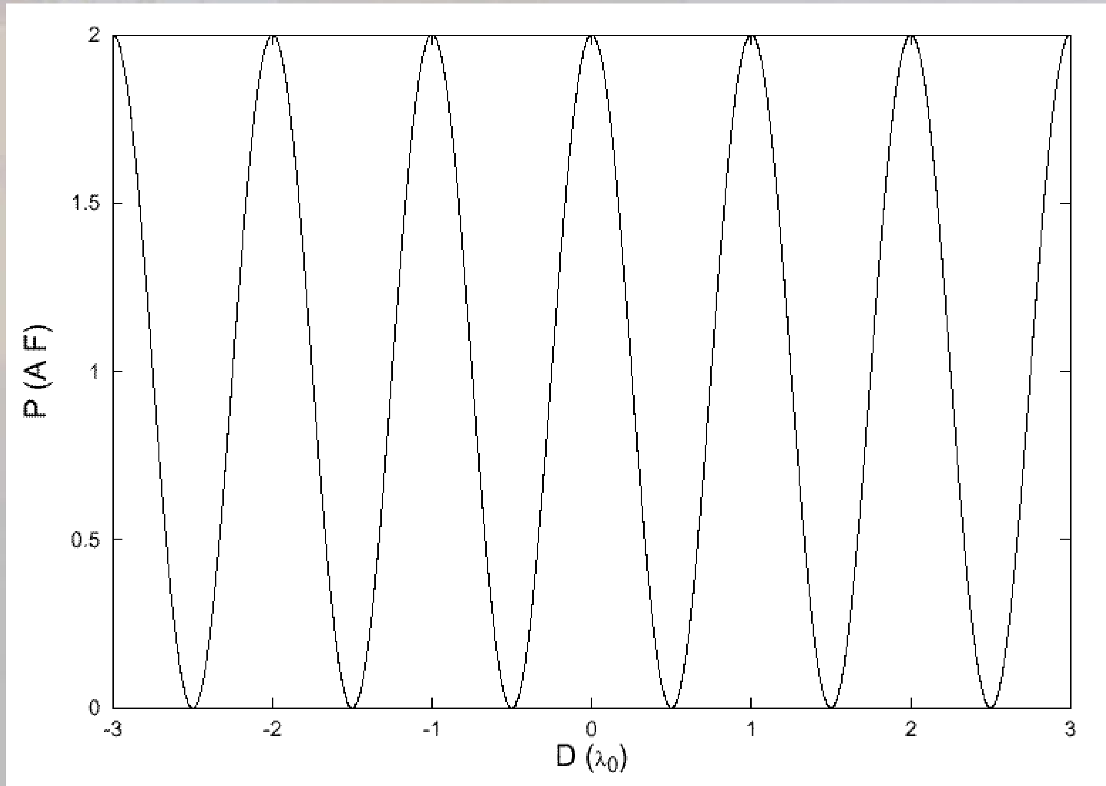
- Hence the time averaged intensity, $\langle\Psi\Psi^*\rangle$, will be given by:

$$\begin{aligned}
\langle\Psi\Psi^*\rangle &\propto \langle\, [\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])] \times [\exp(-ik[\hat{s}.B + d_1]) + \exp(-ik[d_2])]\, \rangle \\
&\propto 2 + 2\cos(\, k\, [\hat{s}.B + d_1 - d_2]\,) \\
&\propto 2 + 2\cos(kD)
\end{aligned}$$

Here we define $D = [\hat{s}.B + d_1 - d_2]$. D is a function of the path lengths, d_1 and d_2, the pointing direction (i.e. where the target is) and the baseline.

What does this output look like?

Detected intensity, $I = \langle \Psi\Psi^* \rangle \propto 2 + 2\cos(k\,[\hat{s}.B + d_1 - d_2])$
$\propto 2 \times [1+\cos(kD)]$, where $D = [\hat{s}.B + d_1 - d_2]$



- The intensity varies co-sinusiodally with kD, with $k = 2\pi/\lambda$.
- Adjacent fringe peaks are separated by
  - $\Delta d_{1 \text{ or } 2} = \lambda$ or
  - $\Delta(\hat{s}.B) = \lambda$ or
  - $\Delta(1/\lambda) = 1/D$.

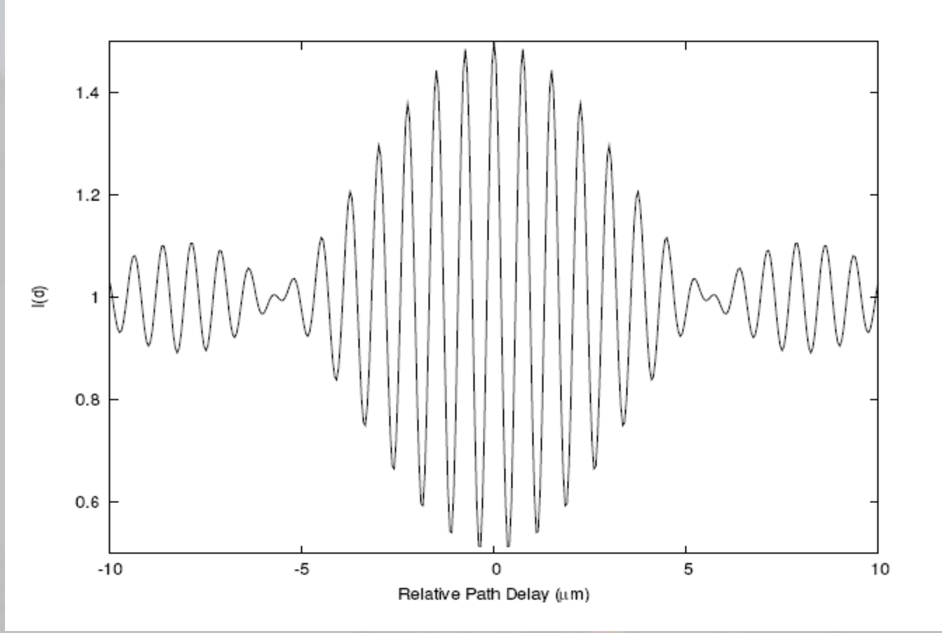
So we get fringes – but what should we measure?

- From an interferometric point of view the key observables are the contrast and location of these modulations in intensity.

- In particular we can identify:
  - The fringe visibility at D=0:

$$V = \frac{[I_{max} - I_{min}]}{[I_{max} + I_{min}]}$$

  - The fringe phase:
    - The location of the white-light fringe as measured from some reference (radians).



The fringe amplitude and phase measure the amplitude and phase of the Fourier transform of the source at one spatial frequency.

Recap & questions

All (2-element summing) interferometers produce a power output that shows a cosinusoidal variation – these are its fringes.

Properties of these fringes encode the amplitude and phase of the Fourier transform of the target.

For every pair of telescopes (i.e. 2-element interferometer) you get one measurement.

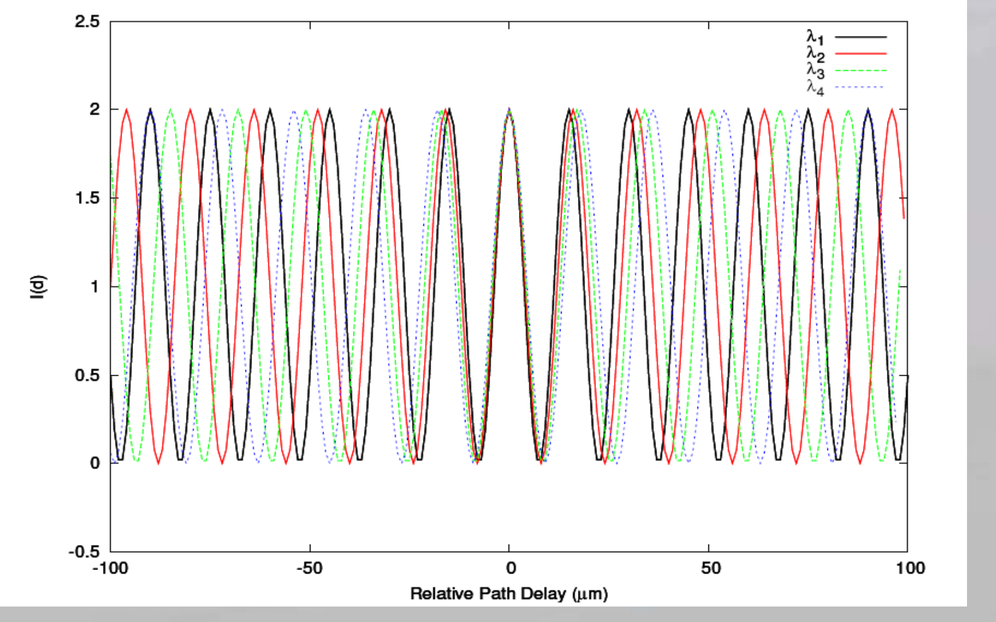
Interferometers are just machines to make these single measurements.

What happens with polychromatic light?

- We can integrate the previous result over a range of wavelengths:
  - E.g for a uniform bandpass of $\lambda_0 \pm \Delta\lambda/2$ (i.e. $\nu_0 \pm \Delta\nu/2$) we obtain:

$$I \propto \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} [2+2\cos(kD)]\, d\lambda$$

$$= \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} 2\,[1+\cos(2\pi D/\lambda)]\, d\lambda$$



What happens with polychromatic light?

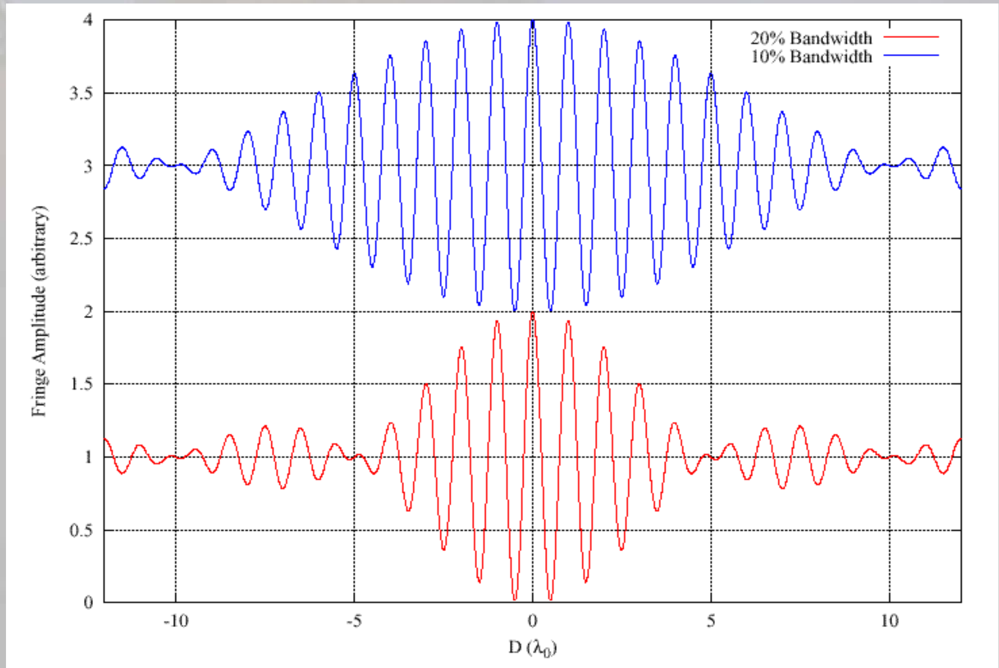
- We can integrate the previous result over a range of wavelengths:
  - E.g for a uniform bandpass of $\lambda_0 \pm \Delta\lambda/2$ (i.e. $\nu_0 \pm \Delta\nu/2$) we obtain:

$$I \propto \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} [2+2\cos(kD)]\, d\lambda$$

$$= \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} 2\,[1+\cos(2\pi D/\lambda)]\, d\lambda$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D\Delta\lambda/\lambda^2_0}{\pi D\Delta\lambda/\lambda^2_0}\cos k_0 D\right]$$

$$= \Delta\lambda\left[1+\frac{\sin \pi D/\Lambda_{coh}}{\pi D/\Lambda_{coh}}\cos k_0 D\right]$$



The fringes are modulated with an envelope with a characteristic width equal to the coherence length, $\Lambda_{coh} = \lambda^2_0/\Delta\lambda$.

Key ideas regarding the interferometric output (i)

- The output of the interferometer is a time averaged intensity.
- The intensity has a co-sinusoidal variation – these are the "fringes".
- The intensity varies a function of (kD), which itself can depend on:
  - The wavevector, $k = 2\pi/\lambda$.
  - The baseline, B.
  - The pointing direction, s.
  - The optical path difference between the two interferometer arms.
- If things are adjusted correctly, then the interferometer output can remain fixed: in that case there will be no fringes.

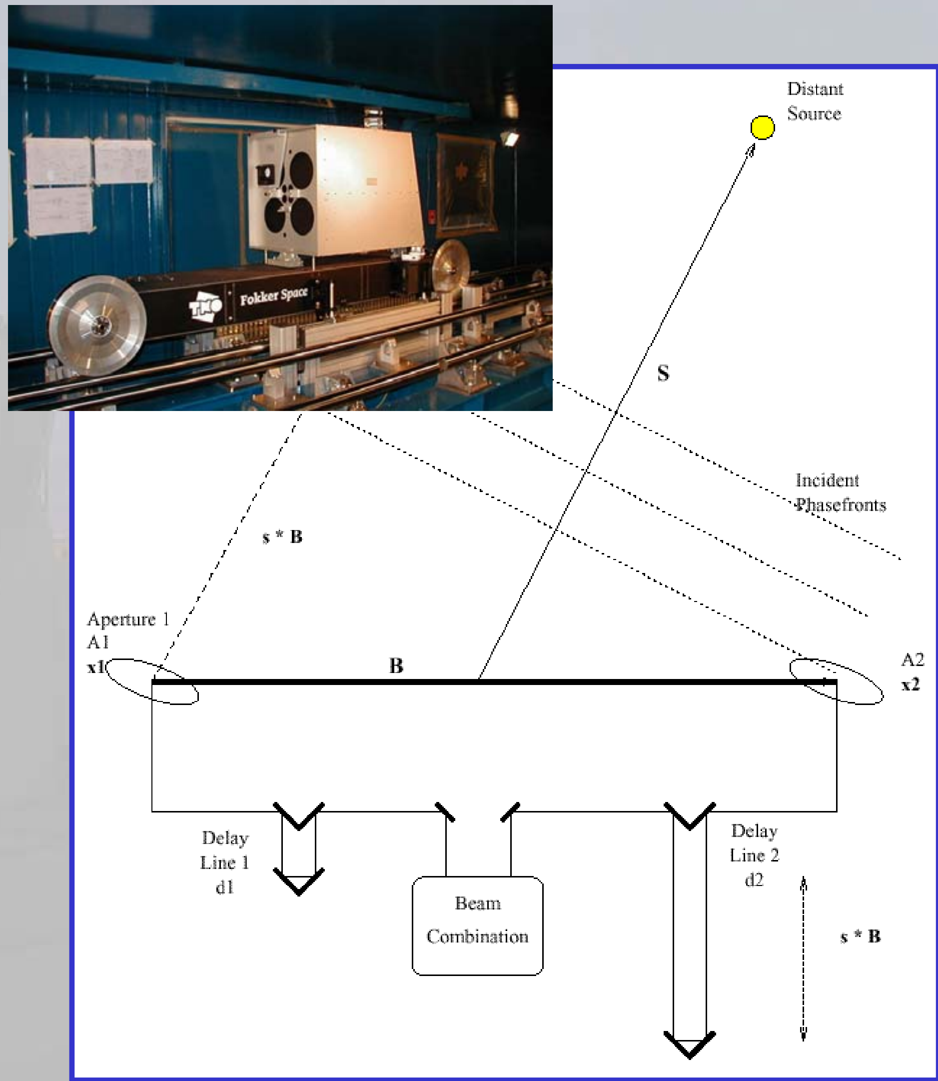
Key ideas regarding the interferometric output (ii)

- The response to a polychromatic source is given by integrating the intensity response for each color.
- This alters the interferometric response and leads to modulation of the fringe contrast:
  - The desired response is only achieved when k [$\hat{s}$.B + d_1 - d_2]) = 0.
  - This is the so called white-light condition.
- This is the primary motivation for matching the optical paths in an interferometer and correcting for the geometric delay.

