THE CCD RIDDLE REVISTED:
SIGNAL VERSUS TIME – LINEAR
SIGNAL VERSUS VARIANCE – NON-LINEAR

Mark Downing[1], Peter Sinclaire [1].

[1]*ESO, Karl Schwartzschild Strasse-2, 85748 Munich, Germany.*

ABSTRACT

The photon transfer curve is one of the most valuable tools for calibrating, characterizing, and optimizing the performance of CCDs and CMOS Imagers. Its primary purpose is to determine the conversion gain of the camera system from which many of the other performance parameters such as read noise, dark current, QE, full well etc. are determined.

Non linearity in the photon transfer curve of back-illuminated CCDs has been reported by the authors in previous papers and confirmed by others even though exhibiting excellent signal linearity. Previous studies have isolated the source of the non linearity to the CCD image area. Spatial autocorrelation analysis showed that the mechanism behind the non linearity was due to a linear change in the way charge is shared between pixels in the image area with signal level. This paper reports on further investigations carried out to explain the phenomenon and describe the mechanism behind the non-linearity.

1. INTRODUCTION

The photon transfer curve (PTC) is one of the most widely used techniques to determine the end-to-end conversion gain (e/ADU) of a camera system. As it is simple to use (only requires the taking of progressive increasing time series of two flats at constant illumination) and does not require complicated or specialized equipment, it is widely used at the telescope to check the health of Charge Coupled Devices (CCDs) and Complementary Metal-Oxide-Semiconductor (CMOS) imagers and their camera system. Most other parameters such as read noise, dark current, quantum efficiency (QE), and full well are determined using this conversion gain.

The method relies on photon events being detected in a statistically independent way by the imager such that the characteristic shot noise of the light source is maintained and thus for a linear (conversion of photons to ADU) system, the mean signal (S) versus variance is related by a constant system conversion gain as follows:

$$K(e-/ADU)=\frac{S(ADU)}{Var(ADU)^2}$$

Previous studies[1][2] by the authors have shown this is not the case in reality and non-linearity in excess of 20% have been observed in backside illuminated CCDs even though having excellent signal linearity. In addition, the non-linearity

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[1] mdowning@eso.org

is greater for thicker devices made from higher resistivity silicon[2]. An investigation to locate the cause of the non-linearity concluded that it was due to the amount of charge collected within a pixel and not due to lateral diffusion of charge in the undepleted region at the back of the imager, the clocking or transport of charge in the image area or serial register to the read out, the output amplifier, or the detector electronics.

Spatial autocorrelation analysis showed that the mechanism behind the non-linearity was due to correlation, a sharing of charge, between pixels and this increases linearly with signal level.

This paper continues the investigation by presenting first a mechanism to explain the phenomena followed by further evidence to support the hypothesis that the effect is due to a change of "sharing of charge" between neighboring pixels as charge build up in the pixels.

Note the investigation to date has been carried out for simplicity (to be able to better isolate the feature at hand) on optical CCDs which have highly linear electron to output conversion. However, all backside illuminated Imagers, whether CCD or CMOS, optical or Infrared, could suffer from this feature if some or all of the volume of the pixel is defined by electric fields and not hard barriers. For the case where the imager has a non-linear signal response, an additional technique [3][4] to first linearize the data must be performed.

2. CHANGE IN "CHARGE SHARING" MECHANISM

The apparent non-linearity observed in the PTC is not due to just the "sharing of charge" between pixels, but to the change in this sharing with signal level. Figure 2-1 contain cross-sections of a backside illuminated CCD with three side by side pixels. This could equally have been a backside illuminated CMOS Imager. In Figure 2-1 (a), the pixels have not yet collected any charge and the electric field from each pixel extends evenly towards the backside to collect charge. When an electron is generated, it will drift towards the potential wells under the influence of the electric fields and be collected. The trapezoid represents the charge cloud as the electron diffuses towards the front-side of the device. In Figure 2-1 (b), the middle pixels has collected much more charge than its two neighbors and its electric field does not extend as far so when an electron is generated, it will drift towards the potential wells with a much broader charge cloud. When compared to the no charge situation, the probability of charge being collected will be less for the middle pixel than its two neighbors.

Results[5] of monte-carlo simulations that use a model whose pixel's collecting capability varies linearly with the number of electrons in a pixel show the same type of characteristic non-linearity in the photon transfer curve. While providing good understanding of and supporting the above hypothesis, the model used was a macro model and not based on physical parameters.

