Testing stellar models against binary stars: a Bayesian approach. Application to the PISA pre-MS models

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Introduction

The mass of a star determines its evolutionary timescales and its observable properties. Stellar masses can be directly measured by the influence of the star on the kinematics of a companion or a circumstellar disk. Combining radial velocities with light curves of eclipsing systems, it is possible to derive mass and radius for both components. If a spectroscopic binary is resolved and an astrometric solution of the orbit is possible, then individual masses can be derived also in this case. By analysis of the stellar types it is possible to derive temperatures and hence luminosities. These determinations are generally more uncertain because they rely on some Spectral Type - to - T_{eff} conversion.

The number of PMS stars for which masses, radii, temperatures and luminosities are well known is quite small (~20), but is going to increase rapidly due to technological developments, specially in interferometry. We are now at a point at which we can start to directly test evolutionary models for the early stages of stellar lives, soon after the end of the main accretion phase.

The Bayesian approach

Stellar observables are compared with theoretical predictions. Following Jørgensen and Lindegren (2005), we find our best-masses and best-ages estimates by first defining the posterior probability:

$$f(\tau,\zeta,m|\mathbf{q}) \propto f_0(\tau,\zeta,m)L(\mathbf{q}|\tau,\zeta,m)$$

where:

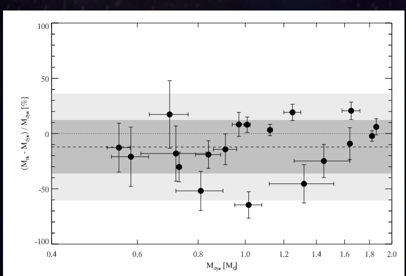
$$L(\mathbf{q}|\tau,\zeta,m) = \left(\prod_{i=1}^{n}\frac{1}{(2\pi)^{1/2}\sigma_i}\right)\times\exp(-\chi^2/2)$$

and

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{q_i^{obs}-q_i(\tau,\zeta,m)}{\sigma_i}\right)^2$$

Here (τ, ζ, m) are the model parameters, i.e. age, metallicity and mass. The q_i^{obs} are the observable quantities (any of M, R, Log L, Log T_{eff}) and the $q_i(\tau, \zeta, m)$ are the predictions of the models for those quantities; our method allows comparison of model and data in the traditional HRD, but also in the Mass-Radius diagram, where observables are more robust and suffer less from systematic errors (see e.g. Lastennet & Valls-Gabaud, 2002 or Mathieu et al., 2007).

Integration of f (τ, ζ, m | q) with respect to τ and ζ gives a function of the mass only, H(m). Maximizing the marginal distribution, H(m), we estimate the most probable mass given the observables q; a similar scheme leads to the best age estimate. We could not include covariance terms in the χ^2, since they are not available in the literature. The additional information on the dynamical mass can be used to set a prior f_0 to reduce the available space of model parameters. Likewise we can set (or not) a prior to impose the coevality of stars in the same system.



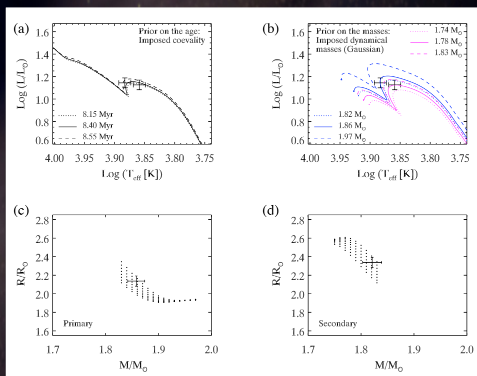
Relative differences between masses predicted using the PISA-PMS database of evolutionary tracks (M_{th}) and dynamical masses (M_{dyn}).
The dashed line represents the average difference, the shaded areas are the 1σ and 2σ scatter around the mean.
Data from Mathieu et al. (2007), see also references therein.

The models

We used stellar tracks calculated with the version of the FRANEC code recently updated in PISA (see Tognelli et al., 2010). The main physical inputs of the code are: OPAL EOS, OPAL opacities for log T > 4.5, Ferguson et al. (2005) opacities for lower T, all calculated for the solar mixture by Asplund et al. (2005). Boundary conditions are obtained interpolating detailed atmosphere models tables in T_{eff}, g and Z (Castelli & Kurucz, 2003 and Brott & Hauschildt, 2005). Convection is treated within the MLT framework, adopting our Sun-calibrated α = 1.68. We calculated models for 12 values of Z ranging from 0.007 to 0.03. Mass and age grids are extremely fine, with 281 mass values between 0.2 and 3.0 $M_\odot$, in steps of 0.01 $M_\odot$ and 1991 age values in the 0.5 - 100 Myr range in 0.05 Myr steps.

The PMS binary RS Cha.

(a): Most probable and +/- 1σ age isochrones, if coevality of the two components is imposed;
(b): Most probable and +/- 1σ masses tracks using Gaussian priors on the dynamical masses;
(c),(d): Masses and Radii predicted by the same models that give the best-fit in the HR diagram (see text).



RS Cha as a test case

We illustrate the application of our technique to the PMS binary RS Cha. In the case of the above figure we restrict ourselves to models calculated for Z = 0.0225, which give the best agreement with the data. This metallicity is at the upper extreme of the measurement range provided by Alecian et al. (2005). We impose priors on both the ages (coevality between the components) and the masses (Gaussian with μ and σ given by the dynamical masses and their errors). After maximizing the marginal probability H(m) and its analogous G(τ), we also derive 68% confidence intervals for the most probable masses and age. The isochrones and tracks of panels (a) and (b) correspond to the most probable value and to the extreme values of the 68% confidence interval.

A good check of the reliability of the result is shown in the (c) and (d) panels. Here we take ages and masses within the confidence intervals calculated from the marginal probability which was derived using q_i = (Log T_{eff} , Log $L/L_\odot$). For those values we plot the corresponding Masses and Radii predicted by the models and compare them to the actual masses and radii of the RS Cha stars.

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