- The narrower the range of wavelengths detected, the smaller is the effect of this "coherence envelope":
  - This is usually quantified via the coherence length, $\Lambda_{coh} = \lambda^2{}_0/\Delta\lambda$.

How well do delay lines have to perform?

- The OPD added can be as large as the maximum baseline:
  - VLTI has opd_{max}~120m.
- The OPD correction varies roughly as B cos(θ) dθ/dt, with θ the zenith angle.
  - VLTI has v_{max}~ 0.5cm/s (though the carriages can move much faster than this).
- The correction has to be better than l_{coh}~ $\lambda^2/\Delta\lambda$.
  - A typical stability is ≤ 14nm rms over an integration time.



Issues with optical/IR delay lines

- Unless very specialized beam-combining optics are used it is only possible to correct the OPD for a single direction in the sky.
  - This gives rise to a FOV limitation: $\theta_{max} \leq [\lambda/B][\lambda/\Delta\lambda]$.

- For an optical train in air, the OPD is actually different for different wavelengths since the refractive index $n = n(\lambda)$.
  - This longitudinal dispersion implies that different locations of the delay line carts will be required to equalize the OPD at different wavelengths!
  - For a 100m baseline and a source 50^{o} from the zenith this ΔOPD corresponds to ~10μm between 2.0-2.5μm.
  - More precisely, this implies the use of a spectral resolution, *R* > 5 (12) to ensure good fringe contrast (>90%) in the K (J) band.

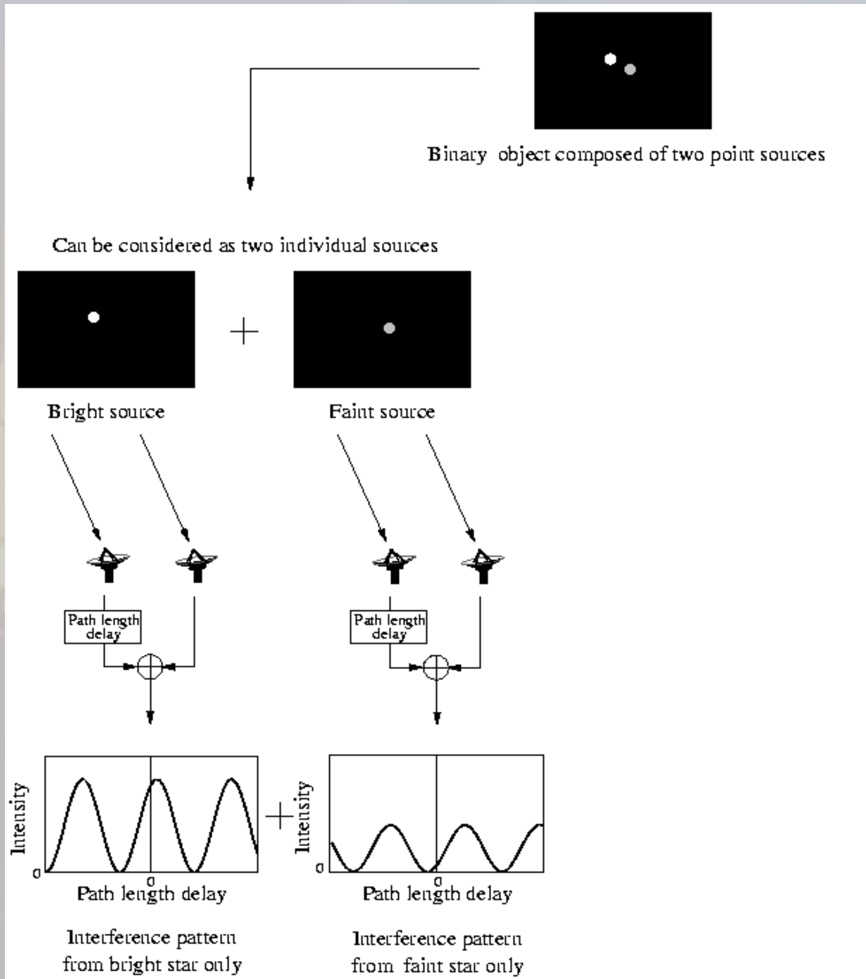
Timeout

Timeout

The fringe modulation encodes information about the Fourier spectrum of the target

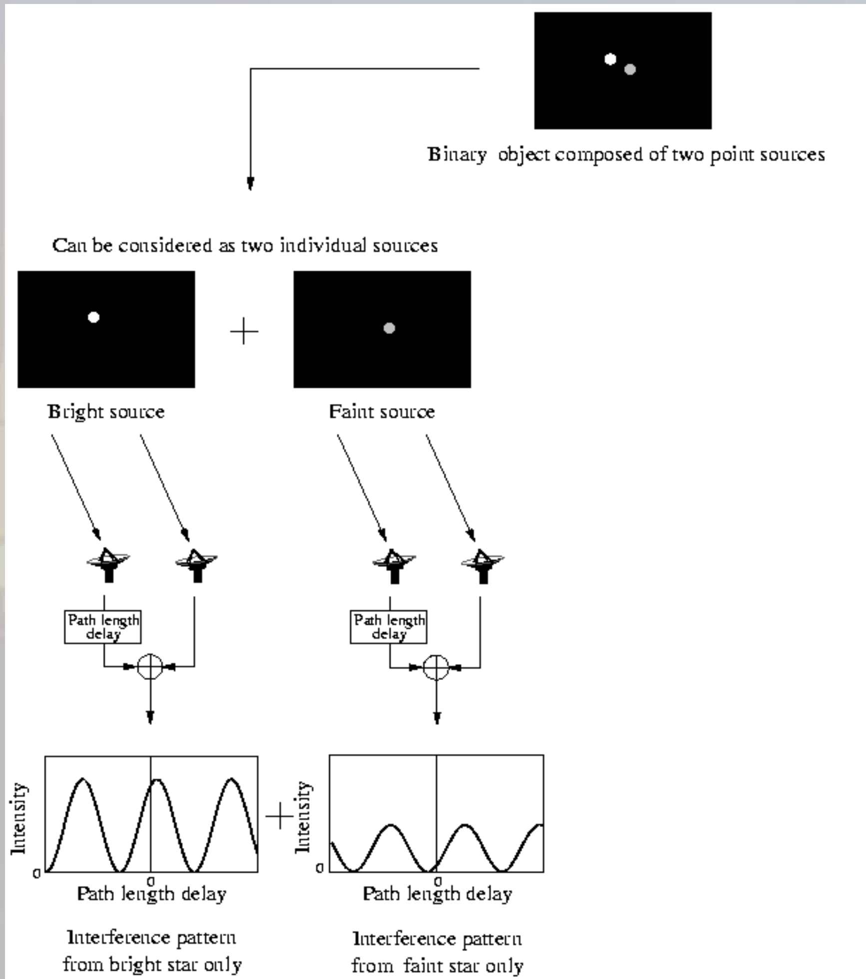
Exactly how does this work?

Heuristic operation of an interferometer



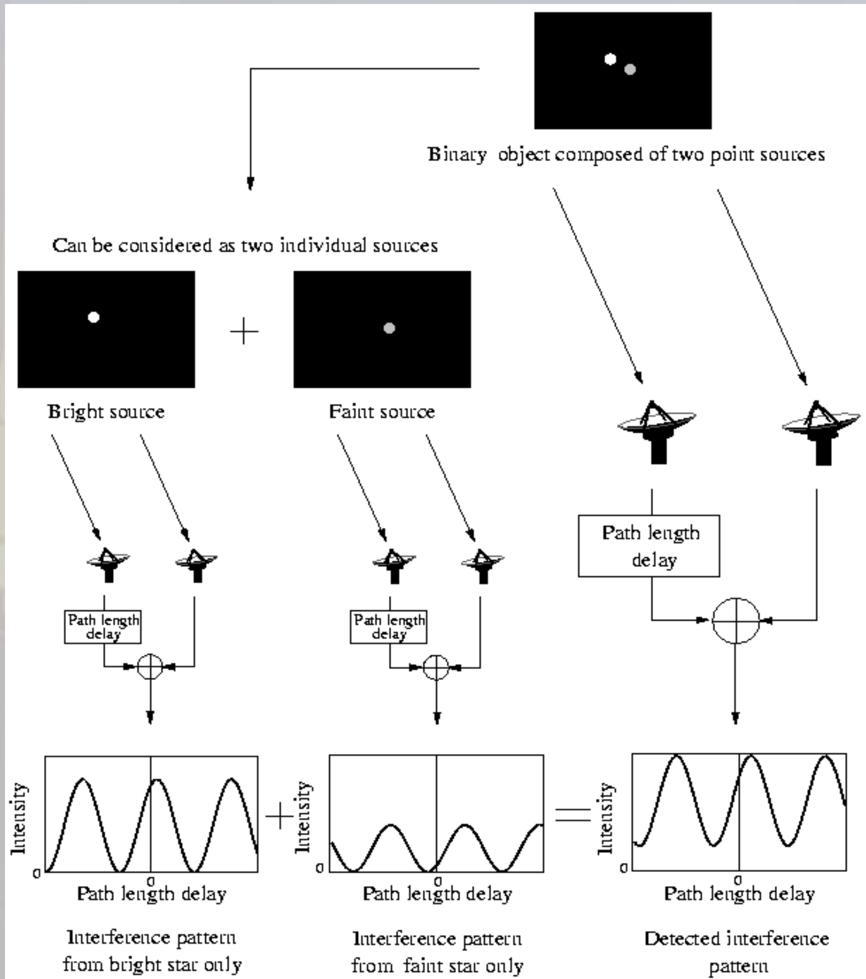
- Each unresolved element of the source produces its own fringe pattern.
- These have unit visibilities and phases that are associated with the location of that source element in the sky:
  - This is the basis for astrometric measurements with interferometers.

Heuristic operation of an interferometer



- The observed fringe pattern from a distributed source is just the intensity superposition of these individual fringe pattern.
- This relies upon the individual elements of the source being "spatially incoherent".

Heuristic operation of an interferometer



- The resulting fringe pattern has a contrast that is reduced with respect to that from each source individually.
- This means we detect less correlated flux.
- The positions of the sources are encoded (in a scrambled manner) in the resulting fringe phase.

Recap & questions

For a given baseline, measurements of the output of an interferometer are made for different values of kD.

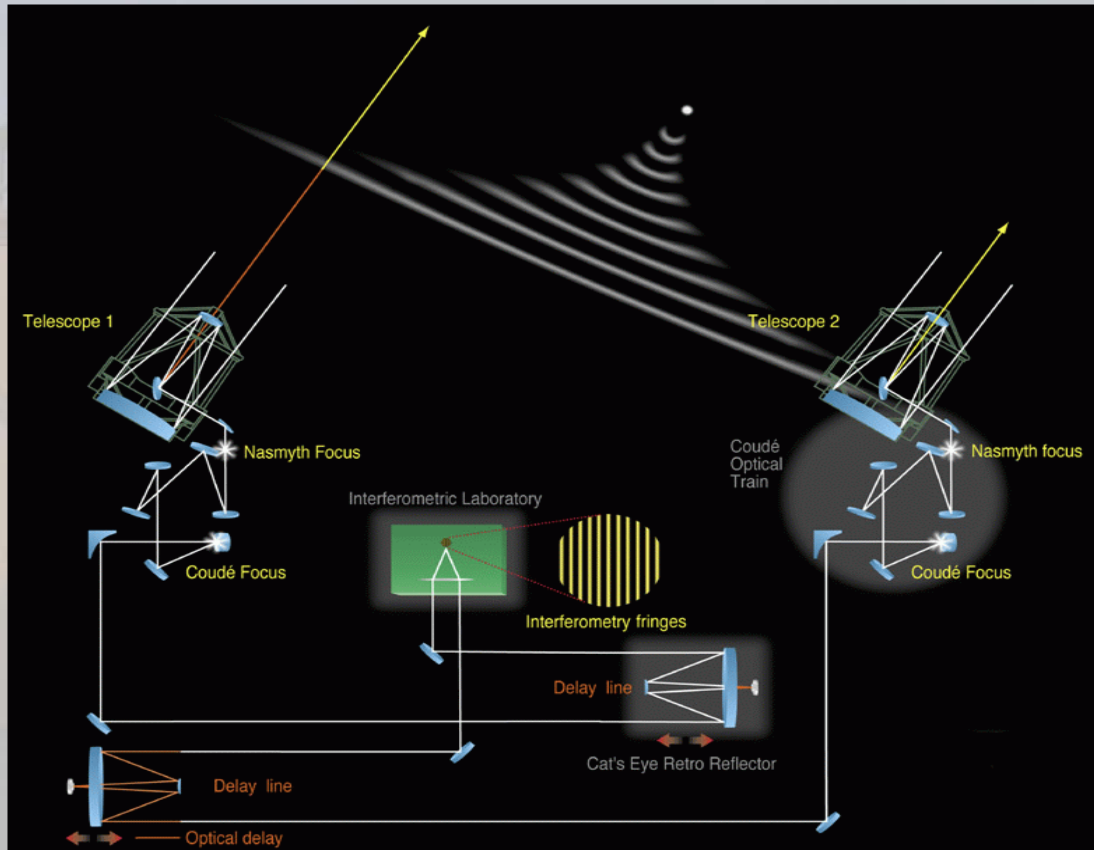
These measurements allow you to recover the Fourier transform $V(u)$, of the target at a single spatial frequency, u, determined by the projected baseline: $u = B_{proj}/\lambda$.

If the projected baseline is large the interferometer probes small scale structures, if short, it probes larger structures.

Even though the Fourier transform is complex and you measure a real signal you recover it fully.

Are we heading in the correct direction?

- Telescopes sample the fields at r_1 and r_2.
- Optical train delivers the radiation to a lab.
- Delay lines assure that we measure when $t_1=t_2$.
- The instruments mix the beams and detect the fringes.



- We measure the fringes.
- We interpret the measurements of the Fourier spectrum of the target.