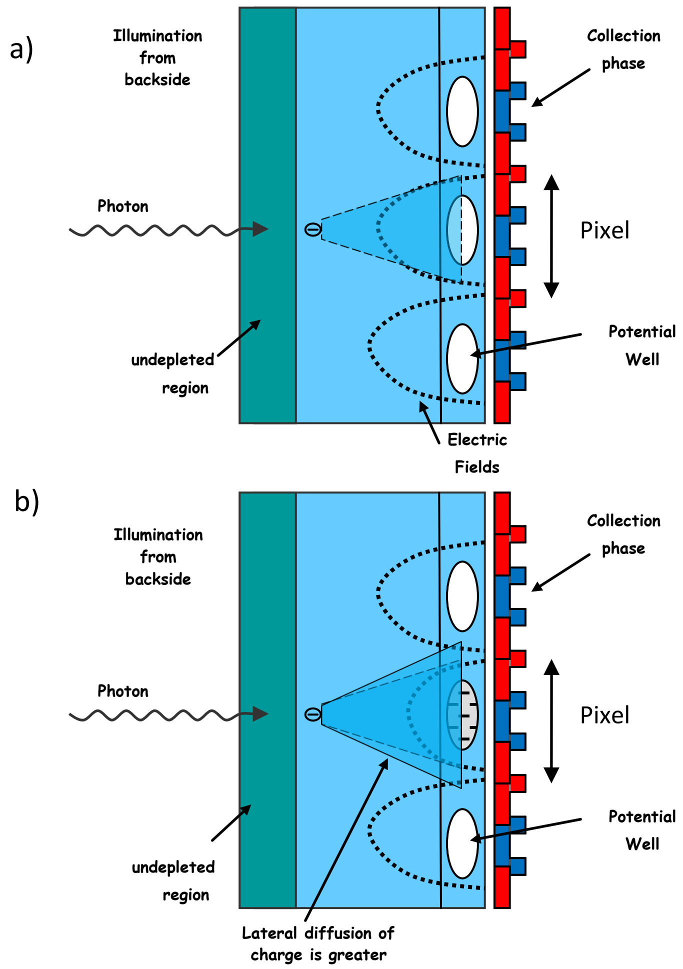


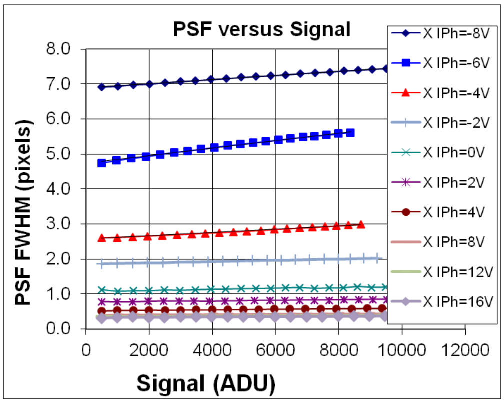
Figure 2-1 : Cross-sections of backside illuminated CCD with three side by side pixels. In a), the pixels have no or even charge and the electric field from each pixel evenly extends towards the backside to collect charge. b) The middle pixel has collected much more charge than its neighbors and its electric fields do not extend as far as the others and as such the laterals diffusion charge is thus much greater; represented by the broader trapezoid.

3. Point Spread Function (PSF)

If charge is shared between pixels and the sharing process increases linearly with signal then one would also expect the PSF (measure of a device's spatial performance) of the device to vary with signal level. In addition, if the effect is due to a reduction in the electric field as charge builds up in the potential well of the pixel, it would also be instructive to investigate what happens as the gate voltage of the pixel, the collection phase voltage, is varied.

The PSF was measured using a standard technique[2] at ESO of projecting a narrow width (2µm) slit onto an e2v[2] Deep Depletion CCD220[6] at a small tilt angle to the grid of pixels and taking images while increasing the illumination intensity.

a)



b)

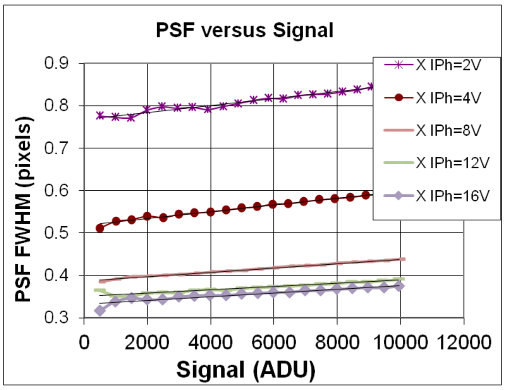


Figure 3-1 : a) Results of measurements of the PSF (in pixels) of a Deep Depletion CCD220 in the row direction versus signal (ADU) as the collection phase voltages (IPh) is varied from -8V to 16V. b) Expansion of range of 2V to 16. The conversion gain is 16e-/ADU in all graphs. The features to note are: i) the PSF varies linearly with signal level and ii) the variation becomes less as the collection phase voltage is increased.