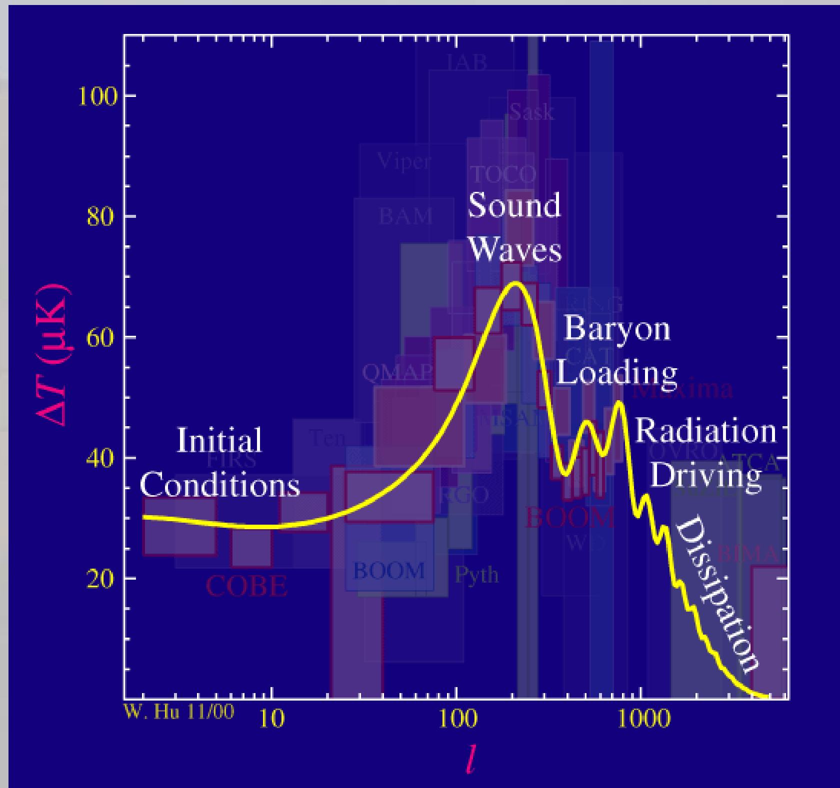
“Science” with interferometers

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

Planning and undertaking interferometric science

When planning – what types of questions are pertinent?

- Which bits of the Fourier spectrum of the target, *V(u,v)*, are the ones you wish to measure?
- How easy will it be to measure these parts?
- How easy will it be for you to interpret these measurements?
- What do we do with the measurements of *V(u, v)* if we wish to make a map of the sky?
- How faint can you go?



Where are the science prizes?

You cannot even start unless you can do FTs in your head

Lets take a break and see if we can learn some "generic" useful stuff about Fourier transforms by looking at some simple 1-d examples

V = Fourier transform of source brightness: amp from fringe contrast, phase from fringe location

v = spatial frequency: derived from projected baseline divided by λ

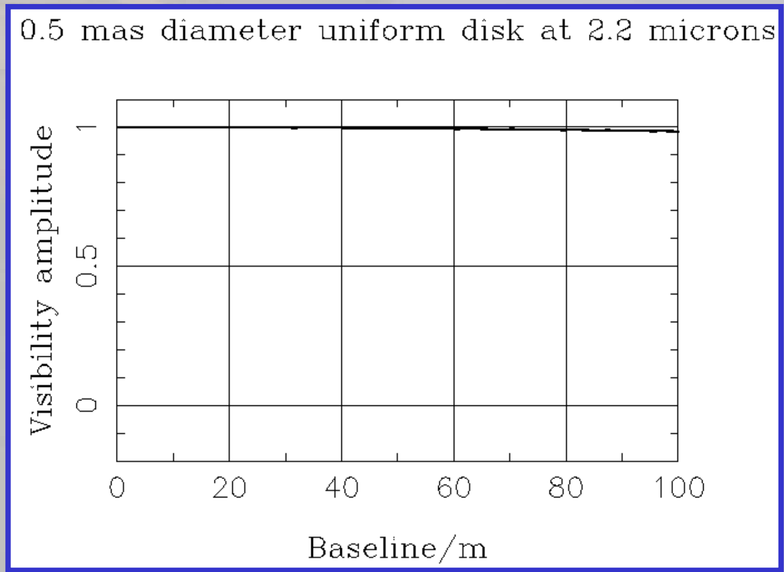
A perfectly unresolved source, not on-axis

$$V(u) \propto \int I(l)\, e^{-i2\pi(ul)}\, dl.$$

Point source of strength A_1 and located at angle l_1 relative to the optical axis.

$$V(u) = \int A_1 \delta(l-l_1)\, e^{-i2\pi(ul)}\, dl \div \text{total flux}$$
$$= e^{-i2\pi(ul_1)}\, .$$

- The visibility amplitude is unity $\forall\, u$.
- The visibility phase varies linearly with u (= B/λ).
- Sources such as this are easy to observe since the interferometer output gives fringes with high contrast.

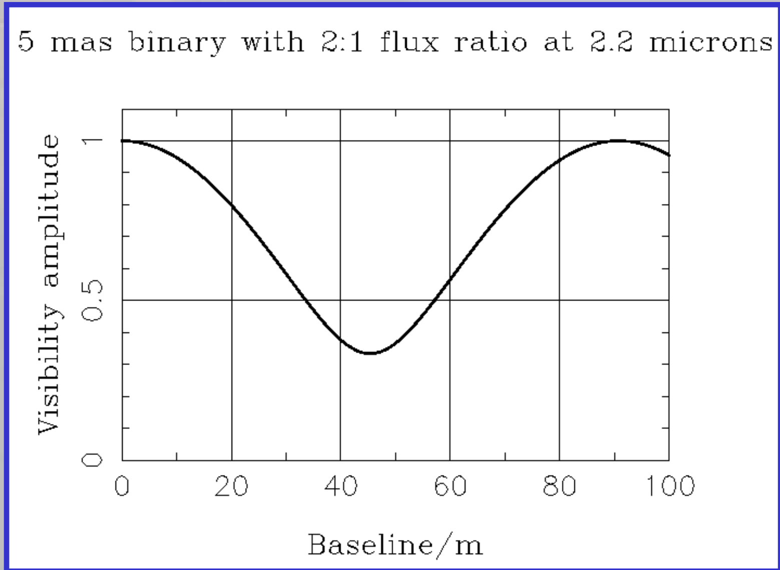


An unequal binary star

A double source comprising point sources of strength A_1 and A_2 located at angles 0 and l_2 relative to the optical axis.

$$V(u) = \int [A_1\delta(l) + A_2\delta(l-l_2)]\, e^{-i2\pi(ul)}\, dl \div \text{total flux}$$
$$= \propto [A_1 + A_2 e^{-i2\pi(ul_2)}] \,.$$

- The visibility amplitude and phase oscillate as functions of *u*.
- To identify this as a binary, baselines from $0 \rightarrow \lambda/l_2$ are required.
- If the ratio of fluxes is large the modulation of the visibility becomes difficult to measure, i.e. the contrast of the interferometric fringes is similar for all baselines.

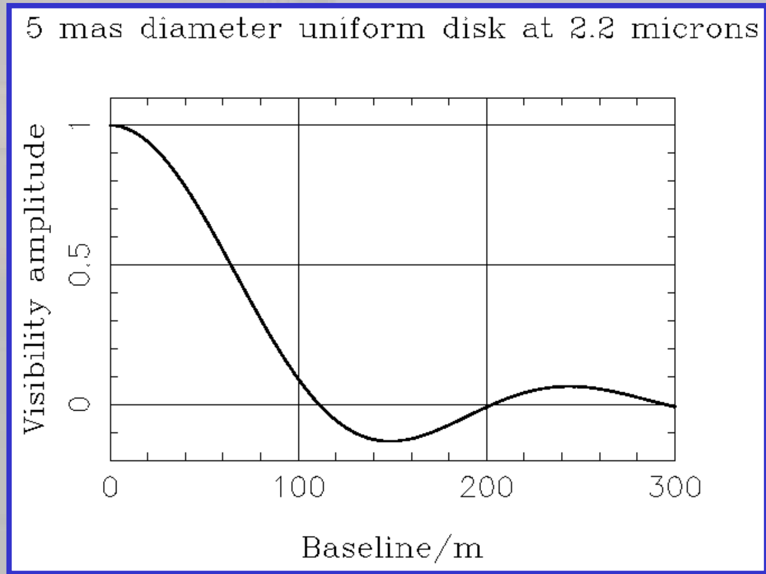


A uniform stellar disc

A uniform on-axis disc source of diameter θ.

$$V(u_r) \propto \int^{\theta/2} \rho\, J_0(2\pi\rho u_r)\, d\rho$$
$$= 2J_1(\pi\theta u_r) \div (\pi\theta u_r)\ .$$

- The visibility amplitude falls rapidly as u_r increases.
- To identify this as a disc requires base-lines from $0 \rightarrow \lambda/\theta$ at least.
- Information on scales smaller than the disc correspond to values of u_r where $V \ll 1$, and is difficult to measure. This is because the interferometer output gives fringes with very low contrast.



How does this help planning observations?

- Compact sources have visibility functions that remain high whatever the baseline, and produce high contrast fringes all the time.
- Resolved sources have visibility functions that fall to low values at long baselines, giving fringes with very low contrast.

$\Rightarrow$ Fringe parameters for fully resolved sources will be difficult to measure.

- To usefully constrain a source, the visibility function must be measured adequately. Measurements on a single, or small number of, baselines may not be enough for unambiguous interpretation.

- Imaging – which necessarily requires information on both small and large scale features in a target – will generally need measurements where the fringe contrast is both high and low.

Recap & questions

When planning an interferometric measurement you <u>must</u> have some idea what the target looks like.

You need to have thought which bits of the Fourier transform of the source are most valuable to measure.

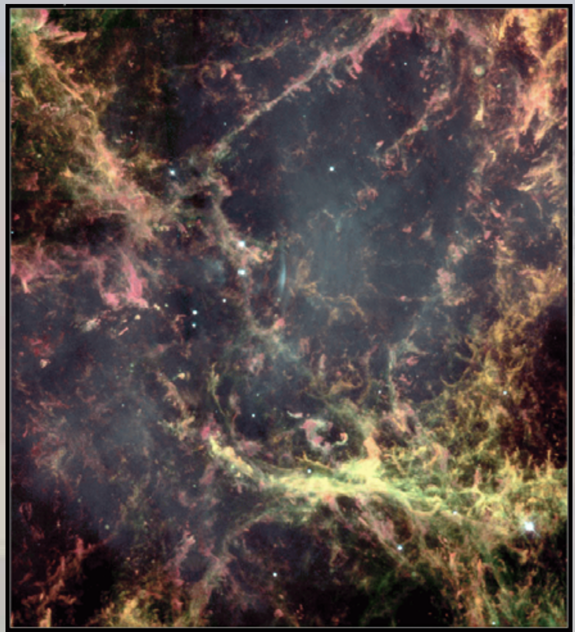
Successful interferometry demands a lot more of the user than conventional imaging.

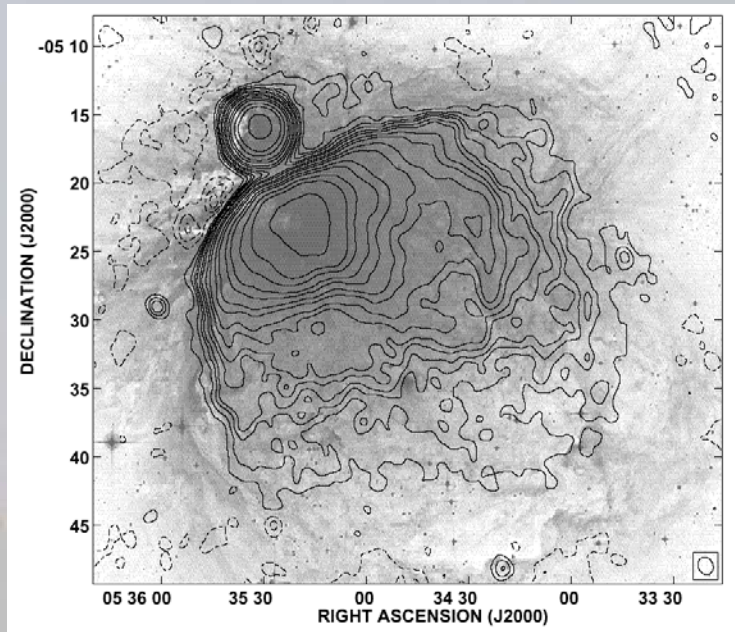
"Science" with interferometers

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

Imaging and sensitivity

Making maps with interferometers – the facts





- Optical HST (left) and 330Mhz VLA (right) images of the Crab Nebula and the Orion nebula. Note the differences in the:
  - Range of spatial scales in each image.
  - The range of intensities in each image.
  - The field of view as measured in resolution elements.

Making maps with interferometers – the "rules"

- The number of visibility data ≥ number of filled pixels in the recovered image:
  - $N_{tel}(N_{tel}-1)/2$ × number of reconfigurations ≥ number of filled pixels.
- The distribution of samples taken of the Fourier plane should be as uniform as possible:
  - To aid deconvolution of the interferometric PSF.
- The range of interferometer baselines, i.e. B_{max}/B_{min}, will govern the range of spatial scales in the map.
- There is no need to sample too finely in the Fourier plane:
  - For a source of maximum extent θ_{max}, sampling very much finer than $\Delta u \sim 1/\theta_{max}$ is unnecessary.