The results (Figure 3-1) clearly show the PSF increasing linearly with signal level and this change with signal level becomes less as the collection phase voltage is increased. The CCD220 was operated during integration with both image phases set to the same voltage, the collection phase voltage. This is possible as the two phase design of the image area of the CCD220 has implants within the pixel to contain the charge. The results in Figure 3-1 are for PSF in the row ("X" designator) direction only. The results in the column direction are almost identical. Increasing the collection phase voltage increases the strength of the electric field in the depleted region and the drift velocity of the electrons. With a higher drift velocity, electrons have less time to 'wander' and end up in the neighboring pixels.

[2] http://www.e2v.com/

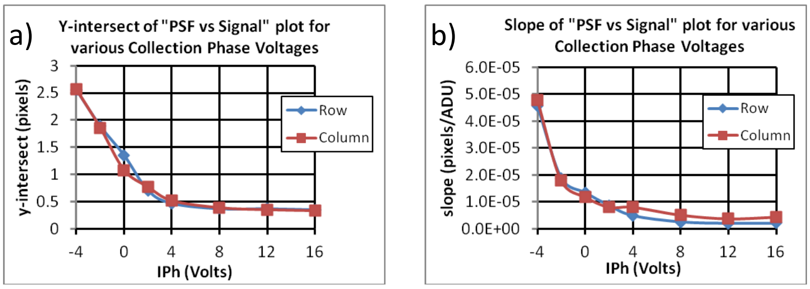


Figure 3-2 : Plot of the slope and y-intersect of the linear fit of the PSF versus signal graphs of Figure 3-1b versus collection phase voltage. Note that the PSF measured in both rows and column directions are similar and there is good correlation between the change of slope and the y-intersect.

The plots (Figure 3-2) of the slope and y-intersect of the linear fit of the PSF curves of Figure 3-1 versus collection phase voltage show a square root relationship shape in agreement with the equation[7] that describe the PSF for a fully depleted CCD,

$$PSF \approx \frac{x_{THICK}}{\sqrt{V_{IP}}} \times 2.2\sqrt{\frac{2kT}{q}}$$

where x_{THICK} is the thickness of the depleted region and V_{IP} is voltage across the device. Results are presented for measurements performed in both the row and column direction of the CCD. Both are very similar. The linear change of PSF with signal level indicates that the thickness (depth) of the depleted region is reduced as electrons build up in the pixel further validating the model.

4. Photon Transfer Curve (PTC)

To compare results, the PTC was determined under the same conditions as the PSF results presented in the previous section. Flat fields of increasing illumination were taken with the same CCD220 using the same collection phase voltages. The plot of signal linearity (Figure 4-1a) shows the device to be very linear at all collection phase voltages. However, as expected, the PTC (Figure 4-1b) is non-linear and the non-linearity decreases as the collection phase voltage is increased. The black trend line shows the expected results if the PTC was completely linear.

The slope, Figure 4-2 (a), of the PTC varies linearly with signal. Thus a 2nd order polynomial,

$$\mathrm{Var} = \mathrm{A} \times \mathrm{S}^2 + \mathrm{B} \times \mathrm{S} + \mathrm{C}$$

must be a good fit to the PTC as demonstrated in Figure 4-2 (b). From previous work[1], it was suggested that the y-intersect of the slope of the PTC was a better estimate of the system gain , K e-/ADU, thus the B term in the polynomial fit can be considered a kind of “gain” term (K = 1/B e-/ADU), the A term a measure of the non-linearity, and √C the read noise.

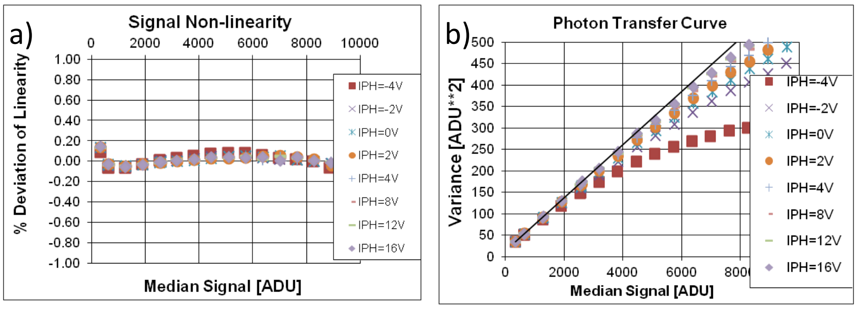


Figure 4-1 : a) Plot of signal non-linearity as the collection phase voltage (IPH) is varied; the device is highly linear. b) Plot of the PTC (signal versus variance) as the collection phase voltage (IPH) is varied. The black line is a linear fit to the low level data. Clearly the non-linearity of the PTC increases as the collection phase voltage is reduced.

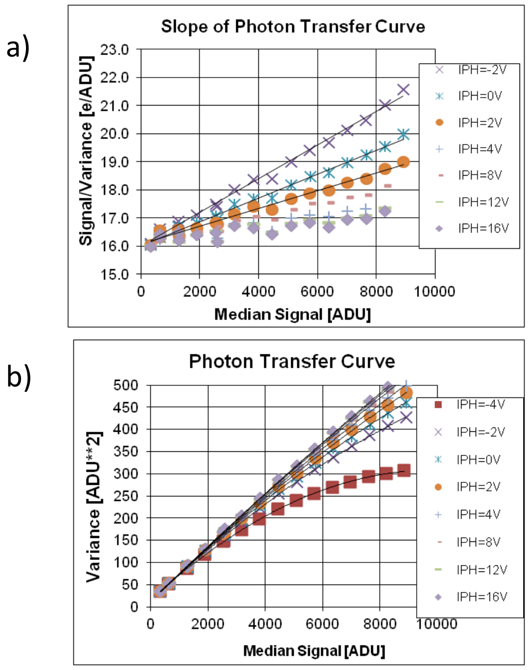


Figure 4-2 : a) Plot of the slopes of the PTC for various collection phase voltages clearly showing the slope varies linearly with signal. b) Plots showing that a second order polynomial is a good fit to the PTC.

The plots (Figure 4-3) of the polynomial fit terms, A and 1/B, versus collection phase voltage show a square root relationship similar to that seen in the PSF results. This provides strong evidence that the same mechanism, increase in charge sharing with signal, is at play and a simple relationship (equation) exist between the non-linearity in the PTC and the change in PSF with signal. Knowing this relationship, one could determine one from the other.

When in operation, it is near impossible to accurately measure the PSF of a detector as this will normally be dominated by the optics of the system. Even during characterization of the detector, this is a difficult and time consuming exercise even if one has the right equipment available. The non-linearity of the PTC is on the other hand much easy to measure (only requires a series of flat fields). For high precision applications such as ultra stable spectrometry and high resolution photometry, any change in the PSF of the detector introduces errors thus a simple method to determine the change in PSF would be welcomed.

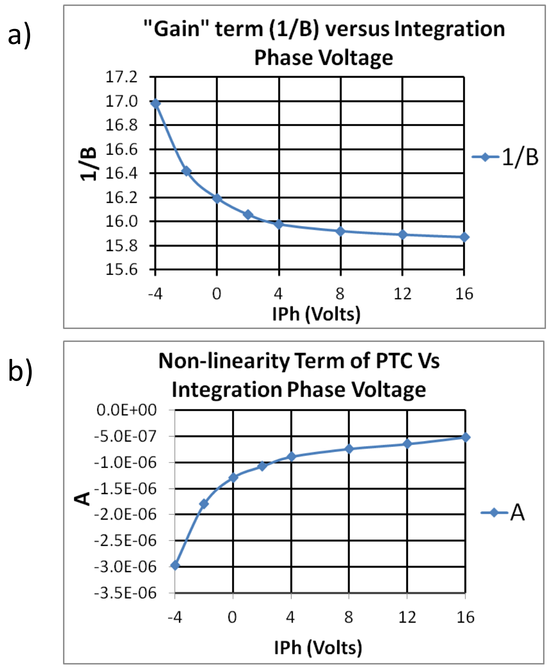


Figure 4-3 : Plot of the polynomial fit terms, a) A and b) "gain" 1/B, of the PTC of Figure 4-2b versus collection phase voltage (IPh). Note that both terms, A and 1/B, show a square root relationship with collection phase voltage.

5. AUTOCORRELATION ANALYSIS

Autocorrelation analysis was performed on the same set of image data as used for the PTC analysis in the previous section. Results (Figure 5-1) shows correlation between pixels as expected and that this correlation increases linearly with signal. Comparing Figure 5-1 which plots % correlation between adjacent pixels in the column direction to the slope of the PTC in Figure 4-2a, one notes the similarity. Note the % correlation values shown in Figure 5-1 need to be multiplied by 4 to include all adjacent pixels.