How maps are actually recovered

- The measurements of the visibility function are secured and calibrated.
- These can be represented as a sampled version of the Fourier transform of the sky brightness:

$$V_{meas}(u, v) = V_{true}(u, v) \times S(u, v).$$

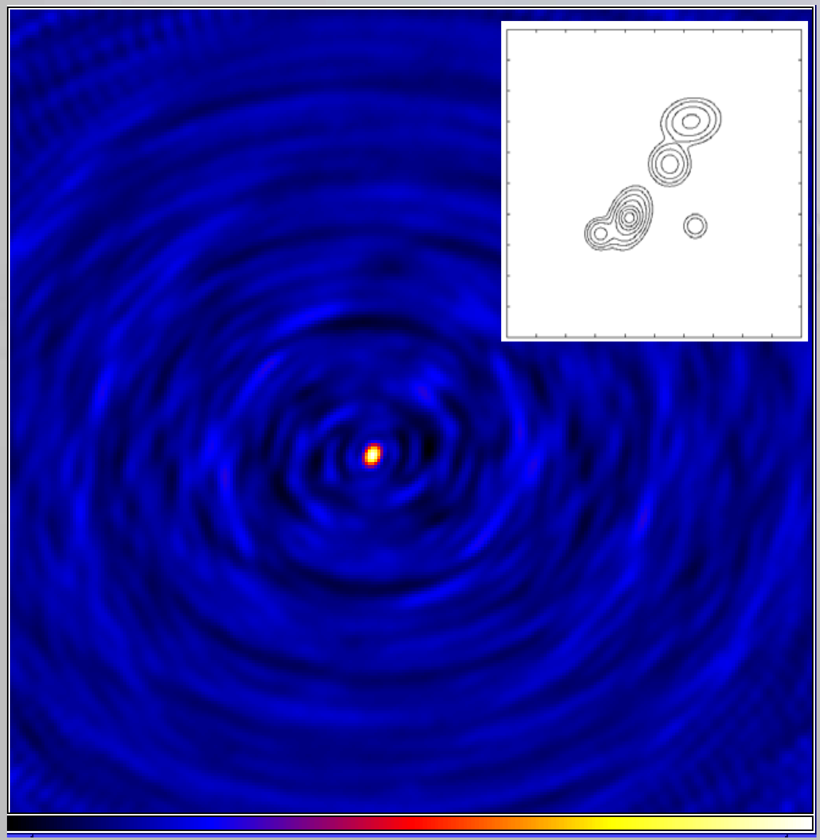
- These data are inverse Fourier transformed to give a representation of the sky, similar to that of a normal telescope, albeit with a strange PSF:

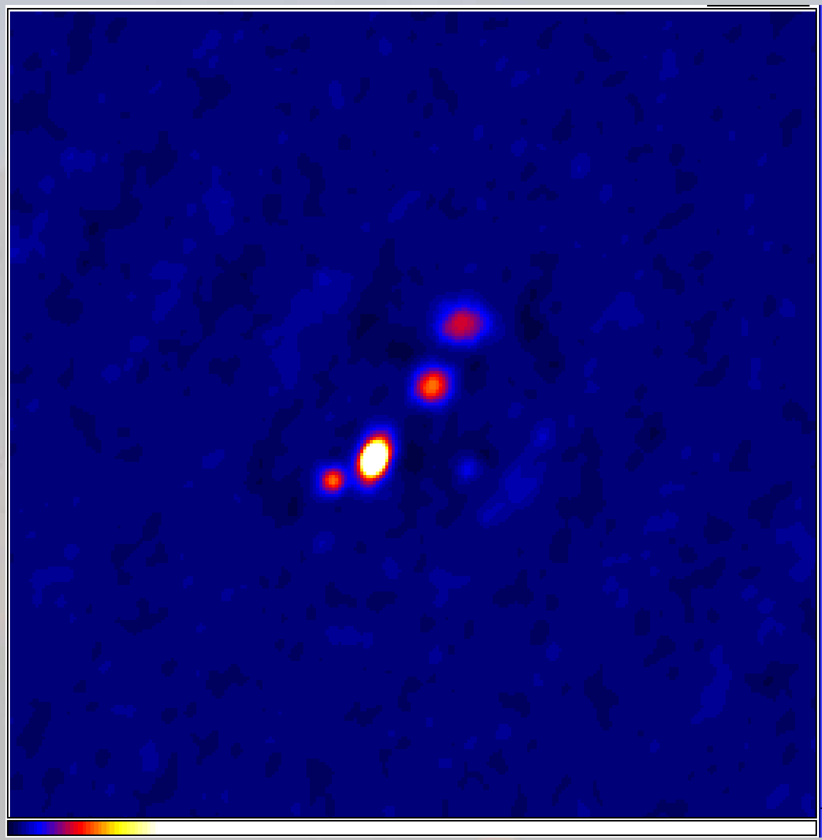
$$\iint S(u, v)\, V_{true}(u, v)\, e^{+i2\pi(ul + vm)}\, du\, dv = I_{norm}(l, m) * B_{dirty}(l, m),$$

where $B_{dirty}(l,m)$ is the Fourier transform of the sampling distribution, or the so-called dirty-beam, and the image is usually referred to as the “dirty” map.

So you get a map that has a “crazy” point spread function

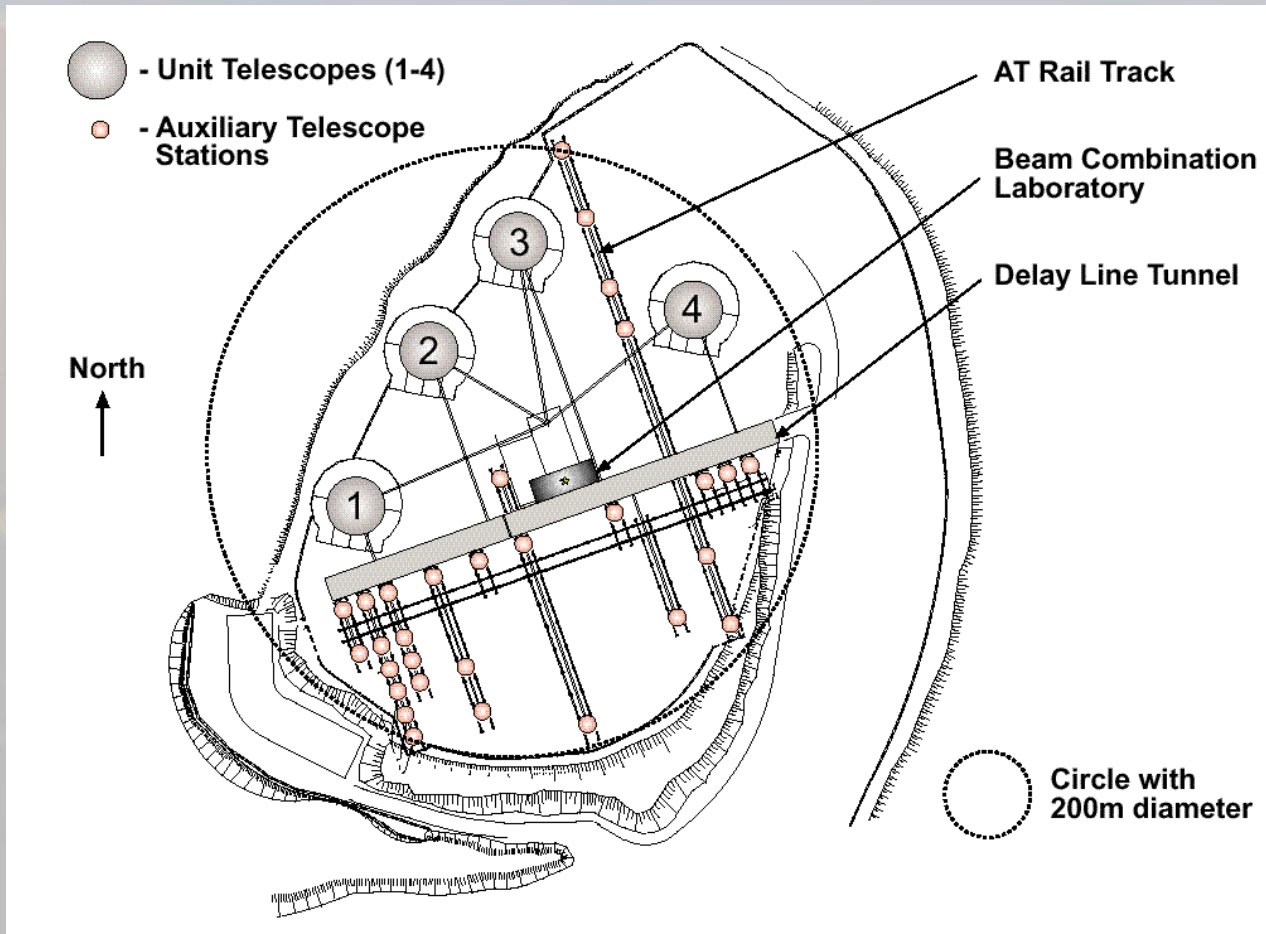
The effect of the sampling distribution



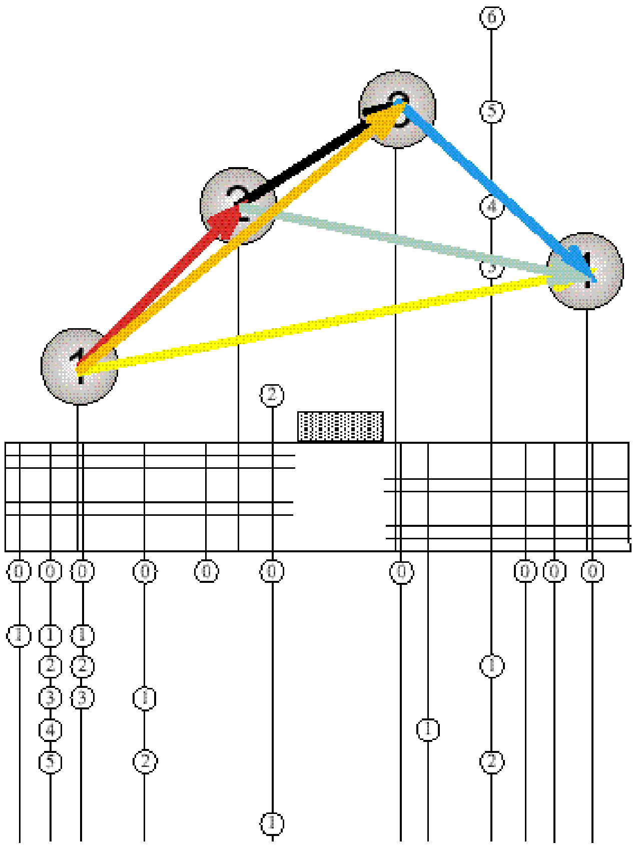


- Interferometric PSF's can often be horrible. Correcting an interferometric map for the Fourier plane sampling function is known as deconvolution and is broadly speaking straightforward.

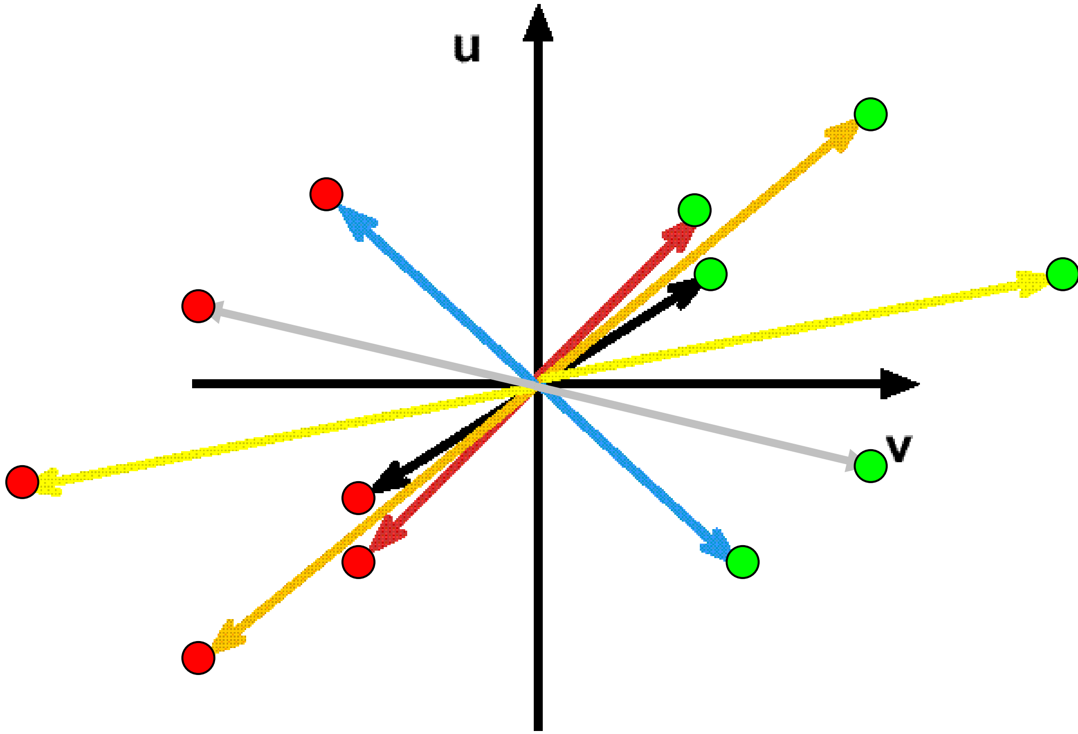
What sort of Fourier plane sampling does the VLTI offer?



The vectors between telescopes are what is important



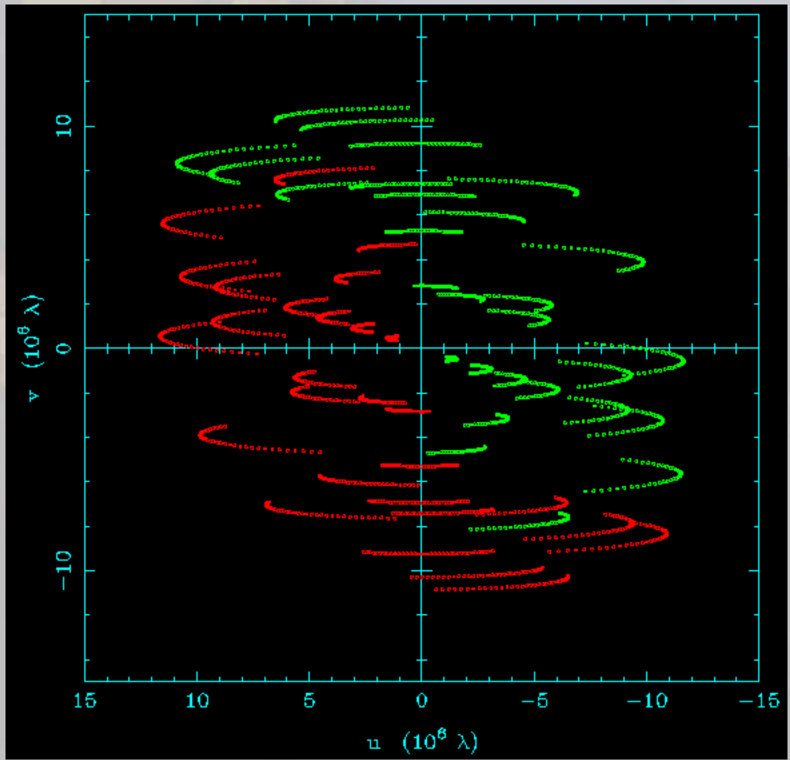
This is the uv-plane:



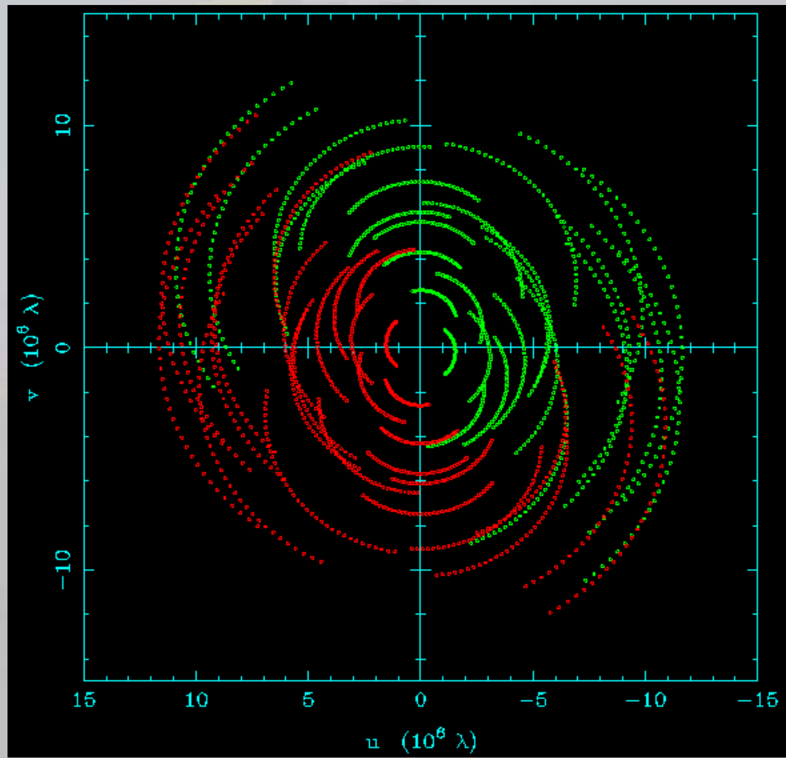
Note: This is the uv-plane for an object at zenith.
In general, the projected baselines have to be used.