Conclusive evidence that the correlation between pixels is the major factor in the non-linearity of the PTC is obtained when one uses the autocorrelation variance as recommended in [1] in the PTC instead of the normal variance. The autocorrelation variance, Var_{AC} , can be calculated from the difference of two images over an area of M x N pixels as follows:

$$Var_{AC} = \frac{\sum_{m=-P,n=-R}^{m=+P,n=+R} \sum_{i=1,j=1}^{i=M-m,j=N-n} V_{i,j} V_{i+m,j+n}}{(M.N-1)}$$

where the interaction between pixels is taken into account of and is contained within pixel distance of P x R. A corrected conversion gain, K_{AC}, can then be calculated as follows:

$$K_{AC} = \frac{\overline{S}}{Var_{AC}}$$

where $\overline{S}$ is the mean unbiased signal. The autocorrelation variance takes into account the cross terms between pixels and thus the correlation; whereas the normal variance assumes no correlation and is a simple sum of the difference of pixel values squared. The plots (Figure 5-2a) of the autocorrelation PTC is very linear and is very similar for all collection phase voltages. The slope (Figure 5-2a) is flat and the gain can be easily calculate; in this case to be 16e-/ADU.

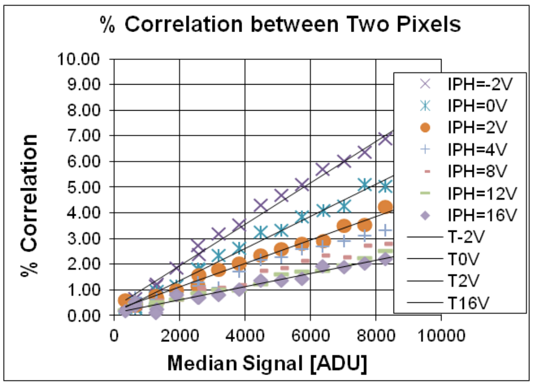
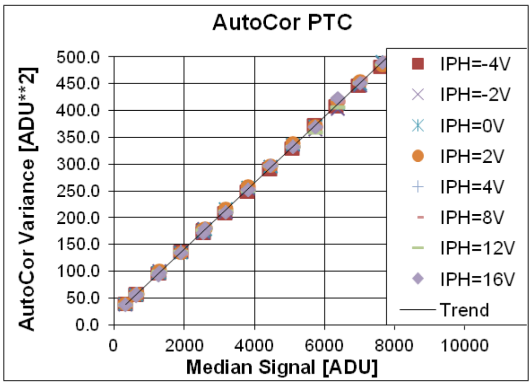


Figure 5-1 : Plot showing % correlation between adjacent pixels in a column. The black trend lines are linear fits to the data. The % correlation needs to be multiplied by 4 to include all adjacent pixels to obtain the total correlation.

a)



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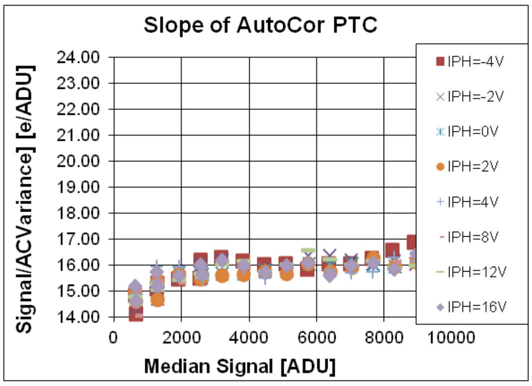

Figure 5-2 : a) Plots of the PTC using the auto covariance instead of the simple variance. The black line is a linear fit of the data. b) Plot of the slope of the PTC for the various collection phase voltages.

6. Conclusion

This paper provided strong evidence that the mechanism behind the non-linearity of the PTC is the reduction in the extent of the electric field as charge builds up in the pixel reducing the pixel's collection competitiveness with respect to its neighbors. Measured linear change of PSF with signal level and 1/√ with collection phase voltage suggests the effect is due to a reduction in the pixel's depletion depth with signal. Results of autocorrelation analysis and noting similar variation with signal and collection phase voltage provides enough evidence to conclude that the same mechanism is behind the change in PSF and the non-linearity of PTC.

7. Future Work

The next step is to accurately determine the relationship between the linear change of PSF and the amount of correlation between pixels so that by taking two Flat Fields, the correlation (using Autocorrelation analysis) can be determined, and from this the change in PSF with signal. Corrections for the change in PSF with signal can then be applied where high accuracy is required.

Of course the ultimate would be to fully understand the physics behind the mechanism and propose changes to the designers of the Imagers.

8. References

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