Earth rotation fills in the Fourier plane

Dec -15

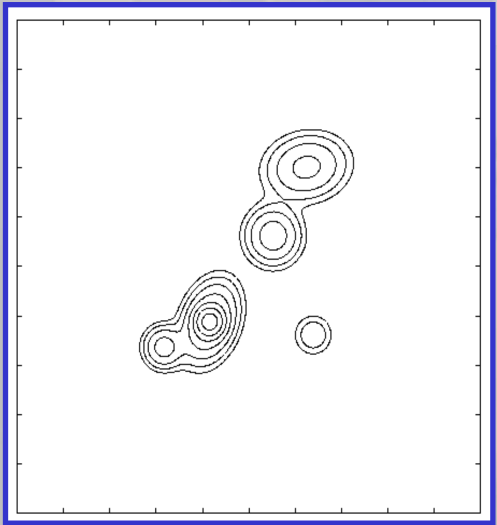


Dec -65

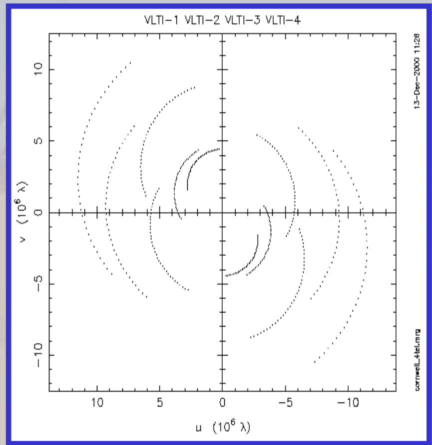


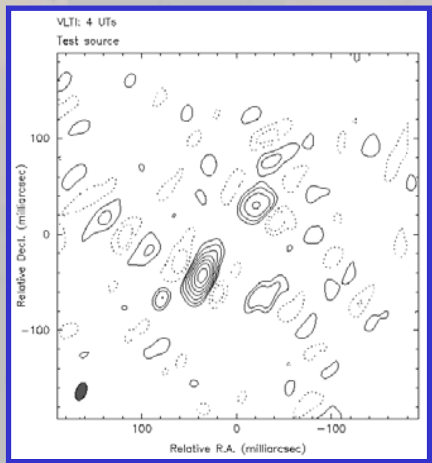
The uv-plane coverage has a big impact on imaging

Model

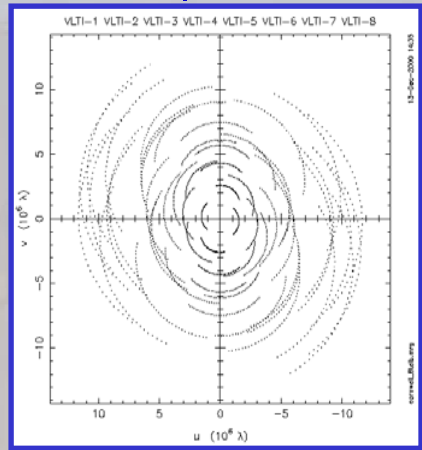


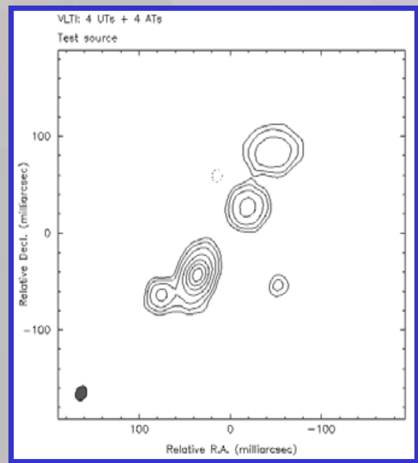
4 telescopes, 6 hours





8 telescopes, 6 hours

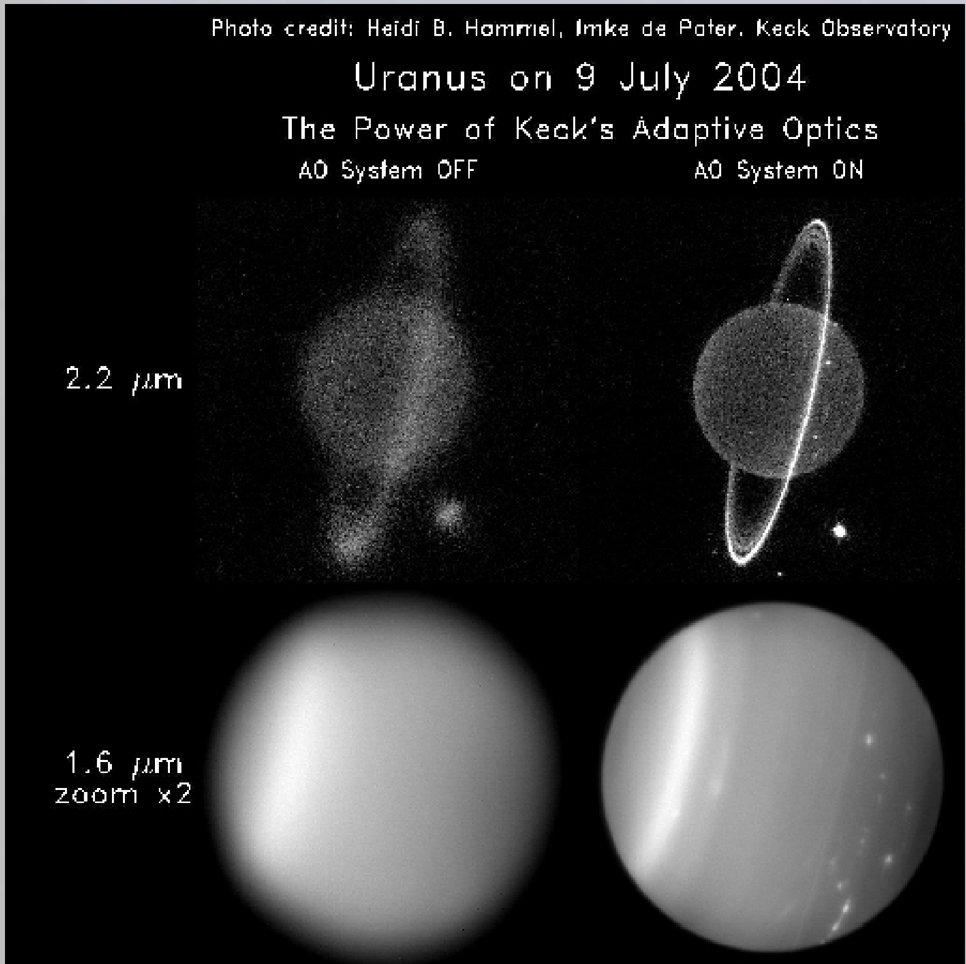




Other useful interferometric rules of thumb

- The FOV will depend upon:
  - The field of view of the individual collectors. This is often referred to as the primary beam.
  - The FOV seen by the detectors. This is limited by vignetting along the optical train.
  - The spectral resolution. The interference condition OPD < $\lambda 2/\Delta\lambda$ must be satisfied for all field angles. Generally $\Rightarrow$ FOV $\leq [\lambda/B][\lambda/\Delta\lambda]$.
- Dynamic range:
  - The ratio of maximum intensity to the weakest believable intensity in the image.
  - $> 10^5$:1 is achievable in the very best radio images, but of order several × 100:1 is more usual.
  - $DR \sim [S/N]_{per\text{-}datum} \times [N_{data}]^{1/2}$
- Fidelity:
  - Difficult to quantify, but clearly dependent on the completeness of the Fourier plane sampling.

What does “sensitivity” mean for interferometry?



A: Whether a guide star is present.

B: Whether a bright enough guide star is present.

C: Whether a bright enough guide star is close enough.

D: How long the AO system stays locked for.

E: How large my telescope is.

F: How sensitive my detector is.

G: How long an exposure time I can sustain.

H: All of the above.

I: None of the above.

So, how do we assess interferometric sensitivity?

- The “source” has to be bright enough to:
  - Allow stabilisation of the interferometer against any atmospheric fluctuations.
  - Allow a reasonable signal-to-noise for the fringe parameters to be build up over some total convenient integration time. This will be measured in minutes.
- Once this achieved, the faintest features one will be able to interpret reliably will be governed by S/N ratio and number of visibility data measured.

- In most cases the sensitivity of an interferometer will scale like some power of the measured fringe contrast × another power of the number of photons detected while the fringe is being measured.

This highlights fringe contrast and throughput as both being critical.
This also highlights the difficulty of measuring resolved targets where V is low.

For those who are interested, the fringe signal-to-noise formula bears some consideration

$$S/N \propto [VN]^2 \,/\, [(N+N_{dark})^2 + 2(N+N_{dark})N^2V^2 + 2(N_{pix})^2(\sigma_{read})^4]^{1/2}$$

with V = apparent visibility, N = detected photons, N_{dark} = dark current, N_{pix} = number of pixels, σ_{read} = readout noise/pixel.

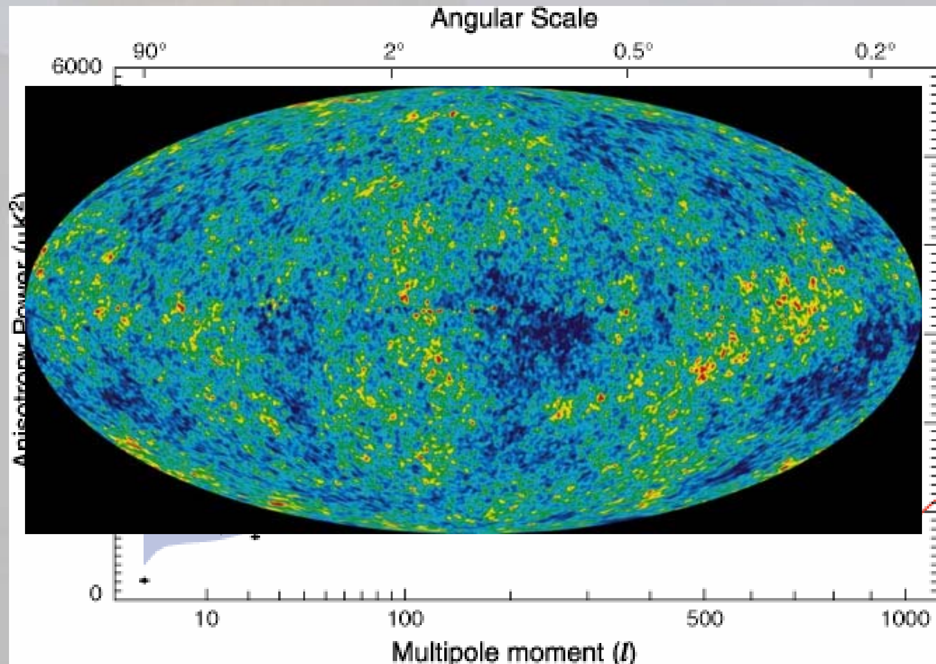
- It depends on the signal coming from the target.
- It depends on the amount of read noise on the detector.
- It depends on the amount of dark/thermal background.
- It depends on the source visibility, i.e. its structure.

This last point is not too unusual really.

It just means that what matters is not the integrated brightness of the target but the surface brightness, i.e. the brightness per unit solid angle on the sky.

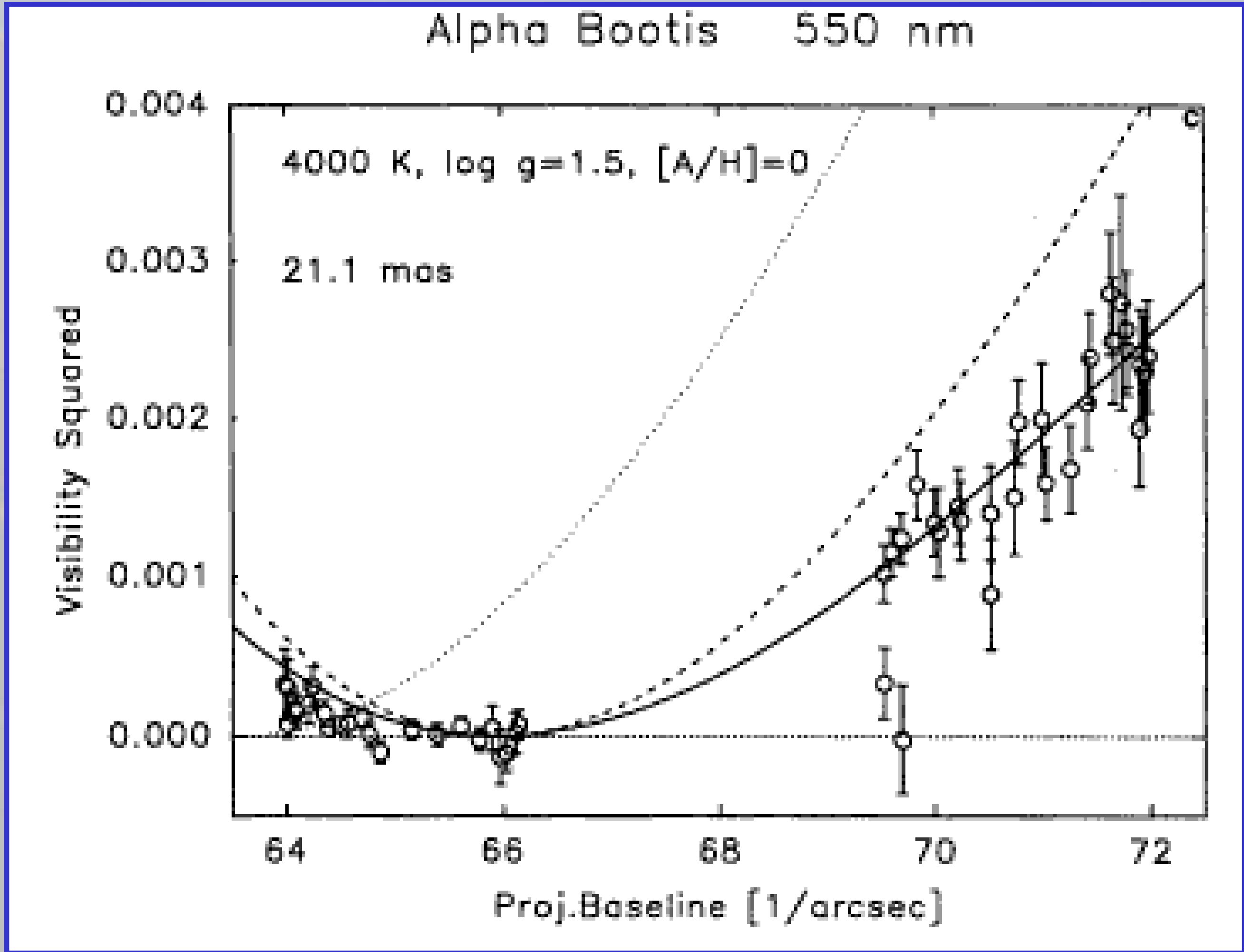
Closing thoughts

- Once we have measured the visibility function of the source, we have to interpret these data. This can take many forms:
  - Small amount of Fourier data:
    - Model-fitting.
  - Moderate amount of Fourier data:
    - Model-fitting.
    - Rudimentary imaging.
  - Large amount of Fourier data:
    - Model-fitting.
    - Model-independent imaging.

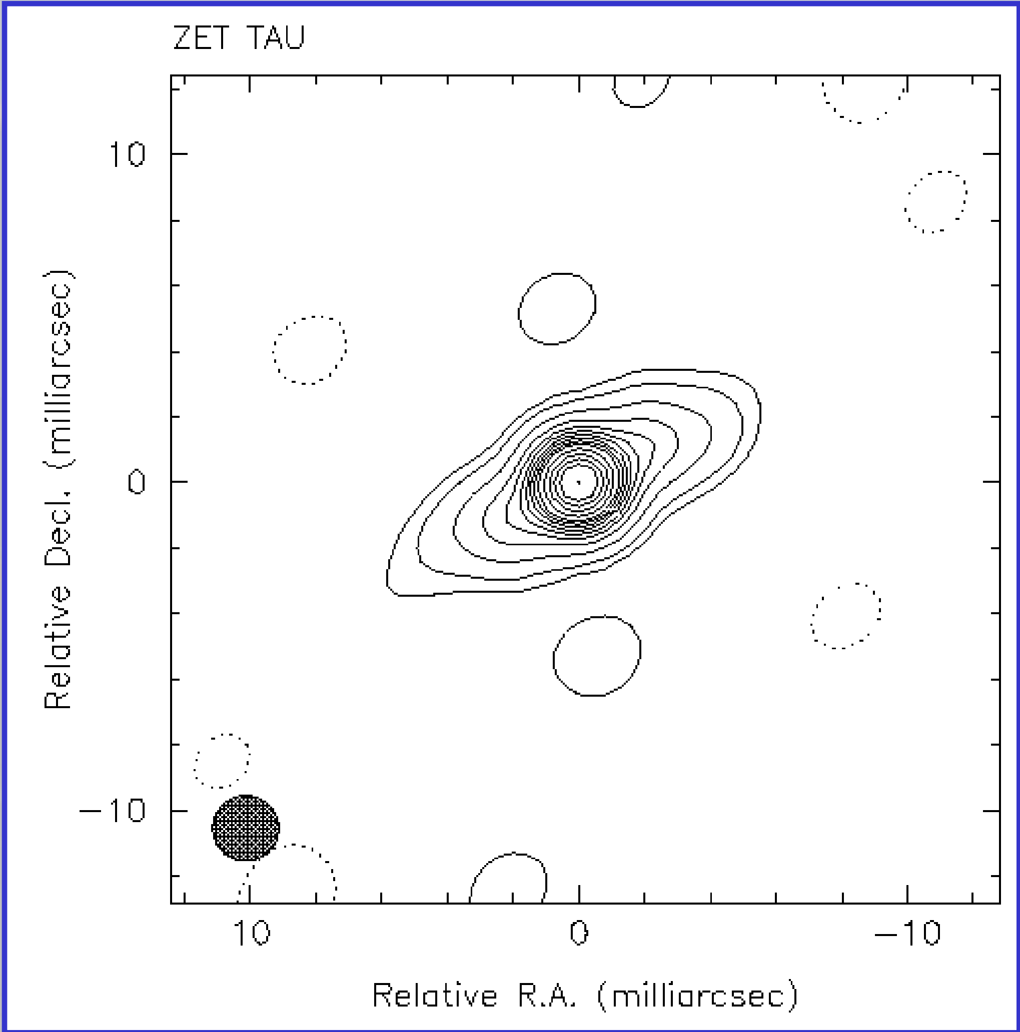


- Don't forget that making an image is not a requirement for doing good science:

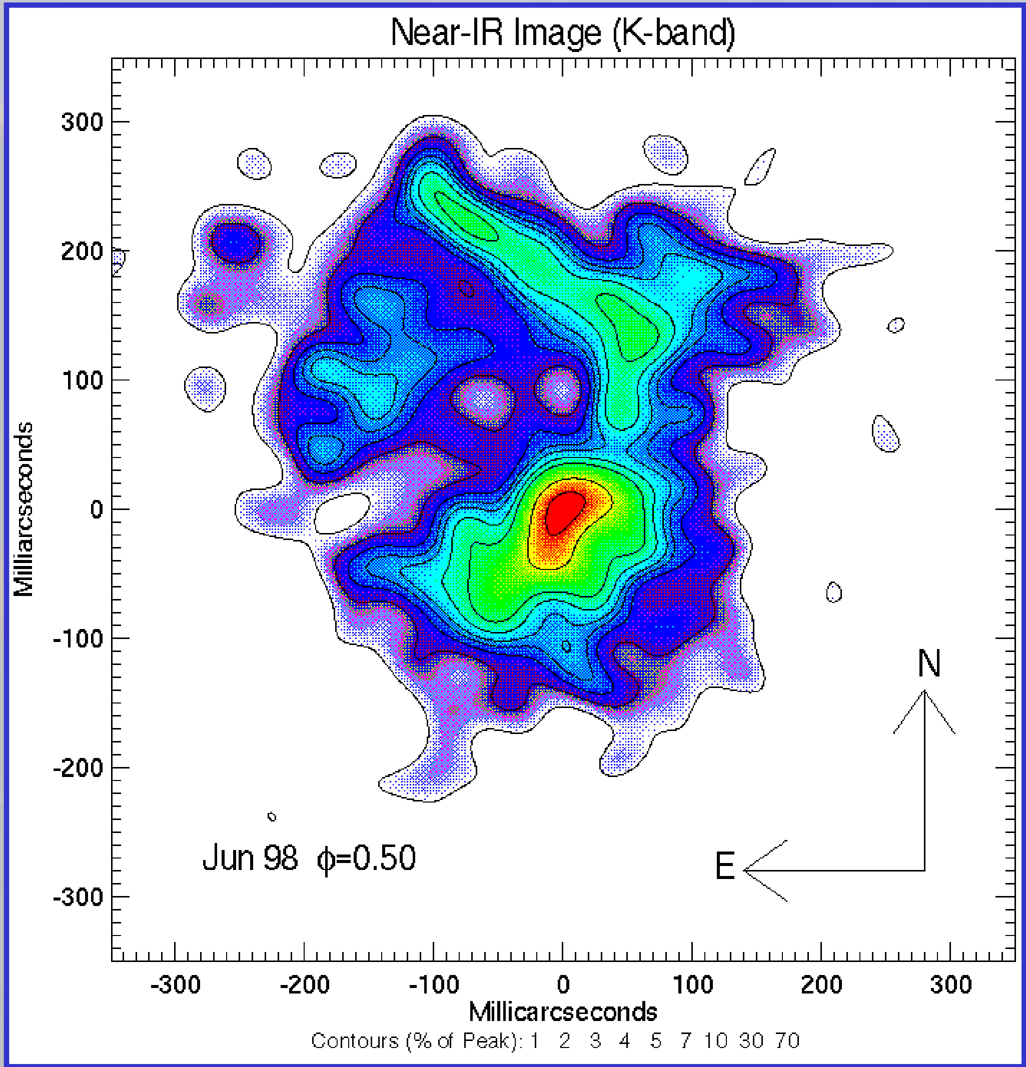
Interferometric science – 2 telescopes



Interferometric science – 5 telescopes



Interferometric science – 21 telescopes



Key lessons to take away

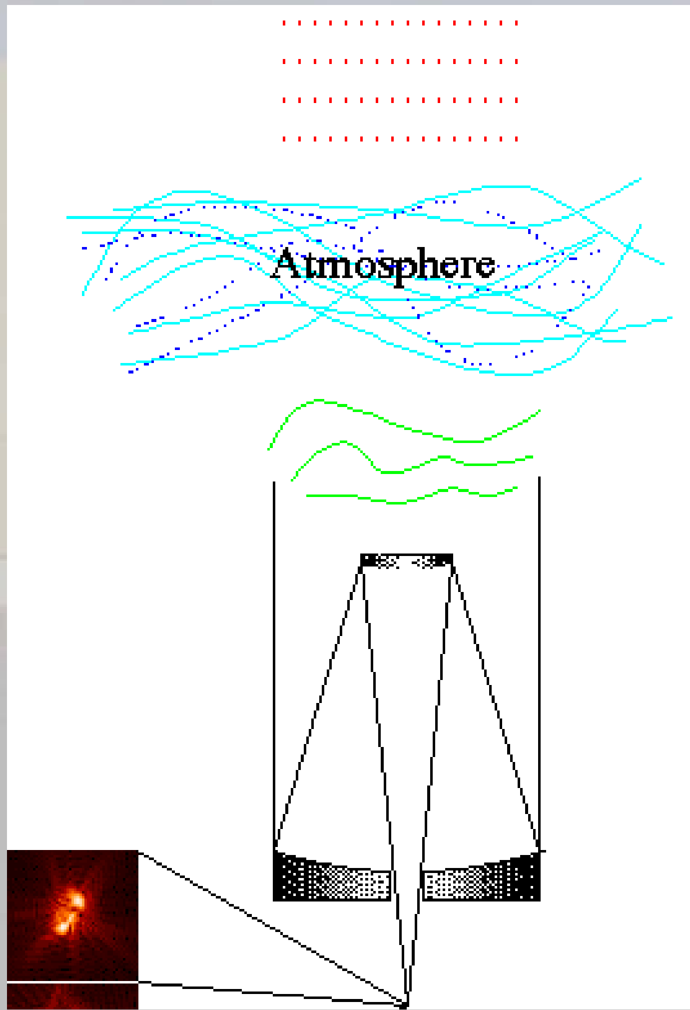
- Alternative methods for describing images:
  - Fourier decomposition, spatial frequencies, physical baselines.
- Interferometric measurements:
  - Interferometer make fringes.
  - The fringe amplitude and phase are what is important.
  - More precisely, these measure the FT of the sky brightness distribution.
  - A measurement with a given interferometer baseline measures a single Fourier component (usually the square of V and its phase are measured).
- Science with interferometers:
  - Multiple baselines are obligatory for studying a source reliably.
  - Resolved targets produce fringes with low contrast – these are difficult to measure well.
  - Once a number of visibilities have been measured, reliable interpretation can take multiple forms – making an image is only required if the source is complex.

The atmosphere

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

How is what we have learnt impacted by the atmosphere?

How can we understand what the atmosphere does?



We visualise the atmosphere altering the phase (but not amplitude) of the incoming wavefronts.

We know that this impacts the instantaneous image.

But we need to understand how this impacts the fringe contrast and phase, which are what we are actually interested in measuring.

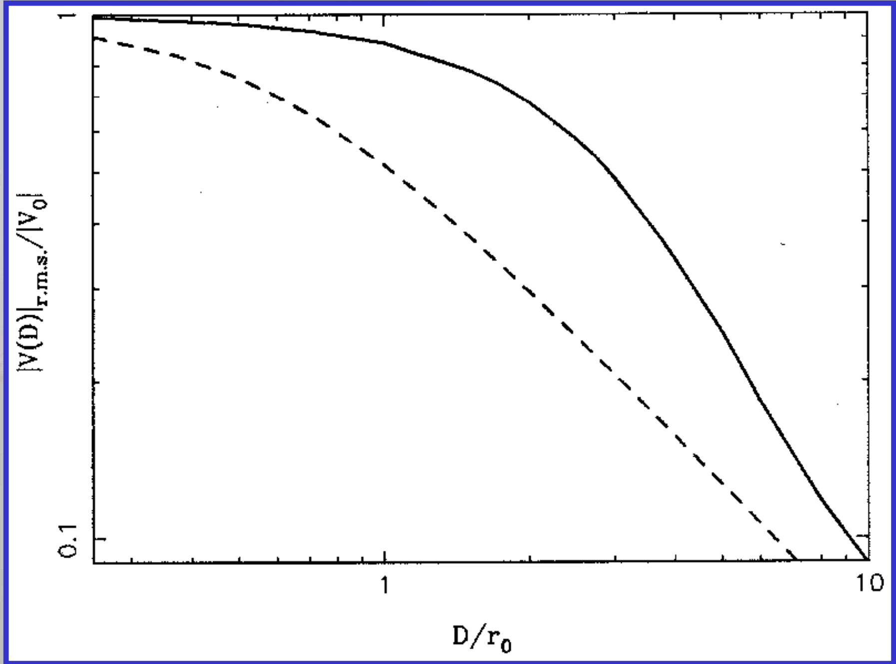
We characterize spatial fluctuations in the wavefront using Fried's parameter: r_0

- The circular aperture size over which the mean square wavefront error is approximately 1 radian^2.
- This scales as $\lambda^{6/5}$.
- Tel. Diameters > or < r_0 delimit different regimes of instantaneous image structure:
  - $D < r_0 \Rightarrow$ quasi-diffraction limited images with image motion.
  - $D > r_0 \Rightarrow$ high contrast speckled (distorted) images.
- Median r_0 value at Paranal is 15cm at 0.5µm.

If you use a telescope with $D \sim r_0$ in diameter, spatial fluctuations are not very important.

Spatial corrugations affect the instantaneous fringe contrast

- Reduces the rms visibility (-----) amplitude as D/r_0 increases.
- Leads to increased fluctuations in V.
- Both imply a loss in sensitivity.
- Calibration becomes less reliable.



- Moderate improvement is possible with tip-tilt correction (——).
- Higher order corrections improve things but more slowly.

The mean fringe contrast now is a function of both the source structure and the atmospheric conditions

How can we mitigate against these perturbations?

- In principle, there are two approaches to deal with spatial fluctuations for telescopes of finite size:

  - Use an adaptive optics system correcting higher order Zernike modes:
    - Can use either the source or an off-axis reference star to sense atmosphere.
    - But need to worry about how bright and how far off axis is sensible.

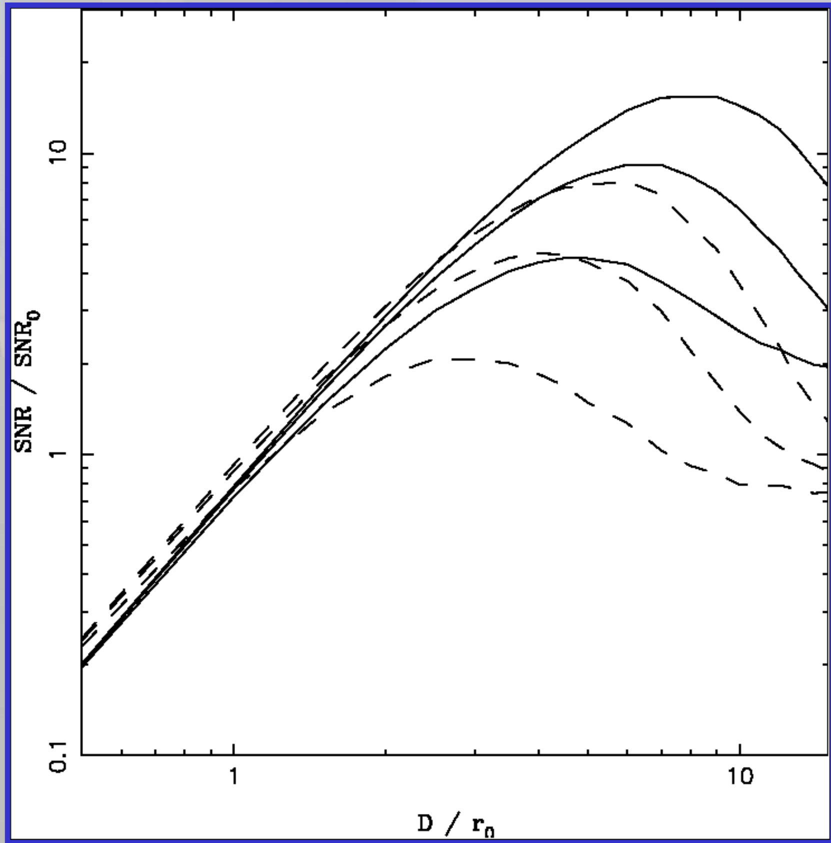
  - Instead, spatially filter the light arriving from the collectors:
    - Pass the light through either a monomode optical fibre or a pinhole.
    - This trades off a fluctuating visibility for a variable throughput.

Many interferometers use both strategies

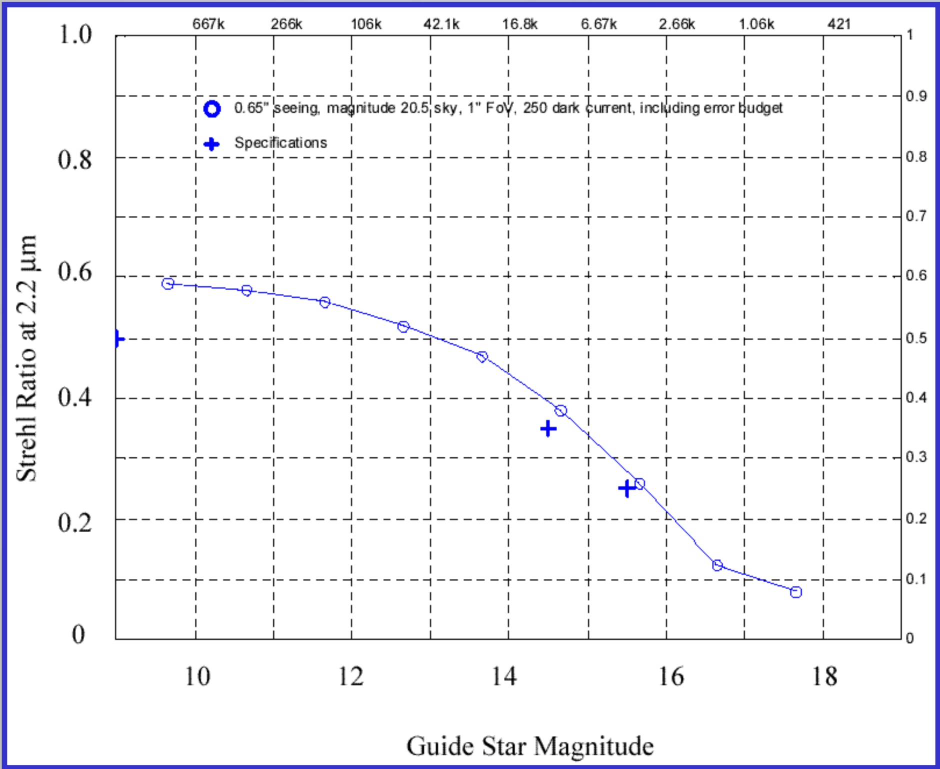
The graph shows how the S/N for fringe contrast measurement scales with telescope size

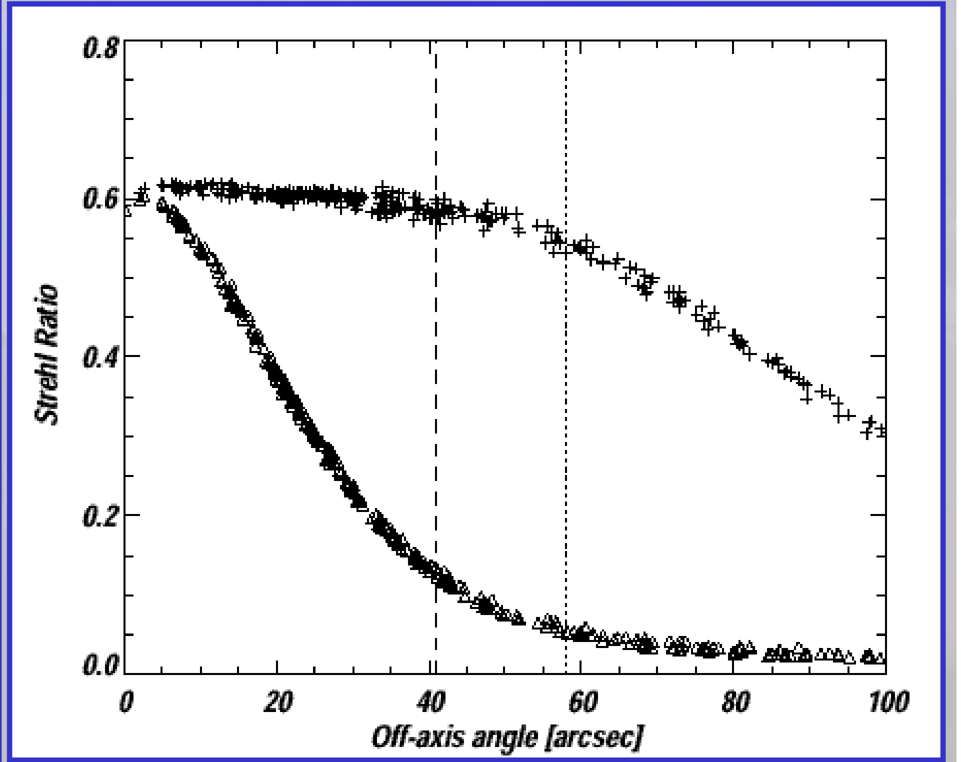
- Solid = with spatial filter
- Dashed = without spatial filter.
- Different curves are for 2, 5 and 9 Zernike mode correction.

- Implications are:
  - For perfect wavefronts $S/N \propto D$.
  - Spatial filtering always helps.
  - Can work with large D/r_0 (e.g. ≤ 10).
  - If D/r_0 is too large for the AO system, make D smaller.



But AO does have its limitations





- Influence of guide-star magnitude. This is for MACAO at the VLTI.
- Influence of off-axis angle. This is for a generic 8m telescope at M. Kea.

NGS adaptive optics systems basically offer modest improvements in sky coverage, and allow photons to be collected faster for bright sources.

Recap & questions

Spatial fluctuations in the atmosphere lead to a reduction in the mean fringe contrast from the intrinsic source-dependent value.

We have to calibrate this effect by looking at source whose visibility we know a priori – usually an unresolved target.

We have to rely upon the instrumental and atmospheric characteristics being identical (in a statistical sense) for the two observations.

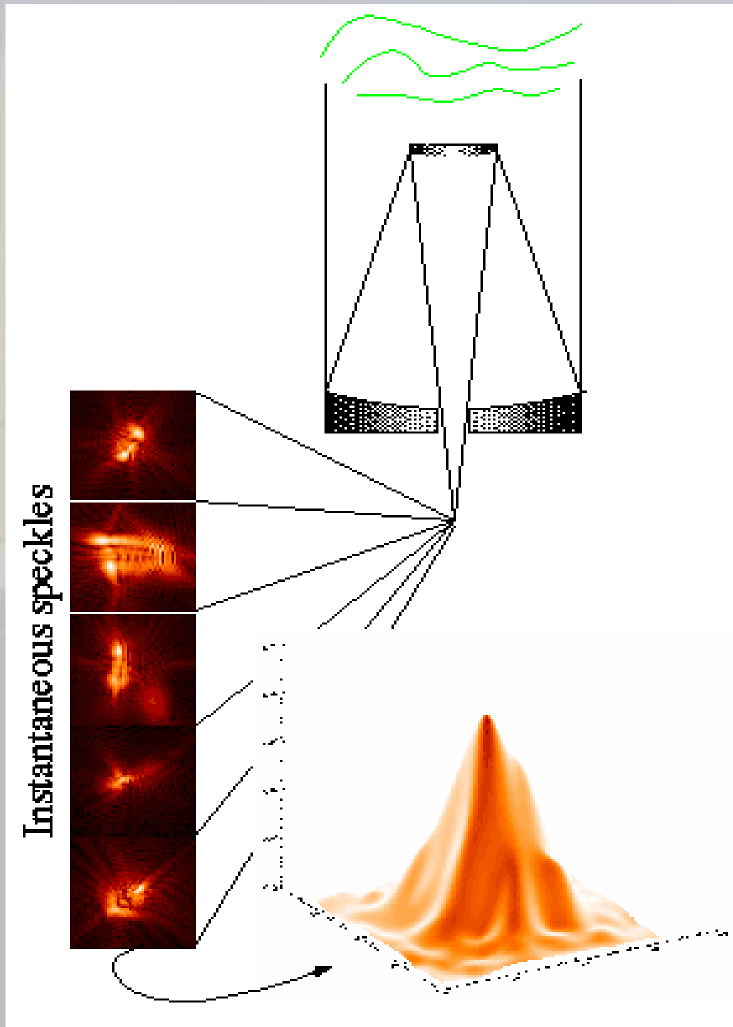
It is possible to moderate the effects of the atmosphere using AO and spatial filtering.

The atmosphere

- Image formation with conventional telescopes
  - The diffraction limit
  - Incoherent imaging equation
  - Fourier decomposition
- Interferometric measurements
  - Fringe parameters
- Science with interferometers
  - Interferometric imaging
  - Useful rules of thumb
  - Interferometric sensitivity
- The atmosphere
  - Spatial fluctuations
  - Temporal fluctuations

Now we need to look what temporal variations of the atmosphere do

The atmosphere – temporal effects



We again visualise the atmosphere altering the phase (but not amplitude) of the incoming wavefronts.

We know that these perturbations change with time.

Again, we need to understand how this impacts the fringe contrast and phase.

We characterize temporal wavefront fluctuations with the coherence time: t_0

- This is the time over which the wavefront phase changes by ~1 radian.
- Related to spatial scale of turbulence and windspeed:
  - If we assume that Taylor's "frozen turbulence" hypothesis holds, i.e. that the timescale for evolution of the wavefronts is long compared with the time to blow past your telescope.
  - Gets us a characteristic timescale $t_0 = 0.314\, r_0/v$, with v a nominal wind velocity. Scales as $\lambda^{6/5}$.
- Typical values can range between 3-20ms at 0.5μm.
  - Data from Paranal show median value of ~20ms at 2.2μm.

We have to make measurements of the fringe parameters in a time comparable to (or shorter) than t_0.

How do these temporal fluctuations affect things?

- Temporal fluctuations provide a fundamental limit to the sensitivity of optical arrays.

– Short-timescale fluctuations blur fringes:
  - Need to make measurements on timescales shorter than $\sim t_0$.

– Long-timescale fluctuations move the fringe envelope out of measurable region.
  - Fringe envelope is few microns
  - Path fluctuations tens of microns.
  - Requires dynamic tracking of piston errors.

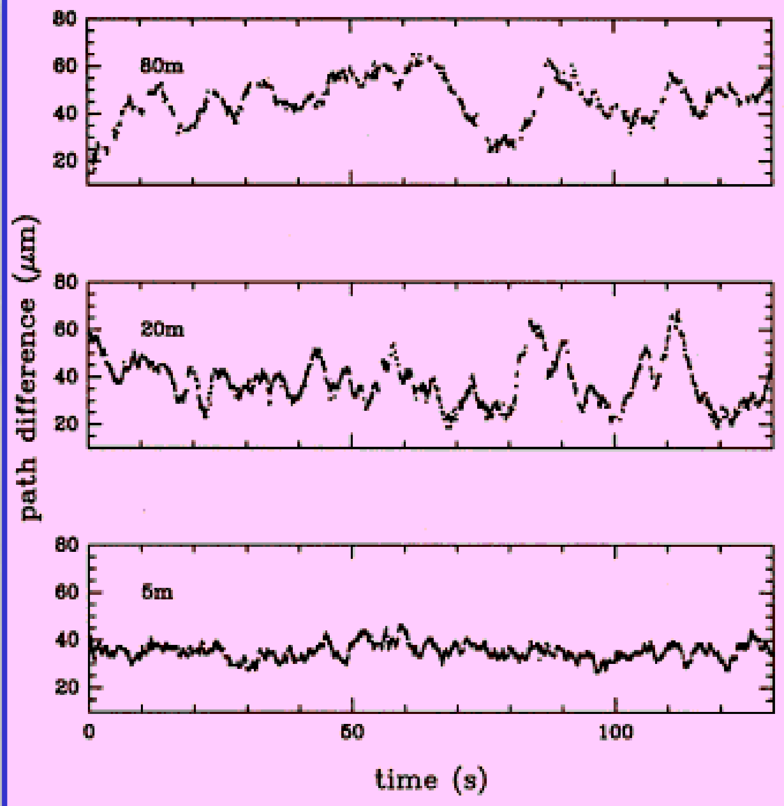


Figure 1. Path offset from the equal optical path position as a function of time for three baselines.

Perturbations to the amplitude and phase of V

- As well as forcing interferometric measurements to be made on short timescales, the other problem introduced by temporal wavefront fluctuations is that they alter the measured phase of the visibility.
- Note also, that if the “exposure time” is too long, the atmosphere reduces the amplitude of the measured visibility too.

Simple Fourier inversion of the coherence function becomes impossible.

- How do we get around the problem of “altered” phases?
  - Dynamically track the atmospheric (and instrumental) OPD excursions at the sub-wavelength level
    - Phase is then a useful quantity.
  - Measure something useful that is independent of the fluctuations.
    - Differential phase.
    - Closure phase.

What you need to know about dynamic fringe tracking

- We can identify several possible fringe-tracking systems:
  1. Those that ensure we are close to the coherence envelope.
  2. Those that ensure we remain within the coherence envelope.
  3. Those that lock onto the white-light fringe motion with high precision.

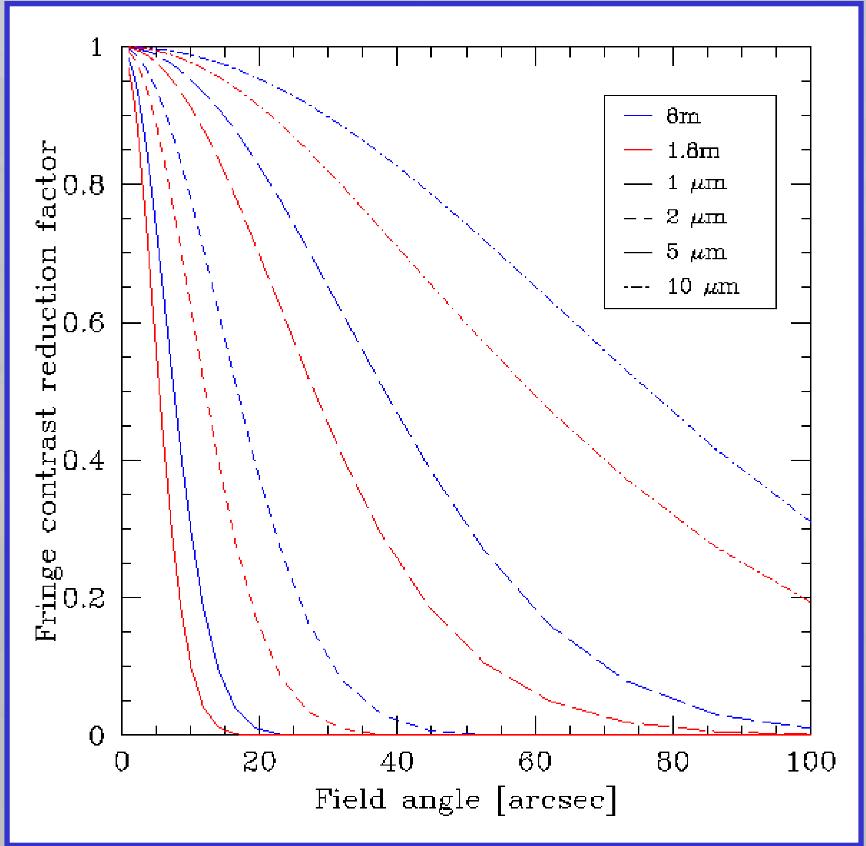
- [1] and [2] still need to be combined with short exposure times for any data taking and the measurement of a "good" observable

- Only [3] allows for direct Fourier inversion of the measured visibility function:
  - You can use the target itself or a very close reference star to track the atmosphere, e.g. as in PRIMA.

[2] is generally referred to as "envelope" tracking or coherencing, while [3] is often called "phase" tracking.

Sky coverage for off-axis phase tracking

- Practical issues:
  - Off-axis wavefront perturbations become uncorrelated as field angle increases and λ decreases.
  - With 1′ field-of-view <1% of sky has a suitably bright reference source (H<12).
  - Metrology is non-trivial.
  - Laser guide stars are not suitable reference sources.



Off-axis reduction in mean visibility for the VLTI site as a function of D and λ.

Good “phase-like” observables

- In the absence of phase tracking systems one can exploit quantities related to the visibility phase that are robust to atmospheric fluctuations.

- These are self-referenced methods - i.e. they use simultaneous measurements of the source itself:
  - Reference the phase to that measured at a different wavelength - differential phase:
    - You make measurements at two wavelengths simultaneously.
    - Depends upon knowing the source structure at some wavelength.
    - Need to know atmospheric path and dispersion.
  - Reference the phase to those on different baselines - closure phase:
    - Independent of source morphology.
    - Need to measure many baselines at once.

Closure phases

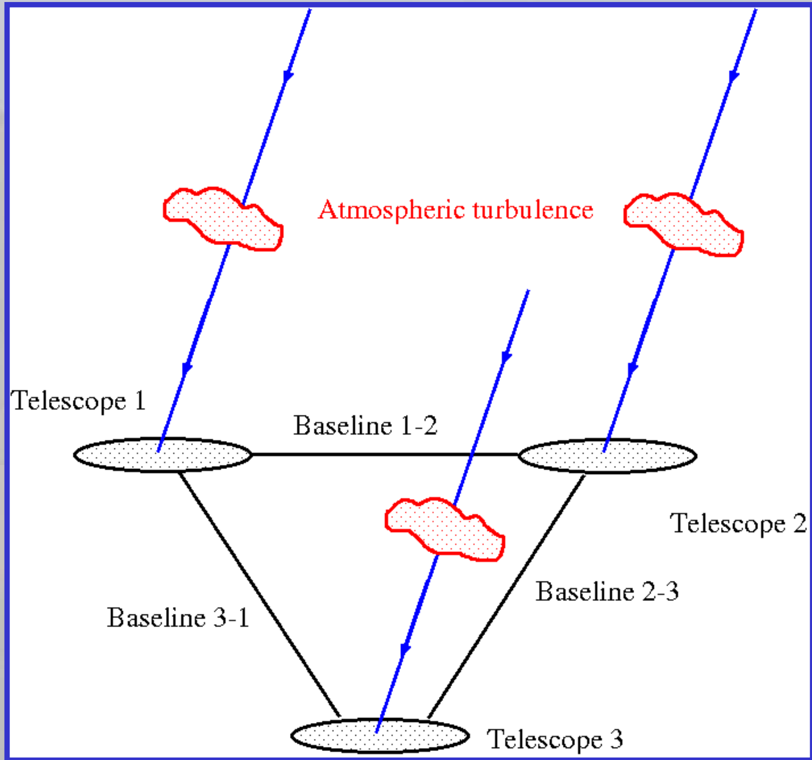
- Measure visibility phase (Φ) on three baselines simultaneously.

- Each is perturbed from the true phase (ϕ) in a particular manner:

$$\Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2$$
$$\Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3$$
$$\Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1$$

- Construct the linear combination of these:

$$\Phi_{12}+\Phi_{23}+\Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$



The error terms are antenna dependent – they vanish in the sum.
The source information is baseline dependent – it remains.
We still have to figure out how to use it!

How can we use these “good” observables?

- Average them (properly) over many realizations of the atmosphere.
- Differential phase, if we are comparing with the phase at a wavelength at which the source is unresolved, is a direct proxy for the Fourier phase we need.
  - Can then Fourier invert straightforwardly.
- Closure phase is a peculiar linear combination of the true Fourier phases:
  - In fact, it is the argument of the product of the visibilities on the baselines in question, hence the name triple product (or bispectrum).

$$V_{12}V_{23}V_{31} = |V_{12}|\,|V_{23}|\,|V_{31}|\exp(I[\Phi_{12} + \Phi_{23} + \Phi_{31}]) = T_{123}$$

  - So we have to use the closure phases as additional constraints in some nonlinear iterative inversion scheme.

Recap & questions

Temporal fluctuations in the atmosphere lead to a reduction in the mean fringe contrast from the intrinsic source-dependent value if the exposure time is too long.

This cannot easily be calibrated, so we try to avoid it.

More importantly, temporal fluctuations change the measured fringes phases in a random manner.

One solution is to monitor (and possibly correct) the atmospheric perturbations in real time.

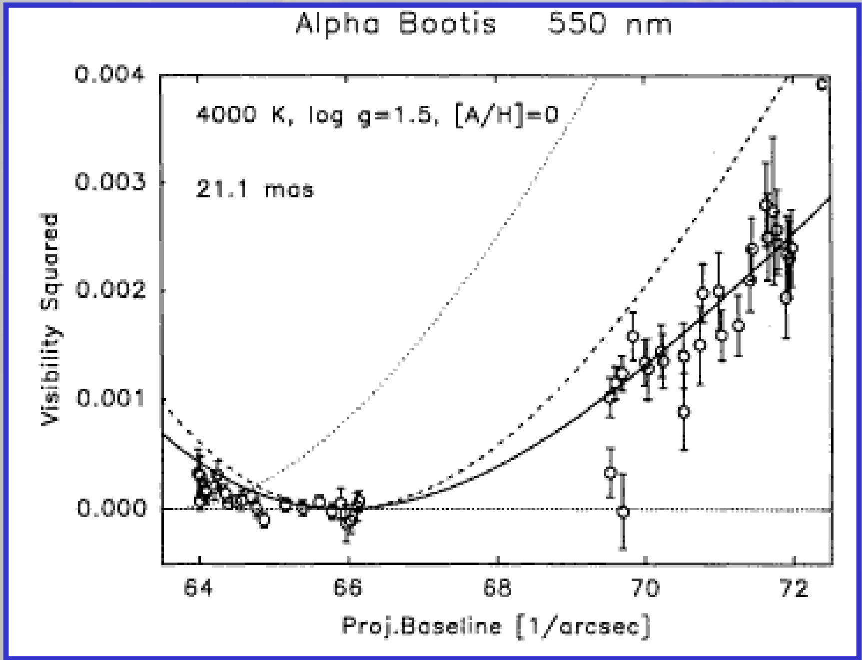
A second strategy is to try to measure combinations of phases that are immune to these perturbations.

We need to calibrate the impact of the atmosphere

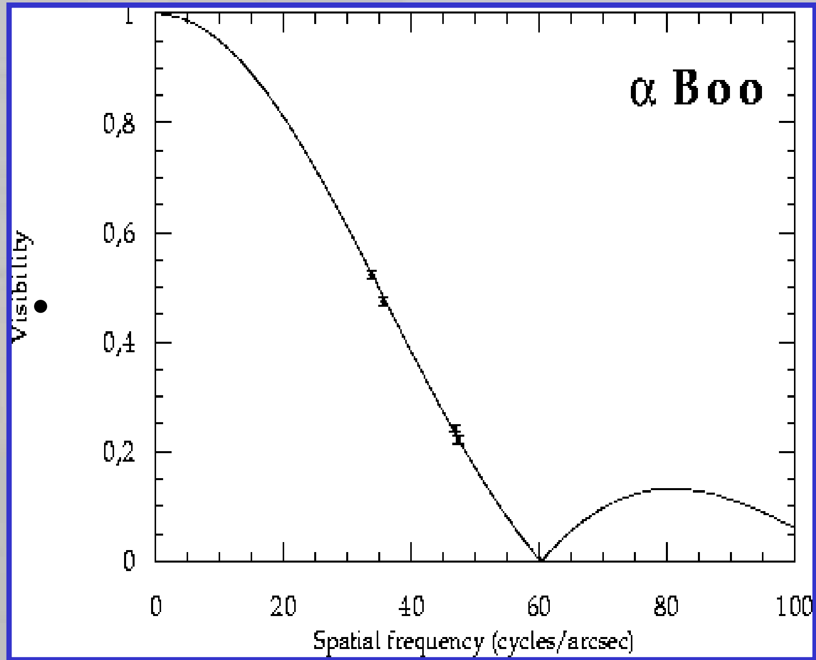
- The basic observables we wish to estimate are fringe amplitudes and phases.
- In practice the reliability of these measurements is generally limited by systematic errors, not the S/N of the fringes.
- There is a crucial need to calibrate the interferometric response:
  - Measurements of sources with known amplitudes and phases:
    - Unresolved targets close in time and space to the source of interest.
  - – Careful design of instruments:
    - Spatial filtering.
  - – Measurement of quantities that are less easily modified by systematic errors:
    - Phase-type quantities.

Examples of real data

- Measurements with the NPOI



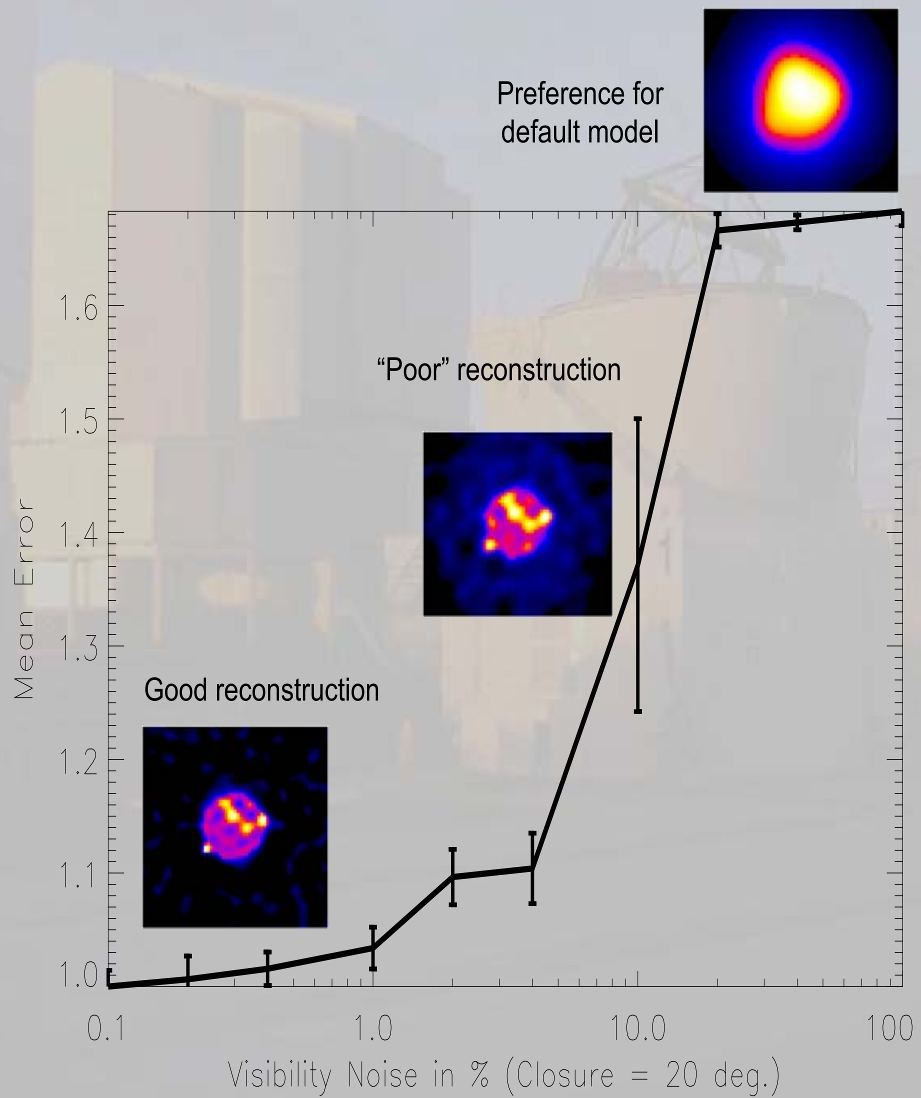
- Measurements with FLUOR



Perrin et al, AA, 331 (1998)

Notice how small these error bars are: compare these to what you get with AMBER and MIDI today.

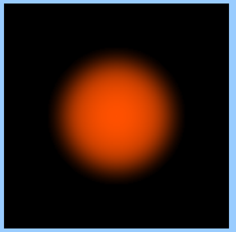
Reconstructions from different quality "fake" data



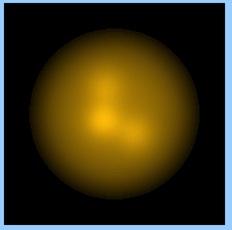
Interestingly, even when the data seem quite poor, you can still learn a lot.

So you need to know what is required for the science.

Final thoughts to take away



- Focus on the Fourier decomposition of a source
- Interferometric measurements:
  - All interferometer make some kind of fringes
  - The fringe contrast and location are the important measurable quantities
  - These capture the complex FT of the sky brightness distribution
  - A given interferometer baseline measures a single Fourier component



- Science with interferometers:
  - Multiple baselines are obligatory for reliable studies
  - Resolved targets give fringes of low contrast – these are difficult to measure well
  - Reliable interpretation can take multiple forms – making an image is only required if the source is complex.

- Impact of the atmosphere:
  - Limits sensitivity dramatically
  - Reduces fringe contrast and alters fringe phase: calibration required
  - Straightforward Fourier inversion becomes difficult